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An Optimal Block Replacement Policy Using Items with Different Reliability***

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Abstract

A block replacement policy using items with different reliability is discussed. We divide system unit failure modes into two modes and use less reliable unit when operating unit fails near the planned preventive replacement time.

In this policy, item A has two failure modes. Mode-1 failure is removed by minimal repair, mode-2 failure by replacement. If mode-2 failure of item A happens in (0, T-δ), failure item A is replaced by new item A. If mode-2 failure of item A happens in (T-δ,T), failure item A is replaced by new item B. Item B should be cheaper and less durable than item A.

Under this policy, we determine the preventive replacement interval T* and the interval δ * of item B replacement which minimize the cost rate per unit time.

I. INTRODUCTION

1.1 Purpose and scope of the study

A preventive maintenance policy is of great importance in reliability theory, since such a policy enables us to reduce the operating cost and the risk of a catastrophic breakdown. Many preventive maintenance policies have been proposed and discussed. In such maintenance policies, it is generally assumed that an unlimited number of spare units is immediately available when replacement is needed.

One of the most elementary and important replacement policies is the well-known block replacement policy. Under the ordinary block replacement policy, operating items are replaced at failure and at planned replacement times by new items. It has been modified because of its drawback that almost new items might be removed at planned replacement times.

In this study, we discuss block replacement policy in which we divide system unit failure modes into two modes and use less reliable unit when operating unit fails near the planned preventive replacement time.

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A preventive maintenance policy has been discussed by many authors. R.E. Barlow and F. Proschan(1) have discussed several replacement problems and derived optimum replacement policies. B.R. Bhat(5) has discussed block replacement policy using used item. In his paper, he gave the theory that failure unit was to be replaced by used item because the replacement cost by new item was very expensive at unit failure time.

S. Yámada and S. Osaki(5) have discussed preventive maintenance policy which minimize the total s-expected cost for replacement. He divides system components into non-essential unit and essential unit. A non-essential unit is minimally reparied at failure. An essential unit is replaced at failure or periodically.

F. Beichelt and K. Fischer (2) have discussed age replacement policy in which two modes of failures are considered. Mode-1 failure is removed by minimal repair, mode-2 by replacement. A more generalized block replacement policy was discussed by F. Beichelt (3) who considered two modes of system failure.

T. Tango (14) has discussed a modified block replacement policy with the purpose of excluding the wastefulness caused by the property that almost new items might be sometimes replaced at planned replacement times under the ordinary block replacement policy.

The purpose of this study is to propose another modified block replacement policy. In this policy, item A has two failure modes: Mode-1 failure is removed by minimal repair, mode-2 by replacement. If mode-2 failure of item A happens in $(0, T\text{-}\delta)$, failure item A is replaced by new item A. If mode-2 failure of item A happens in $(T\text{-}\delta, T)$, failure item A is replaced by item B. Item B should be cheaper and less durable than item A.

Under this policy, we determine the preventive replacement interval T^* and the interval δ^* of item B replacement which minimize the cost rate per unit time.

1.2 System configuration

During the running of a system, two failure modes of item A can happen;

Mode-1 failures are removed by minimal repair.

Mode-2 failures are removed by replacement.

Mode-1 failures can be interpreted as slight and easily fixed; whereas mode-2 failures might be total breakdowns. Operating items are replaced by item A at planned preventive replacement times. If mode-1 failure of item A happens during the system operation, failure item A is removed by minimal repair. If mode-2 failure of item A happens in $(0, T\text{-}\delta)$, failure item A is replaced by new item A. If mode-2 failure of item A happens in $(T\text{-}\delta, T)$, failure item A is replaced by new item B. In this system, item B should be cheaper and thus less durable than item A. If item B fails before preventive replacement time, failure item B is replaced by new item B.

In this system, we assume that failures are instantly detected and replaced.

1.3 Characteristics of maintenance policies

In many situations, failure of a unit during actual operation is costly or dangerous. If the unit is characterized by a failure rate that increases with age, it may be wise to replace it before it has aged too greatly. We shall be interested in minimizing the average expected cost per unit time.

Maintenance actions can be classified into two types; corrective and preventive maintenance action. Corrective maintenance action is generally taken when the system fails during the system operation. Generally, we remove the failed components by replacement in preventive maintenance action.

