# 강제 대류에서 수증기의 찬물 표면에서의 응축

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Forced Convection Condensation of

Vapor on A Cold Water

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# 초 록

2 차원 채널 입구에서의 꿰떼 난류 유동하는 찬 물 위를, 같은 방향으로 빠르게 난류 유동하는 수증기의 응축은 액체필름 초기상태의 과냉 정도에 의하여서 응축능력이 정하여진다. 수증기와 액체의 채널 입구에서의 균일한 속도 및 온도, 그리고 채널 입구에서 액체와 증기가 차지하는 체적비, 즉 액체 필름과 채널 높이를 알고 있을 때, 하류로 유동하면서 응축이 일어나는 현상을 예측하는 모델을 제안하고, 실험치와 비교한 것이다.

채널 입구에서 윗쪽으로는 더운 기체, 아래쪽으로는 찬 액체가 평행한 방향으로 유동하면서 접촉하고 평균적인 액체필름의 두께와 단열된 채널 벽체를 가정하여서, 기본방정식으로 연속방정식, 운동방정식을 세우고, 에너지와 운동량 전달 메카니즘 사이에 유사성이 존재한다고 가정하였으며, 전단응력의 크기는 필자의 모델을 적용하였다. 기본방정식을 기체 속도, 액체 속도, 필름의 두께, 압력에 대해서 수치해를 구하여서 동일조건 하에서 실험한 데이터와 비교하였다. 수증기와 액체 경계면에서의 전단응력은 매우 좋은 일치를 보여주고 있다.

## 1. Introduction

The solution of turbulent condensation problems in forced-convection relied on empirical or semiempirical correlation methods. It is not easy to calculate the interfacial shear stress because the interfacial shear stress is coupled with the interfacial mass transfer due to condensation (see reference Lee<sup>5)</sup>). The problem of turbulent condensation in cocurrent, horizontal stratified steam/water flows between adiabatic

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parallel plates is considered in this study. In contrast to film condensation due to the cold walls, direct-condensation occurs as a result of initial liquid subcoolings and the primary thermal resistance is related to a thermal energy exchange mechanism which is dependent on the overall characteristics of the film itself. The problem of turbulent condensation of vapor on a fairly thick layer of subcooled (cold)liquid has not been studied extensively.

Past experiments include the measurements for the stratified horizontal cocurrent flow of Linehan, Petrick and El-Wakil<sup>8)</sup> which was limited to a subcooled thin liquid and measurements by Thomas 100 of condensation on a turbulent liquid pool from quiescent stream. Young, Yang and Novotyny 12) reported the high heat transfer coefficient for a steam jet condensing in a coaxial water flow. Lee, Bankoff, Yuen, Jensen and Tankin6 measured velocity profiles in steam flow and water layer thickness in the one-component condensation at the interface between the fast moving steam flow and the slower water flow. Jensen20 measured interfacial shear stress by laser doppier velocimetry system and Jensen<sup>2)</sup> and Yuen<sup>11)</sup> reported a heat transfer correlation. A predicted procedure is outlined here to calculated the heat and mass transfer at the liquid-vapor interphase for the condensation of a superheated vapor on a subcooled film in terms of bulk variables.

### 2. Physical and Analytical Model

#### A. Flow Model

The governing equations are based on the following assumptions: the flow is steady incom-

pressible, turbulent and one-dimensional; the mass transfer is due to condensation, that is, the flow has no entrainment; actual time varying film thickness is replaced with an averaged film thickness; the interfacial temperature is equal to the saturation temperature; and the vapor phass is superheated, but remains constant.

