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A CLASS OF FUNCTIONS α-PRESTARLIKE OF ORDER β

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1. Introduction

Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n \tag{1.1}$$

which are analytic in the unit disk $\mathcal{U}=\{z : |z|<1\}$.
And let $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of analytic and univalent functions $f(z)$ in the unit disk $\mathcal{U}$. A function $f(z)$ of $\mathcal{S}$ is said to be starlike of order α if and only if

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha \qquad (z\in\mathcal{U}) \tag{1.2}$$

for some $\alpha(0\leqq\alpha<1)$. We denote the class of all starlike functions of order α by $\mathcal{S}^*(\alpha)$. Further a function $f(z)$ of $\mathcal{S}$ is said to be convex of order α if and only if

$$\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha \qquad (z\in\mathcal{U}) \tag{1.3}$$

for some α $(0\leqq\alpha<1)$. And we denote the class of all convex functions of order α by $\mathcal{K}(\alpha)$. It is well-known that $f(z)\in\mathcal{K}(\alpha)$ if and only if $zf'(z)\in\mathcal{S}^*(\alpha)$. Note that $\mathcal{S}^*(0)\equiv\mathcal{S}^*$ and $\mathcal{K}(0)\equiv\mathcal{K}$ for $\alpha=0$.

These classes $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ were first introduced by Robertson [3], and latter were studied by Schild [5], MacGregor [1] and Pinchuk [2].

Now, the function

$$S_\alpha(z)=\frac{z}{(1-z)^{2(1-\alpha)}} \tag{1.4}$$

is the well-known extremal function for $\mathcal{S}^*(\alpha)$. Setting

$$C(\alpha,n)=\frac{\prod_{k=2}^{n}(k-2\alpha)}{(n-1)!} \qquad (n=2,3,4,\ldots), \tag{1.5}$$

$S_\alpha(z)$ can be written in the form

$$S_\alpha(z)=z+\sum_{n=2}^{\infty}C(\alpha,n)z^n. \tag{1.6}$$

Then we can see that $C(\alpha,n)$ is decreasing in α and satisfies

$$\lim_{n\to\infty} C(\alpha, n) = \begin{cases} \infty & (\alpha<1/2) \\ 1 & (\alpha=1/2) \\ 0 & (\alpha>1/2). \end{cases} \tag{1.7}$$

Let $f*g(z)$ denote the convolution or Hadamard product of two functions $f(z)$ and $g(z)$, that is, if $f(z)$ is given by (1.1) and $g(z)$ is given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \tag{1.8}$$

then

$$f*g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n. \tag{1.9}$$

Let $\mathcal{R}(\alpha, \beta)$ be the subclass of $\mathcal{A}$ consisting of functions $f(z)$ such that $f*S_\alpha(z) \in \mathcal{S}^*(\beta)$ for $0 \leqq \alpha < 1$ and $0 \leqq \beta < 1$. Further let $\mathcal{C}(\alpha, \beta)$ be the subclass of $\mathcal{A}$ consisting of functions $f(z)$ satisfying $zf'(z) \in \mathcal{R}(\alpha, \beta)$ for $0 \leqq \alpha < 1$ and $0 \leqq \beta < 1$. $\mathcal{R}(\alpha, \beta)$ is called to be the class of functions α-prestarlike of order β and was introduced by Sheil-Small, Silverman and Silvia [6].

Let $\mathcal{T}$ denote the subclass of $\mathcal{A}$ consisting of functions whose nonzero coefficients, from the second on, are negative. That is, an analyic function $f(z)$ is in the class $\mathcal{T}$ if it can be expressed as

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \qquad (a_n \geqq 0). \tag{1.10}$$

Further we denote by $\tilde{\mathcal{R}}(\alpha, \beta)$ and $\tilde{\mathcal{C}}(\alpha, \beta)$ the classes obtained by taking intersections, respectively, of the classes $\mathcal{R}(\alpha, \beta)$ and $\mathcal{C}(\alpha, \beta)$ with $\mathcal{T}$.

The class $\tilde{\mathcal{R}}(\alpha, \beta)$ was recently studied by Silverman and Silvia [7]. In this paper, we study the class $\tilde{\mathcal{C}}(\alpha, \beta)$ by using the results for $\tilde{\mathcal{R}}(\alpha, \beta)$ given by Silverman and Silvia [7].

2. Coefficient inequalities

We need the following result by Silverman and Silvia [7].

