PUBLICATIONS OF THE KOREAN ASTRONOMICAL SOCIETY
1:9~12, 1984

Some Physical Parameters of Globular Clusters

II. Dynamical Masses of Six Globular Clusters

SUH, YOUNG-RAN AND CHUN, MUN-SUK
Department of Astronomy and Meteorology, Yonsei University
(Received February 28, 1984)

ABSTRACT

Using King's model, we derived the dynamical masses of six globular clusters. The masses of clusters were calculated from the dynamical length parameters combined with the central velocity dispersion. The dynamical masses are all in the range from $2.5\times10^5 M_\odot$ to $1.4\times10^6 M_\odot$. The $(M/L_V)_\odot$ values lie between 1.0 and 1.2, which are typical for galactic clusters.

I. INTRODUCTION

The dynamical masses are necessary to discuss the dynamical properties of globular clusters. Globular clusters are the oldest objects in the galaxy and so are of considerable importance for furthering our understanding of star formation early in the life of the Galaxy. In paper I (Chun *et al.* 1980) we showed that the dynamical length parameters define the dynamical structure of globular clusters (core radius, tidal radius and concentration factor).

To determine dynamical masses, the cental velocity dispertions were combined with the dynamical length paprameters. Absolute integrated magnitudes with the masses of globular clusters give M/L values. This mass to light ratio of clusters is important to study the dynamical evolution of clusters. In Table 1 we presented the integrated magnitude from

Table 1. Structure parameters

NGC	$r_c(pc)$	$r_t(pc)$	$C=\log r_t/r_c$	m_v	Reference
362	0.54	25.26	1.67±0.04	6.42	Kr
5024	2.63	69.18	1.42±0.06	7.69	KM
5139	4.18	67.79	1.21±0.06	3.54	Kr
5904	0.92	59.40	1.81±0.06	5.85	KM
7078	0.39	32.44	1.92±0.03	6.29	KM
7089	1.01	45.12	1.65±0.04	6.46	KM

Kr : Kron (1974)
KM : Kron and Mayall (1960)

the photoelectric observations. The aim of this paper is the study of the dynamical masses and M/L of globular clusters.

II. MASSES AND MASS-TO-LIGHT RATIOS

a) From King's Model

To calculate the dynamical mass of cluster, we need to know their velocity dispersions and the dynamical length parameters of clusters. From King's theoretical surface density distribution, we determined the dynamical length parameters (r_c, r_t, C). The velocity dispersion at the center of a cluster is determined by using high-dispersion coude spectra of the integrated light. Since we did not observe high-dispersion coude spectra, the value of the central velocity dispersion comes from Peterson and King (1975).

Using the equation of King (1966), the dynamical masses can be derived. The dimensionless mass μ is

$$\mu=\int_0^{R_t} 4\pi R^2(\rho/\rho_0)dR \qquad (1)$$

where the dimensionless tidal radius $R_t=r_t/r_c$, the dimensionless radius $R=r/r_c$, and the central density $\rho_0=(0.89\pi G^2j^2r_c2)^{-1}$. G is the gravitational constant and the modulus of precision $j=(2\langle V^2\rangle)^{-1}$ from the form of the velocity distribution function used by King (1966). He assumed the velocity distribution of globular clusters is everywhere isotropic. The dynamical mass is then

$$M=\rho_0 r_c^3\mu \qquad (2)$$

The masses derived using equation (2) are tabulated in column (2) of Table 2. The errors given with the masses are from fitting the uncertainty and the error for the core radius r_c. The fitting errors were discussed in paper I. The uncertainty of the velocity dispersion is not included in the error for masses.

b) $r^{\frac{1}{4}}$ Law/Virial Theorem Method

For comparision with the dynamical masses from King's model, the masses for globular

Table 2. Masses and mass-to-light ratios of globular clusters

NGC	King Model				$r^{1/4}$law/virial theorem		
	$\langle V^2\rangle^{1/2}$ (kms^{-1})	$M_\odot\times10^6$	$(L_v)_\odot\times10^6$	$(M/L_v)_\odot$	$R_{1/2}(pc)$	$M_\odot\times10^6$	$(M/L)_\odot$
362	9.28	0.25±0.11	0.22±0.10	1.14±0.50	2.30	0.38	1.69
5024	5.57	0.30±0.13	0.27±0.13	1.11±0.48	9.73	0.61	2.20
5139	10.96	1.33±0.58	1.26±0.55	1.06±0.46	25.07	6.05	4.80
5904	6.76	0.27±0.12	0.26±0.11	1.04±0.45	4.15	0.39	1.48
7078	10.25	0.34±0.15	0.35±0.15	0.97±0.42	1.77	0.37	1.07
7089	9.15	0.44±0.19	0.36±0.16	1.22±0.53	3.66	0.62	1.73

clusters have also been derived using the $r^{\frac{1}{4}}$ law/virial theorem method. The effective radius $R_{\frac{1}{2}}$ is derived from $r^{\frac{1}{4}}$ Law (de Vaucouleurs 1948, 1953),

$$\log f(r) = -Ar^{\frac{1}{4}} + B \quad (3)$$

where A and B are arbitrary constants and $f(r)$ is the surface brightness at radius r from the center. This empirical equation coincides well with the surface brightness distribution of elliptical galaxies. The effective radius $R_{\frac{1}{2}}$ which is derived from the fitted r curve is close to the actual half light radius determined by integration of the surface density distribution. The effective radius is determined as the distance at which the surface brightness has fallen by log 3.33 below the extrapolated central surface brightness.