Continuity

$$W_1 = \rho_1 U_1 S, W_g = \rho_g U_g (H - S) \cdots (1) (2)$$

Momentum

$$\frac{d}{dx}(\alpha_1'W_1U_1) - U_i\frac{dW_1}{dx} = -S\frac{dP_1}{dx} + \tau_1 - \tau_{wb}$$
(3)

$$\frac{d}{dx}(\alpha_{\varepsilon}'W_{\varepsilon}U_{\varepsilon}) - U_{i}\frac{dW_{\varepsilon}}{dx} = -(H-S)\frac{dP_{\varepsilon}}{dx}$$
$$-(\tau_{wt} + \tau_{i}) \cdots (4)$$

where

$$\dot{m}'' = \frac{dW_1}{dx} = \frac{dW_g}{dx}$$

 $lpha_{\it g}'=$  velocity distribution coefficient for turbulent vapor flow

 $a_1'=$  velocity distribution coefficient for turbulent liquid flow

 $U_{\rm g}$  = average vapor velocity

 $U_1$  = average liquid velocity

S =thickness of liquid layer

H = height of channel

The above equations are ordinary, nonlinear differential equations for the dependent variables  $U_g$ ,  $U_l$ , S,  $P_g$ . In order to solve the equations, relationships for the condensation rate  $dW_{l}/dx(=\dot{m}'')$ , liquid pressure  $P_1$ , the shear stresses  $\tau_i$ ,  $\tau_{wb}$  and  $\tau_{wt}$  and the interfacial velocity  $U_i$  must be specified. The con-

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densation rate(m'') can be evaluated from energy transfer mechanism. Most of the properties of vapor are estimated at saturated temperature except enthalpy of the superheated vapor. Viscosity of liquid is estimated at mean liquid temperature, which is a function of position (x) and increases gradually due to the direct condensation at the interface and adiabatic walls. Yuen's measurements implies that the one-dimensional Bernoulli relationship(steady state marcroscopic mechanical energy balance) may be applicable for liquid flow by neglecting turbulent kinetic energy terms.

#### B. Interfacial Shear Stress Model

Lee<sup>5)</sup> derived a general form of the interfacial shear stress for stratified two-phase flows from momentum integral equation of the vapor flow,

It is shown that the first term on the right-hand side is dominant for rapid condensation problem. When there is no condensation ( $\dot{\mathbf{m}}'' = 0$ ),  $C_1$  is same as the adiabatic shear stress. Turbulent plane Coutte flow (Reichardt<sup>9)</sup>) is found in the most of the liquid layer region by Lim's experiments<sup>7)</sup>, but the universal velocity distribution is locally assumed in the liquid-vapor and the liquid-solid interphase region.

$$0 < y^{+} < 5$$
  $U_{1}^{+} = y^{+}$   
 $5 < y^{+} < 30$   $U_{1}^{+} = -3.05 + 5 L_{1} y^{+}$ 

$$30 < y^+ < \delta^+/2$$

$$U_1^+=5.5+2.5l_n(\frac{2y^+}{2-2y^+/\delta^+})$$

where

$$U_1^+ = U_1/u^*$$
,  $y^+ = yu^*/\nu_1$  and  $u^* = \sqrt{\tau_1/\rho_1}$ .

Using

$$\tau_1 = (\mu_1 + \mu_{\bar{g}}) \frac{dU_1}{dv}$$

and solving for  $\epsilon_m (= \mu_g^e/\rho_1)$ , where  $\mu_g^e$  is eddy viscosity , one obtains:

$$\epsilon_m/\nu_1 = \frac{1}{dU^+/dy^+} - 1$$

For the laminar zone

$$\epsilon_m = 0 \left( \mu_{\rm g}^{\rm e} / \mu_1 \ll 1 \right)$$

For the buffer zone

$$\epsilon_m/\nu_1 = \nu^+/5 - 1$$

For the turbulent zone

$$\epsilon_m/\nu_1 = v^+/2.5/[1+1/(\delta^+/v^+-1)]$$

# C. Energy Transfer Mechanism and Condensation Rate

Assuming  $q_i \simeq q_l$  and integrating

$$q_l = (k_1 + k_e) \frac{dT_1}{dv},$$

the following equation can be obtained:

$$\frac{1}{h_x} = \frac{T_i - T_1}{q_i} = \int_0^{\delta_1} \frac{\nu_1 \, dy^+}{\rho_1 \, C_{\rho_1} \, (\alpha_1 + \frac{\epsilon_m}{P_{r_i}}) \, u^*}$$

where 
$$P_{rt} = \epsilon_m / \epsilon_h$$
  $\epsilon_h = k_e / \rho_1 C_{\rho_1}$   $\alpha_1 = k_1 / \rho_1 C_{\rho_1}$   $\delta^+ = \delta u^* / \nu_1$ 