LEMMA. *Let the function $f(z)$ be defined by* (1.10). *Then $f(z)$ is in the class $\tilde{\mathcal{R}}(\alpha, \beta)$ if and only if*

$$\sum_{n=2}^{\infty} (n-\beta) C(\alpha, n) a_n \leqq 1-\beta. \tag{2.1}$$

The result is sharp.

THEOREM 1. *Let the function $f(z)$ be defined by* (1.10). *Then $f(z)$ is in the class $\tilde{\mathcal{C}}(\alpha, \beta)$ if and only if*

$$\sum_{n=2}^{\infty} n(n-\beta) C(\alpha, n) a_n \leqq 1-\beta. \tag{2.2}$$

The result is sharp.

Proof. Since $f(z) \in \tilde{\mathcal{C}}(\alpha, \beta)$ if and only if $zf'(z) \in \tilde{\mathcal{R}}(\alpha, \beta)$, we have the theorem by replacing a_n with na_n in Lemma. Further we can see that the function $f(z)$ given by

$$f(z)=z-\frac{1-\beta}{n(n-\beta)C(\alpha,n)}z^n \qquad (n\geqq 2) \tag{2.3}$$

is an extremal function for the theorem.

COROLLARY 1. *Let the function $f(z)$ defined by* (1.10) *be in the class* $\tilde{\mathscr{C}}(\alpha,\beta)$. *Then*

$$a_n\leqq\frac{1-\beta}{n(n-\beta)C(\alpha,n)} \tag{2.4}$$

for $n\geqq 2$. Equality holds for function $f(z)$ given by (2.3).

REMARK. In view of Lemma and Theorem 1, we know that $\tilde{\mathscr{C}}(\alpha,\beta)\subset\tilde{\mathscr{R}}(\alpha,\beta)$.

THEOREM 2. *Let*

$$f_1(z)=z \tag{2.5}$$

and

$$f_n(z)=z-\frac{1-\beta}{n(n-\beta)C(\alpha,n)}z^n \qquad (n\geqq 2). \tag{2.6}$$

Then $f(z)$ is in the class $\tilde{\mathscr{C}}(\alpha,\beta)$ if and only if it can be expressed in the form

$$f(z)=\sum_{n=1}^{\infty}\lambda_n f_n(z), \tag{2.7}$$

where $\lambda_n\geqq 0$ for $n\in N=\{1,2,3\ldots\}$ and

$$\sum_{n=1}^{\infty}\lambda_n=1. \tag{2.8}$$

Proof. Assume that

$$\begin{aligned} f(z)&=\sum_{n=1}^{\infty}\lambda_n f_n(z)\\ &=z-\sum_{n=2}^{\infty}\frac{1-\beta}{n(n-\beta)C(\alpha,n)}\lambda_n z^n\\ &=z-\sum_{n=2}^{\infty}a_n z^n, \end{aligned} \tag{2.9}$$

where

$$a_n=\frac{1-\beta}{n(n-\beta)C(\alpha,n)}\lambda_n. \tag{2.10}$$

Then we observe that

$$\begin{aligned} \sum_{n=2}^{\infty}n(n-\beta)C(\alpha,n)a_n&=\sum_{n=2}^{\infty}(1-\beta)\lambda_n\\ &=(1-\beta)(1-\lambda_1)\\ &\leqq 1-\beta. \end{aligned} \tag{2.11}$$

This gives that $f(z)$ belongs to the class $\tilde{\mathscr{C}}(\alpha,\beta)$ by means of Theorem 1.

Conversely, assume that $f(z)$ is in the class $\tilde{\mathscr{C}}(\alpha,\beta)$ for $0\leqq\alpha<1$ and $0\leqq\beta<1$. Then we have

$$a_n\leqq\frac{1-\beta}{n(n-\beta)C(\alpha,n)}\qquad(n\geqq2)\tag{2.12}$$

by means of Theorem 1. Setting

$$\lambda_n=\frac{n(n-\beta)C(\alpha,n)}{1-\beta}a_n\qquad(n\geqq2)\tag{2.13}$$

and

$$\lambda_1=1-\sum_{n=2}^{\infty}\lambda_n,\tag{2.14}$$

we have the representation (2.7). This completes the proof of the theorem.

3. Distortion theorems

As a consequence of Theorem 2, we have the following distortion theorems for $f(z)$ belonging to $\tilde{\mathscr{C}}(\alpha,\beta)$.

