

STUDIES ON GENERALIZED NONEXPANSIVE MAPS

JONG SOOK BAE

The purpose of this thesis is to study fixed points of generalized nonexpansive maps. A generalized nonexpansive map is a natural generalization of a nonexpansive map and a contractive type map. Recently the subject has grown out of works on metric geometry and nonlinear analysis.

In chapter I, we state some historical remarks on nonexpansive and generalized nonexpansive maps.

In chapter II, we prove several fixed point theorems for generalized nonexpansive maps. In particular, §1 is devoted to classify generalized nonexpansive maps and to obtain fixed point theorems in each case without assuming normal structure. In §2, we obtain a fixed point theorem for generalized nonexpansive selfmaps of a weakly compact convex subset of a Banach space having asymptotic normal structure, and prove new fixed point theorems for some variations of generalized nonexpansive maps, and common fixed point theorems for a commuting family of generalized nonexpansive maps. In §3, we prove that any Banach space with uniformly normal structure is reflexive.

In chapter III, we study nonexpansive and generalized nonexpansive multivalued maps, and obtain some iteration schemes to find fixed points. In particular, in §1 we show that Ishikawa's method can be applied to multivalued nonexpansive maps to obtain an asymptotically regular iteration, and in §2, we adopt the idea of Senter and Dotson, and obtain an iteration scheme to find fixed points of generalized nonexpansive multivalued maps.

Chungnam National University
Daejeon 300-31, Korea

This thesis submitted to Seoul National University, December 1983. Degree approved February 1984. Supervisor: Professor Sehie Park.