 $T_1$  = average temperature of liquid  $k_x^e$  = eddy thermal conductivity

Result of the above integration may be given as

$$S_{lx}^{t} = \frac{h_{x}}{\rho_{1} C_{\rho 1} u^{*}} = \frac{1}{2l^{t}} \cdots (6)$$

where

$$0 < \delta^{+} < 10 \quad t^{+} = \frac{\delta^{+}}{2} P_{r}$$

$$0 < \delta^{+} < 60 \quad t^{+} = 5 P_{r} + 5 P_{rt} \, l_{n} \left[ 1 + \frac{P_{r}}{P_{rt}} \right]$$

$$(\frac{\delta^{+}}{10} - 1) \, ]$$

$$60 < \delta^{+} \qquad t^{+} = 5 P_{r} + 5 P_{rt} \, l_{n} (1 + 5 \frac{P_{r}}{P_{rt}})$$

$$+ 2.5 P_{rt} \, \left[ l_{n} \left( \frac{\delta^{+}}{60} \right) + l_{n} \left( 2 - 60 / \delta^{+} \right) \right]$$

The above derivation is somewhat similar to the works of Bae et al. In general,  $P_{rt}$  is a function of the Reynolds number, Prandtl number and the coordinate normal to the wall or interphase (Jischa and Rieke3). However, in the present calculations the turbulent Prandtl number was as signed the constant value of 0.9 as used by Jones and Renz4, Calculating t+ and combining the interfacial shear stress model, friction velocity  $u^*$  can be given and local heat transfer coefficient  $h_x$  and condensation rate (m'') also can be obtained. A prediction method is outlined here to calculate the heat, mass tranfer and interfacial shear stress at the liquid-vapor interphase for the forced -convection turbulent condensation of a superheated vapor on a subcooled film in terms of bulk variables.

## 3. Methods of Solution

From the interfacial shear stress model, a simplified equation is obtained:

$$\tau_i = \rho_l \, u^{*2} \cong \mathring{m}''(U_g - U_i)$$

Combining the energy transfer equations and definition of local heat transfer coefficient, the following relationship is given

$$h_{x} = \frac{\dot{n}''(U_{g} - T_{s}C_{pl})}{T_{c} - T} = \frac{\rho_{l} C_{pl} u^{*}}{t^{+}}$$

and

$$u^* = \frac{C_{pl} (T_i - T_l) (U_g - U_i)}{(H_g - C_{pl} T_s) t^+} \dots (7)$$

$$\dot{m}'' = \frac{h_x(T_i - T_l)}{H_g - C_{pl}T_s}$$
 (8)

where  $H_{\ell}$  is enthalpy of vapor relative to 0 °C datum. Therefore, we can estimate condensate rate  $(\dot{m}'')$ , interfacial shear stress  $(\rho_l u^{*2})$ . Approximately

$$\alpha'_{\sigma} \approx 1$$
 and  $U_i = 2U_l$ .

By using the above relationship, continuity and momentum equations can be rewritten as liquid momentum

$$S\frac{dP}{dx} + \alpha_i' \rho_i U_i S \frac{dU_i}{dx} + \rho_i g S \frac{dS}{dx} = \tau_i - \tau_{wb}$$
$$- (\alpha_i' U_i - U_i) \dot{m}'' \qquad (9)$$

vapor momentum

$$(H-S)\frac{dP}{dx} + \rho_g U_g (H-S)\frac{dU_g}{dx}$$

$$= -(\tau_{bt} + \tau_i) + \dot{m}'' (U_g - U_i) \cdots \cdots (\omega)$$

continuity

(liquid)

$$S\frac{dU_l}{dx} + U_l \frac{dS}{dx} = \frac{\dot{m}''}{\rho_l}$$
 (11)

(vapor)

$$(H-S)\frac{dU_g}{dx}-U_g\frac{dS}{dx}=-\frac{\dot{m}''}{\rho_g}\cdots (12)$$

where m'',  $\alpha'_i$  and  $\tau_1$  are given,

$$\tau_{wb} = \frac{C_f}{2} \rho_l \ U_l^2$$

$$\tau_{wt} = \frac{C_f}{2} \rho_g U_g^2$$

 $C_f$  is given by the standard turbulant boundary layer shear law for a single phase flow.