THEOREM 3. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathscr{C}}(\alpha,\beta)$. Then*

$$|f(z)|\geqq\mathrm{Max}\left\{0,\ |z|-\frac{1-\beta}{4(1-\alpha)(2-\beta)}|z|^2\right\}\tag{3.1}$$

and

$$|f(z)|\leqq|z|+\frac{1-\beta}{4(1-\alpha)(2-\beta)}|z|^2\tag{3.2}$$

for $z\in\mathscr{U}$. The results are sharp.

Proof. By Theorem 2, we can know that

$$|f(z)|\geqq\mathrm{Max}\left\{0,\ |z|-\underset{n\in N-\{1\}}{\mathrm{Max}}\frac{1-\beta}{n(n-\beta)C(\alpha,n)}|z|^n\right\}\tag{3.3}$$

and

$$|f(z)|\leqq|z|+\underset{n\in N-\{1\}}{\mathrm{Max}}\frac{1-\beta}{n(n-\beta)C(\alpha,n)}|z|^n\tag{3.4}$$

for $z\in\mathscr{U}$. Let

$$G(\alpha,\beta,|z|,n)=\frac{1-\beta}{n(n-\beta)C(\alpha,n)}|z|^n.\tag{3.5}$$

Since

$$C(\alpha,n+1)=\frac{n+1-2\alpha}{n}C(\alpha,n)\tag{3.6}$$

for $|z|\neq0$ and $n\geqq2$, we can see that

$$G(\alpha,\beta,|z|,n)\geqq G(\alpha,\beta,|z|,n+1)\tag{3.7}$$

if and only if

(3.8) $H(\alpha,\beta,|z|,n)=(n+1)(n+1-\beta)(n+1-2\alpha)-n^2(n-\beta)|z|\geqq 0.$

It is easy that $H(\alpha,\beta,|z|,n)$ is a decreasing function of $\alpha(0\leqq\alpha<1)$ for fixed $\beta(0\leqq\beta<1)$, $n\geqq 2$ and $|z|<1$, $H(1,\beta,|z|,n)$ is a decreasing function of $|z|$ ($|z|<1$) for fixed $\beta(0\leqq\beta<1)$ and $n\geqq 2$, and $H(1,\beta,1,n)$ is an increasing function of $\beta(0\leqq\beta<1)$ for fixed $n\geqq 2$. Hence we show that

$$(3.9)\qquad \begin{aligned} H(\alpha,\beta,|z|,n) &\geqq H(1,\beta,|z|,n)\\ &\geqq H(1,\beta,1,n)\\ &\geqq H(1,0,1,n)=n^2-n-1>0 \end{aligned}$$

for $n\geqq 2$. Thus we can see that the function $G(\alpha,\beta,|z|,n)$ is decreasing in n ($n\geqq 2$), hence further,

$$(3.10)\qquad \begin{aligned} |f(z)| &\geqq \mathrm{Max}\left\{0,\ |z|-\frac{1-\beta}{2(2-\beta)C(\alpha,2)}|z|^2\right\}\\ &=\mathrm{Max}\left\{0,\ |z|-\frac{1-\beta}{4(1-\alpha)(2-\beta)}|z|^2\right\} \end{aligned}$$

and

$$(3.11)\qquad \begin{aligned} |f(z)| &\leqq |z|+\frac{1-\beta}{2(2-\beta)C(\alpha,2)}|z|^2\\ &=|z|+\frac{1-\beta}{4(1-\alpha)(2-\beta)}|z|^2 \end{aligned}$$

for $z\in\mathcal{U}$.

Finally, the bounds of the theorem are attained for function $f(z)$ given by

$$(3.12)\qquad f(z)=z-\frac{1-\beta}{4(1-\alpha)(2-\beta)}z^2.$$

COROLLARY 2. *Let the function $f(z)$ defined by (1.10) be in the class $\tilde{\mathcal{C}}(\alpha,\beta)$. Then $f(z)$ is included in a disk with its center at the origin and radius r given by*

$$(3.13)\qquad r=1+\frac{1-\beta}{4(1-\alpha)(2-\beta)}.$$

THEOREM 4. *Let the function $f(z)$ defined by (1.10) be in the class $\tilde{\mathcal{C}}(\alpha,\beta)$. Then*

$$(3.14)\qquad |f'(z)|\geqq \mathrm{Max}\left\{0,\ 1-\frac{1-\beta}{2(1-\alpha)(2-\beta)}|z|\right\}$$

and

$$(3.15)\qquad |f'(z)|\leqq 1+\frac{1-\beta}{2(1-\alpha)(2-\beta)}|z|$$

for $0\leqq\beta<1$, and either $0\leqq\alpha\leqq(5-\beta)/2(3-\beta)$ or $|z|\leqq(3-\beta)/2(2-\beta)$. The results are sharp.