A commonly considered replacement policy is the policy based on age. Under this policy, a unit is always replaced at the time of failure or T hours after its installation, whichever occurs first; T is a constant unless otherwise specified. If T is a random variable, we shall refer to the policy as a random age replacement policy. Under a policy of block replacement the system units are replaced at times kT (k=1,2,....) and at failure. Under this policy, we replace a block or group of units in a system at prescribed time kT (k=1,2,...) independent of the failure history of the system. System failure rate is as good as new after replacement. If the failure rate is increasing, replacement should be considered. Almost replacement problems can be treated by the techniques of renewal theory.

II. ANALYSIS OF SYSTEM

2.1 Notations

$h_i(x)$	:	Failure rate of the item i, i = A, B
$p(x)$	:	Fraction of mode-2 failure of item A at age x
$\bar{p}(x)$	:	Fraction of mode-1 failure of item A at age x, $p(x) = 1-\bar{p}(x)$
Y	:	Time to the first mode-2 failure of item A without preventive maintenance
$G_A(t)$	:	cdf of Y
$G_B(t)$	:	cdf of item B failure time t
Z_t	:	Number of mode-1 failure of item A during the time interval (0, min (Y, t))
T	:	Fixed preventive replacement interval
M(t)	:	S-expected number of minimal repairs in (0, t)
$N_i(t)$	:	S-expected number of replacement of item i in (0, t), i = A, B
δ	:	Replacement-by-item B interval
$r(T-\delta)$	:	Mean residual life function after point $T-\delta$
C_{A1}	:	Cost of a minimal repair of item A
C_{A2}	:	Cost of a failure replacement by item A
C_B	:	Cost of a failure replacement by item B
C_p	:	Cost of a preventive replacement by item A
$C_1(T,\delta)$	:	S-expected cost of minimal repairs per unit time
$C_2(T,\delta)$	:	S-expected cost of replacements per unit time
$C(T,\delta)$	:	S-expected maintenance cost per unit time

2.2 Model description

System failure modes and its maintenance policy are shown in Figure 1.

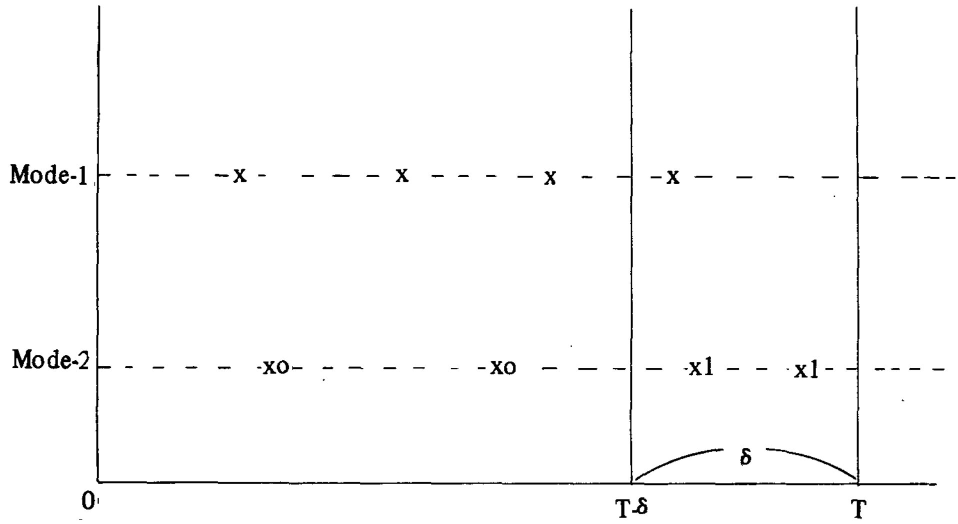

Fig. 1. Failure modes and maintenance policy

where x : minimal repair point of item A

xo : replacement point of item A

x1 : replacement point of item B

As can be seen from Figure 1, two modes of item A failure are possible: mode-1 failure and mode-2 failure. Mode-1 failures can be interpreted as slight and easily fixed, whereas mode-2 failures might be total breakdowns.

Characteristics of this model are as follows;

1) Operating items are replaced by item A at times kT (k=1,2, ...)
2) Fraction of mode-2 failure of item a is p(x) at age x.
3) Mode-1 failures of item A are removed by minimal repair.
4) Mode-2 failure of item A are removed by replacement.
5) If mode-2 failure of item A happens in $(0, T-\delta)$, failure item A is replaced by new item A, and if in $(T-\delta, T)$, that is replaced by new item B.
6) A cost C_{A1} is suffered for each mode-1 failure of item A, a cost C_{A2} for each mode-2 failure of item. A.
7) A cost C_B is suffered for each item B replacement.
8) A cost C_p is suffered for each preventive replacement.