The above system of four simultaneous first order ordinary differential equations from equation (9) to equation (12) for four dependent bulk variables of  $P_{g}(x)$ ,  $U_{g}(x)$ ,  $U_{l}(x)$  and S(x) can be written as an initial value problem.

(A) 
$$\vec{F} = \vec{G}$$

This equation can be solved if determinant of the coefficient of matrix A is not equal to zero. Bulk temperature  $T_I$  is estimated at each step.

## 4. Results and Discussion

Turbulent condensation in cocurrent stratified horizontal flows at two dimensional entrance are solved simultaneously for vapor pressure  $(P_{\ell})$ , average vapor velocity  $(U_{\ell})$ , average liquid velocity  $(U_{\ell})$  and average film thickness (S) by using Couette plane turbulent flow model, analogy between heat and momentum transfer, and Lee's interfacial shear stress model.

Jensen (2) measured the heat and mass transfer at the liquid-vapor interphase for the condensation of a superheated vapor on a subcooled "thick" film. Numerical results are compared with Jensen's experiments.

Conditions of measurements(Jensen) is listed here:

 $H = .0635 \, m \, \text{width} = .3048 \, m$ 

$U_g(m/s)$	$U_l(m/s)$	S(m)	$T_{g}(^{\circ}\mathbb{C})$	$T_l(^{\circ}\mathbb{C})$
14.45	.311	.1583	141.7	26.3

A qulitatively good agreement are shown in Figs. 2, 3, 4. When cold liquid film is thick, condensation rate is strong and significant amount of vapor becomes liquid, then interfacial shear stress at the down stream becomes weak and the Couette turbulent flow assumption for the liquid flow may be deviated, since liquid film becomes thick (see the Fig. 2) and interface is weakly controlled by the vapor flow.

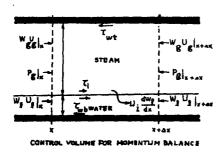
If the experimental data for "slightly thin" film is available, the comparison will be more interesting.

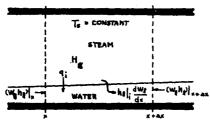
Fig. 4 shows a very good agreement with Jensen's measurements

#### 5. Conclusion

A predicted method is outlined here to colculate the heat and mass transfer at the liquid-vapor interphase for the condensation of a superheated vapor on a subcooled film in terms of bulk variables.

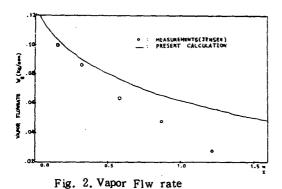
When initial conditions of velocities and temperatures for both phases, thickness of liquid film and pressure are given, this present model gives a qualitatively good agreement with measurements. If the more experimental data is available, the more comparison will be desirable.





CONTROL VOLLIME FOR ENERGY BALANCE

Fig. 1. Model used for developing continuity, energy, and momentum balances.



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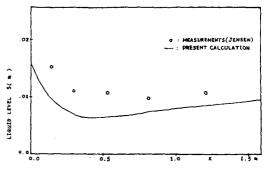


Fig. 3. Height of Liquid Flow

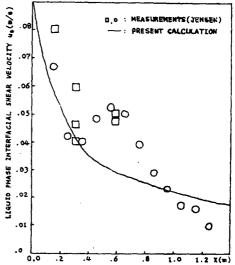


Fig. 4. Calculated and measured liquid phase interfacial
(circles and squares are experimental data from gas phase force balance and LDA measurements)

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