Proof. We note that

(3.16) $$|f'(z)|\geqq \mathrm{Max}\left\{0,\ 1-\underset{n\in N-\{1\}}{\mathrm{Max}}\frac{1-\beta}{(n-\beta)C(\alpha,n)}|z|^{n-1}\right\}$$

and

(3.17) $$|f'(z)\leqq 1+\underset{n\in N-\{1\}}{\mathrm{Max}}\frac{1-\beta}{(n-\beta)C(\alpha,n)}|z|^{n-1}\}$$

by means of Theorem 2. It suffices to prove that

(3.18) $$G_1(\alpha,\beta,|z|,n)=\frac{1-\beta}{(n-\beta)C(\alpha,n)}|z|^{n-1}$$

is decreasing in n $(n\geqq 2)$. We can see that, for $|z|\neq 0$,

(3.19) $$G_1(\alpha,\beta,|z|,n)\geqq G_1(\alpha,\beta,|z|,n+1)$$

if and only if

(3.20) $$H_1(\alpha,\beta,|z|,n)=(n+1-\beta)(n+1-2\alpha)-n(n-\beta)|z|\geqq 0.$$

$H_1(\alpha,\beta,|z|,n)$ is decreasing in $\alpha(0\leqq\alpha\leqq(5-\beta)/2(3-\beta))$ for fixed $\beta(0\leqq\beta<1)$, $|z|(|z|<1)$ and $n\geqq 2$. Thus we obtain that

(3.21) $$\begin{aligned}H_1(\alpha,\beta,|z|,n)&\geqq H_1((5-\beta)/2(3-\beta),\beta,|z|,n)\\&=n(n-\beta)(1-|z|)+\frac{(1-\beta)(n-2)}{3-\beta}\\&\geqq 0\end{aligned}$$

for $0\leqq\beta<1$, $|z|<1$ and $n\geqq 2$.

Next, $H_1(\alpha,\beta,|z|,n)$ is decreasing in $|z|$ $(|z|<1)$ and increasing in n $(n\geqq 2)$. Hence we can see that

(3.22) $$\begin{aligned}H_1(\alpha,\beta,|z|,n)&\geqq H_1(1,\beta,|z|,n)\\&\geqq H_1(1,\beta,(3-\beta)/2(2-\beta),2)\\&=0.\end{aligned}$$

This gives two estimates (3.14) and (3.15) we require.

Finally the bounds of the theorem are attained for function $f(z)$ given by (3.12).

4. Radii of starlikeness and convexity

Since $f(z)$ defined by (1.10) is univalent in the unit disk $\mathcal{U}$ if $\sum_{n=2}^{\infty}na_n\leqq 1$, we can see that $f(z)$ defined by (1.10) belongs to the class $\mathcal{S}$ if $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$ with the aid of Theorem 1.

THEOREM 5. *Let the function $f(z)$ defined by* (1. 10) *be in the class $\tilde{\mathcal{C}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$. Then $f(z)$ is starlike of order $\delta(0\leqq\delta<1)$ in the disk $|z|<r_1$, where*

(4.1) $$r_1=\inf_{n\in N-\{1\}}\left\{\frac{n(n-\beta)(1-\delta)C(\alpha,n)}{(1-\beta)(n-\delta)}\right\}^{1/(n-1)}.$$

The result is sharp.

Proof. We employ the same technique as used by Sarangi and Uralegaddi [4]. Note that

(4.2)
$$\left|\frac{zf'(z)}{f(z)}-1\right|\leqq\frac{\sum_{n=2}^{\infty}(n-1)a_n|z|^{n-1}}{1-\sum_{n=2}^{\infty}a_n|z|^{n-1}}\leqq 1-\delta$$

if and only if

(4.3)
$$\sum_{n=2}^{\infty}\left(\frac{n-\delta}{1-\delta}\right)a_n|z|^{n-1}\leqq 1.$$

By virtue of Theorem 1, we need only find values of $|z|$ for which

(4.4)
$$\left(\frac{n-\delta}{1-\delta}\right)|z|^{n-1}\leqq\frac{n(n-\beta)C(\alpha,n)}{1-\beta}$$

for $n\geqq 2$, which will be true when $|z|\leqq r_1$. Further we can see that the result is sharp for function $f(z)$ given by

(4.5)
$$f(z)=z-\frac{1-\beta}{n(n-\beta)C(\alpha,n)}z^n \qquad (n\geqq 2).$$

This completes the proof of the theorem.