The failure rate functions for this maintenance policy are shown in Figure 2 and Figure 3. Figure 2 illustrates the failure rate when no mode-2 failure of item A in $(T-\delta, T)$.

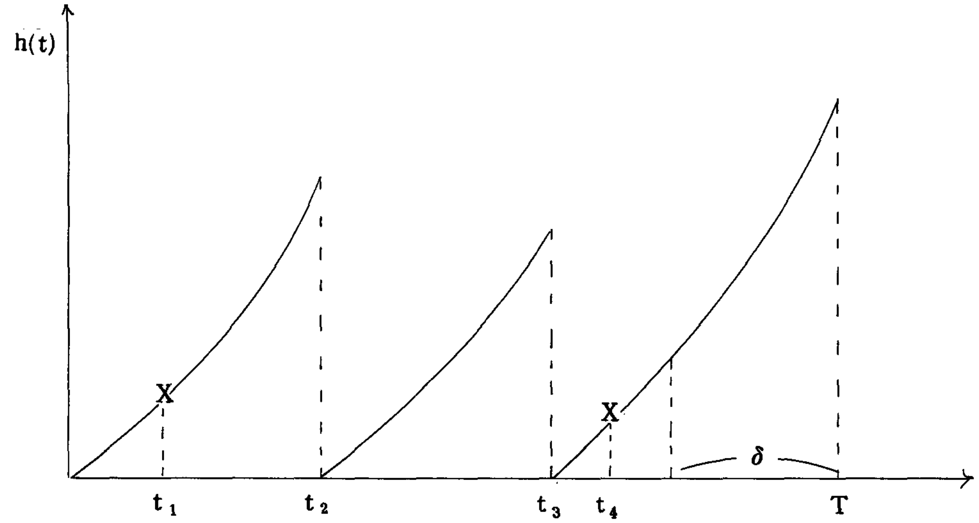

Fig. 2. Failure rate when no mode-2 failure of item A in (T-δ, T)

where t_1, t_4 : minimal repair point,
t_2, t_3 : replacement point of item A,
T: preventive replacement point by new item A.

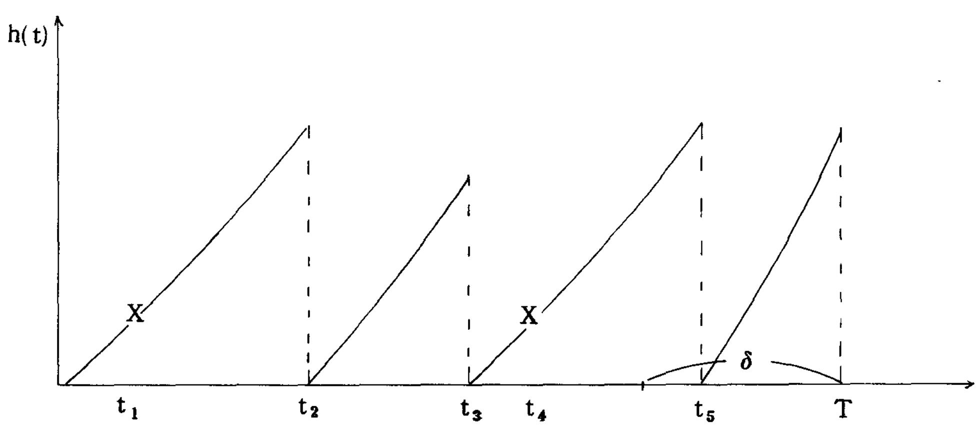

Fig. 3. Failure rate when mode-2 failure of item A happens in (T-δ, T),

where t_1,t_4 : minimal repair point
t_2,t_3 : replacement point of item A
t_5 : replacement point of item B
T : preventive replacement point by new item A

2.3 Assumptions

The following basic assumptions are made.