Using $R_{\frac{1}{2}}$ the masses can be derived from the virial theorem

$$2T + U = 0. \quad (4)$$

where T is the total kinetic energy and U is the total potential energy of a cluster. As the clusters can be assumed spherical, the total kinetic energy of a cluster is represented by $T = 1/2M\langle V^2\rangle_t$, where $\langle V^2\rangle_t$ is the mean square of total velocity dispersion. The total velocity dispersion is assumed constant throughout the cluster and the value is assumed to be three times of the observed velocity dispersion. The total potential energy U was given by integrating $r^{\frac{1}{4}}$ law with the assumption that mass-to-light ratio throughout the system of spherical galaxies is constant. This was discussed by Poveda (1958). The total potential energy $U = -0.33GM^2/R_{\frac{1}{2}}$, where the unit of the constant is CGS. From equation (4) the mass is

$$M = 670R_{\frac{1}{2}} \langle V^2\rangle_t \quad (5)$$

where the units of $R_{\frac{1}{2}}$ and $\langle V^2\rangle_t$ are parsecs and $(\mathrm{km\ sec^{-1}})^2$ respectively. The masses calculated using equation (5) are tabulated in column (5) of Table 2.

c) Mass-to-light Ratio

The visual luminosities were calculated by taking $(M_V)_\odot = 4.79$ (Allen 1963) and the absolute magnitudes M_V from Table 1. The values are tabulated in column (3) of Table 2 and expressed in solar units ($(L_V)_\odot = 1$). The calculated errors for the luminosities come from the distance uncertainty. The mass-to-light ratios $(M/L_V)_\odot$ are given in column 5 and 8 of Table 2 using both the King's model and $r^{\frac{1}{4}}$ law/virial Theorem mass values.

III. CONCLUSION

The calculated masses of globular clusters show that the mass values from the virial theorem are greater than those using King's model by a factor of $\frac{1}{2}$. Mass to light ratios increase with same tendency by a factor of 2. This mainly comes from the difference of the velocity dispersion, where the virial theorem assumes the constant velocity dispersion

Table 3. Mass estimates of other authors

NGC	$M_{\odot}\times10^{6}$	$(M/L_{v})_{\odot}$	$M_{\odot}\times10^{6}$	$(M/L_{v})_{\odot}$
362	0.25	1.14	0.19(1)	0.90(1)
5024	0.30	1.11	—	—
5139	1.33	1.06	0.71(2)	0.50(2)
5904	0.27	1.04	—	—
7078	0.34	0.97	0.13(3)	1.00(3)
7089	0.44	1.22	0.14(3)	1.00(3)

(1) Illingworth (1976)
(2) Dickens and Woolley (1967)
(3) Bahcall and Hausman (1977)

throughout the cluster while King's model regards the constant velocity dispersion only in the central part of a cluster.

We compared our results with the various authors (Illingworth 1976 for NGC 362, Bahcall and Hausman 1977 for NGC 7078 and NGC 7089) in Table 3. The results show that our masses are very close to the other estimates within a factor of 2. However Dickens and Woolley's result for NGC 5139 (1967) which was obtained from the Maxwellian velocity distribution truncated with mass function arbitrary did not match well with the dynamical mass.

As we assume the initial mass function is same for all clusters, we can expect the mass to light ratio is similar for all clusters unless there was a great dynamical change within a cluster. In Table 2 we showed that the average value of mass to light ratio of six clusters is nearly 1, which is similar to that derived by Illingworth (1975). Therefore we conclude that the globular clusters are in a dynamical equilibrium state.

REFERENCES

Allen, C.W., 1963, Astrophysical Quantities (2nd ed., University of London: Athlone Press)
Bahcall, N.A., and Hausman, M.A., 1977, Ap. J., **213**, 93.
Chun, M.S., Suh, Y.R., and Lee, Y.B., 1980, J.K.A.S., 13, 27. (paper I)
Dickens, R.J., and Woolley, Sir R., 1967, Roy. Obs. Bull., No. 128.
Illingworth, G.D., 1976, Ap.J., **204**, 73.
King, I.R., 1966, A.J., **71**, 64.
Kron, G.E., 1974, Private Communication.
Kron, G.E., and Mayall, N.U., 1960, A.J., **65**, 581.
Peterson, C.J., and King, I.R., 1975, A.J., **80**, 427.
Poveda, A., 1958, Bol. Obs. Tonanzintla Y. Tacubaya, No. 170.
Vaucouleurs, G. de., 1948, Ann. d'p., **11**, 247.
Vaucouleurs, G. de., 1953, M.N.R.A.S., **113**, 134.