COROLLARY 3. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathscr{C}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$. Then $f(z)$ is univalent and starlike for $|z|<r_2$, where*

(4.6)
$$r_2=\inf_{n\in N-\{1\}}\left\{\frac{(n-\beta)C(\alpha,n)}{1-\beta}\right\}^{1/(n-1)}.$$

The result is sharp.

THEOREM 6. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathscr{C}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$. Then $f(z)$ is convex of order $\delta(0\leqq\delta<1)$ in the disk $|z|<r_3$,* where

(4.7)
$$r_3=\inf_{n\in N-\{1\}}\left\{\frac{(n-\beta)(1-\delta)C(\alpha,n)}{(1-\beta)(n-\delta)}\right\}^{1/(n-1)}.$$

The result is sharp.

Proof. Since $f(z)$ is convex of order δ if and only if $zf'(z)$ is starlike of order δ, we have the theorem by replacing a_n with na_n in Theorem 5. Further the result is sharp for function $f(z)$ given by

(4.8)
$$f(z)=z-\frac{1-\beta}{n^2(n-\beta)C(\alpha,n)}z^n \qquad (n\geqq 2).$$

COROLLARY 4. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathscr{C}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$. Then $f(z)$ is univalent and convex for $|z|<r_4$, where*

(4.9) $$r_4=\inf_{n\in N-\{1\}}\left\{\frac{(n-\beta)C(\alpha,n)}{n(1-\beta)}\right\}^{1/(n-1)}.$$

The result is sharp.

5. Modified Hadamard product

Let $f(z)$ be defined by (1.10) and $g(z)$ be defined by

(5.1) $$g(z)=z-\sum_{n=2}^{\infty}b_nz^n \qquad (b_n\geqq 0).$$

Then we denote by $f*g(z)$ the modified Hadamard product of $f(z)$ and $g(z)$, that is,

(5.2) $$f*g(z)=z-\sum_{n=2}^{\infty}a_nb_nz^n.$$

THEOREM 7. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathcal{R}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$. Then the modified Hadamard product $f*f(z)$ is also in the same class $\tilde{\mathcal{R}}(\alpha,\beta)$.*

Proof. In view of Lemma, we can see that

(5.3) $$\sum_{n=2}^{\infty}(n-\beta)C(\alpha,n)a_n{}^2\leqq\frac{(1-\beta)^2}{2(1-\alpha)(2-\beta)}$$
$$\leqq 1-\beta$$

for $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/2(2-\beta)$. This proves that $f*f(z)\in\tilde{\mathcal{R}}(\alpha,\beta)$

THEOREM 8. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathcal{C}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(7-3\beta)/4(2-\beta)$. Then the modified Hadamard product $f*f(z)$ is also in the same class $\tilde{\mathcal{C}}(\alpha,\beta)$.*

The proof of Theorem 8 is obtained by using the same technique as in the proof of Theorem 7 with the aid of Theorem 1.

THEOREM 9. *Let the function $f(z)$ defined by* (1.10) *be in the class $\tilde{\mathcal{R}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/4$. Further let the function $g(z)$ defined by* (5.1) *be in the class $\tilde{\mathcal{C}}(\alpha,\beta)$ with $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/4$. Then the modified Hadamard product $f*g(z)$ is in the class $\tilde{\mathcal{C}}(\alpha,\beta)$.*

Proof. By using Lemma, we have

(5.4) $$a_n\leqq\frac{1-\beta}{2(1-\alpha)(2-\beta)}$$

for $n\geqq 2$. Hence, with the aid of Theorem 1, we obtain that

(5.5) $$\sum_{n=2}^{\infty}n(n-\beta)C(\alpha,n)a_nb_n$$
$$\leqq\frac{1-\beta}{2(1-\alpha)(2-\beta)}\sum_{n=2}^{\infty}n(n-\beta)C(\alpha,n)b_n$$

$$\leqq \frac{(1-\beta)^2}{2(1-\alpha)(2-\beta)}$$
$$\leqq 1-\beta$$

for $0\leqq\beta<1$ and $0\leqq\alpha\leqq(3-\beta)/4$. Consequently we have the theorem.

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