1) All failure events are s-independent.
3) The system is renewed in case of preventive replacement and mode-2 failures.
4) System failure rate is not disturbed by minimal repair.
5) All maintenance actions take only negligible times.
6) $G_A(t)$ and $G_B(t)$ are IFR (increasing failure rate).
7) $p(x_1) \leqslant p(x_2)$ if $0 \leqslant x_1 \leqslant x_2$
8) $0 < C_{A1} < C_p < C_{A2}, C_{A1} < C_B < C_{A2}$
9) $r(t\text{-}\delta) \leqslant \delta$

2.4 Method of Analysis

An explicit formula for the cost function can be obtained in the following procedure.

1) Let $C_1(T, \delta)$ be the minimal repair cost of item A in the fixed interval $(0, T)$.This cost function $C_1(T, \delta)$ can be expressed as the product of cost C_{A1} per failure and the s-expected number of mode-1 failures of item A; that is,

$$C_1(T, \delta) = C_{A1} \times M(T) \qquad (1)$$

where M(T) is the s-expected number of mode-1 failures of item A.

2) Let $C_2(T, \delta)$ be the replacement cost of item A in the fixed interval $(0, T)$. The cost function $C_2(T, \delta)$ can be expressed as the product of cost C_{A2} per replacement and the s-expected number of item A replacements; that is,

$$C_2(T, \delta) = C_{A2} \times N_A(T) \qquad (2)$$

where $N_A(T)$ is the s-expected number of item A replacements.

3) Let $C_3(T, \delta)$ be the replacement cost of item B in the fixed interval $(0, T)$. The cost function $C_3(T, \delta)$ can be expressed as the product of cost C_B per replacement and the s-expected number of item B replacement: that is,

$$C_3(T, \delta) = C_B \times N_B(T) \qquad (3)$$

where $N_B(T)$ is the s-expected number of item B replacements.

Using equations (1), (2), and (3), the objective cost function per unit time can be expressed as follows;

$$C(T, \delta) = (C_1(T, \delta) + C_2(T, \delta) + C_3(T, \delta) + C_p) / T \qquad (4)$$

By numerical procedure, we can find the T^*, δ^* to minimize the equation (4). We consider the following procedure to find the T^*, δ^*.

(1) : We specifiy the relations between δ and T.

(2) : We determine the value of T^i to minimze $C(T, \delta)$ for relation i. Then, let $C^i(T, \delta)$ be the minimal cost for relation i.

(3) : We compare the values of cost $C^i(T, \delta)$ for various relations, then find T^*, δ^* having minimal cost, $C^*(T, \delta)$ among $C^i(T, \delta)$'s.

The procedure of dtermination of T and δ is shown in Figure 4.

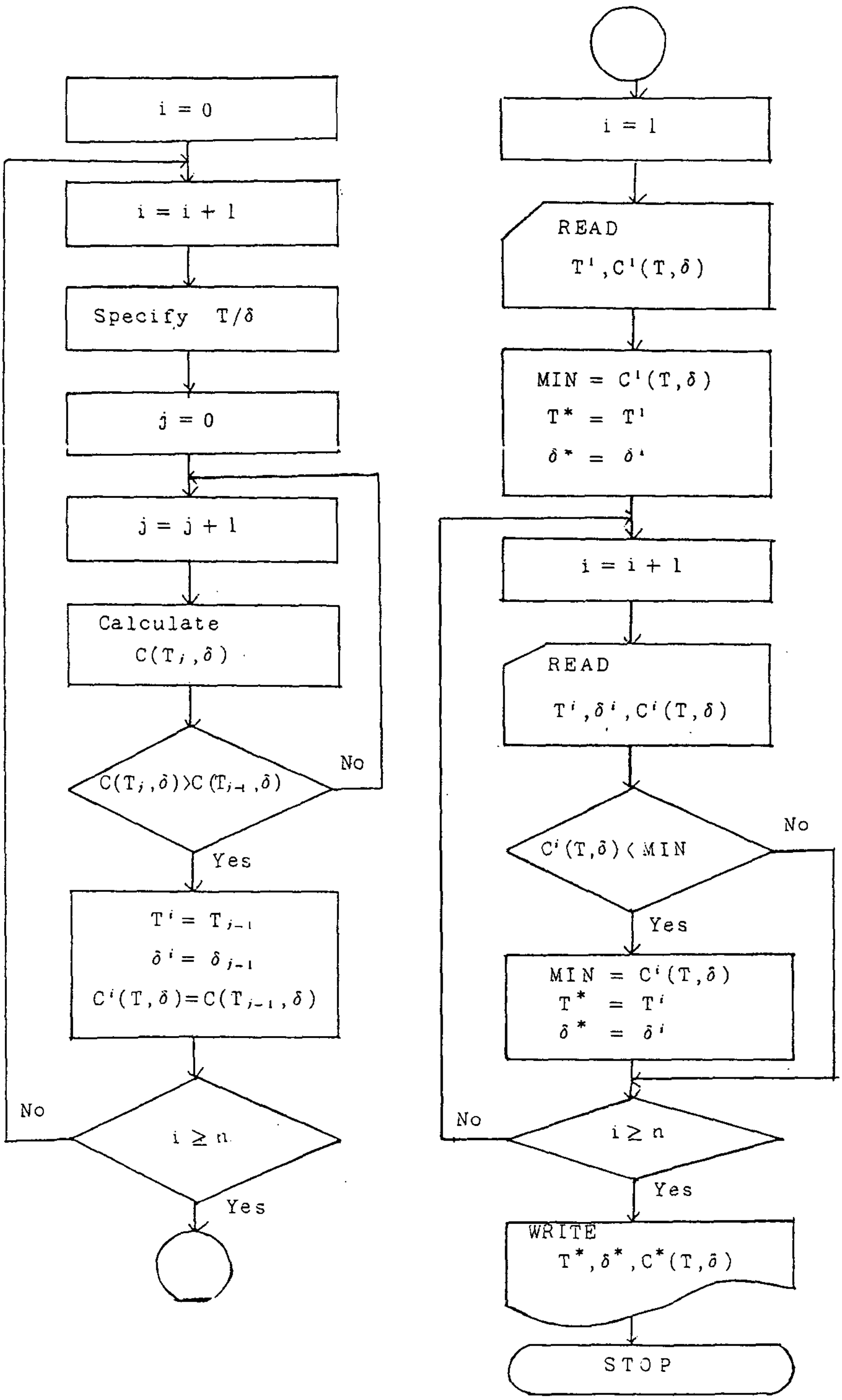

Fig. 4. Procedure of determination of T* and δ*

2.5 Determination of the optimal block replacement time

1) Mean residual life function

Let $r(T\text{-}\delta)$ be the mean residual life function after the time $T\text{-}\delta$, then $r(T\text{-}\delta)$ can be expressed as follows:

$$r(T\text{-}\delta) = E(Y-(T\text{-}\delta) \mid Y > T\text{-}\delta) \quad \ldots\ldots (5)$$

Since $E(Y-(T\text{-}\delta) \mid Y > T\text{-}\delta) = \frac{1}{1-G_A(T\text{-}\delta)} \int_{T\text{-}\delta}^{\infty} (1-G_A(Z))\,dz \quad \ldots\ldots (6)$

we have from (5)

$$r(T\text{-}\delta) = \frac{1}{1-G_A(T\text{-}\delta)} \int_{T\text{-}\delta}^{\infty} (1-G_A(Z))\,dz \quad \ldots\ldots (7)$$

where $G_A(t)$ is cdf of Y.

2) Cost for the minimal repair

Let M(t) be the s-expected number of mode-1 failures of item A in (0, t). Considering the relation between time to the first mode-2 failure Y and minimal repair number Z_t during the time interval (0, min (Y, t)), we can obtain the following equation for M(t),

$$M(t) = E(Z_t \mid Y \geq t)\,\bar{G}_A(t) + \int_0^t (E(Zx \mid Y{=}x) + M(t-x))dG_A(x) \quad \ldots\ldots (8)$$

where $\bar{G}_A(t) = \mathrm{Exp}\left(-\int_0^t p(x)h_A(x)dx\right) \quad \ldots\ldots (9)$

In equation (8), $E(Z_t \mid Y \geq t)$ is the s-expected number of minimal repairs during the time interval (0, T) provided that time to the first mode-2 failure Y is greater than time t. $E(Z_t|Y \geq t)$ can be expressed as follows:

$$E(Z_t \mid Y \geq t) = \int_0^t p(x)h_A(x)dx \quad \ldots\ldots (10)$$

$E(Zt \mid Y < t)$ can be derived as follows:

$$E(Z_t \mid Y < t) = \frac{1}{G_A(t)} \int_0^t \int_0^y P(x)h_A(x)dx\, dG_A(y) \quad \ldots\ldots (11)$$

then, $E(Z_t|Y < t)$ can be expressed as following:

$$E(Z_t \mid Y < t) = \frac{1}{G_A(t)} \int_0^t E(Z_x \mid Y = x)\, dG_A(x) \quad \ldots\ldots (12)$$

We obtain from equation (8) and (12),

$$M(t) = E(Z_t \mid Y < t)G_A(t) + E(Z_t|Y \geq t)\,\bar{G}_A(t) + \int_0^t M(t-x)dG_A(x) \quad \ldots\ldots (13)$$

Recall that we can derived from equation (10) and (11)

$$E(Z_t \mid Y < t)G_A(t) + E(Z_t \mid Y \geq t)\bar{G}_A(t)$$
$$= \int_0^t \bar{\bar{G}}_A(x)h_A(x)dx - G_A(t) \quad \ldots\ldots (14)$$

Thus by inserting (14) in (13), we obtain

$$M(t) = \int_0^t \bar{G}_A(x)h_A(x)dx - G_A(t) + \int_0^t M(t-x)dG_A(y) \quad \ldots\ldots (15)$$

In this system model, we can take minimal repairs for mode-1 failures of item A until the time $r(T\text{-}\delta)$ after $T\text{-}\delta$, that is, $T\text{-}\delta + r(T\text{-}\delta)$. Therefore the s-expected number of minimal repairs during the time interval (0, T) can be expressed as follows using (15).

$$M(T\text{-}\delta + (T\text{-}\delta)) = \int_0^{T\text{-}\delta + r(T\text{-}\delta)} G_A(x)h_A(x)dx - G_A(T\text{-}\delta + r(T\text{-}\delta))$$

$$+\int_{0}^{T-\delta + r(T-\delta)} M(T-\delta + r(T-\delta) - x)\, dG_A(x) \quad \text{.......} \quad (16)$$

Thus the minimal repair cost of item A in $(0, T)$, $C_1(T, \delta)$ can be defined as follows

$$C_1(T, \delta) = C_{A1} \times M(T-\delta + r(T-\delta)) \quad \text{.......} \quad (17)$$

3) Cost for the replacement of item A

Let $N_A(t)$ be the s-expected number of mode-2 failures of item A in $(0, t)$. The moments of mode-2 failures generate a renewal process with the renewal function $N_A(t)$. $N_A(t)$ can be defined as follows : $N_A(t) = \sum_{k=1}^{\infty} G_A^{(k)}(t)$ .... (18)

Using the fact that $G_A^{(K+1)}(t) = \int_0^t G_A^{(k)}(t-x)\, dG_A(x)$ we obtain the following equation:

$$N_A(t) = G_A(t) + \sum_{k=1}^{\infty} \int_0^t G_A^{(k)}(t-x) dG_A(x) \quad \text{.......} \quad (19)$$

Therefore $N_A(t) = G_A(t) + \int_0^t N_A(t-x) dG_A(x)$ .... (20)

In this system model, we can take replacement for the mode-2 failure of item A in $(0, T-\delta)$. Thus, the s-expected number of replacement of item A can be expressed as follows using (20);

$$N_A(T-\delta) = G_A(T-\delta) + \int_0^{T-\delta} N_A(T-\delta - x) dG_A(x) \quad \text{.......} \quad (21)$$

Thus, the replacement cost of item A in $(0, T)$, $C_2(T, \delta)$ can be defined as following:

$$C_2(T, \delta) = C_{A2} \times N_A(T-\delta) \quad \text{.......} \quad (22)$$

4) Cost for the replacement of item B

Let $N_B(t)$ be the s-expected number of item B failures in $(0, t)$. The moments of failures generate a renewal process with the renewal function $N_B(t)$.

Therefore,

$$N_B(t) = G_B(t) + \int_0^t N_B(t-x) dG_B(x) \quad \text{.......} \quad (23)$$

where $G_B(t)$ is the cdf of item B failure time t.

In this system model, we can take replacements for item B failures in $(T-\delta, T)$. The replacements of item B include the replacement of item B by occuring of mode-2 failure of item A after $T-\delta$ and the replacement of failure item B.

The interval of item B failure is $T-(T-\delta + r(T-\delta))$, that is $\delta - r(T-\delta)$.

Therefore, the s-expected number of failures of item B can be expressed as following using(23):

$$N_B(\delta - r(T-\delta)) = G_B(\delta - r(T-\delta)) + \int_0^{\delta - r(T-\delta)} N_B(\delta - r(T-\delta) - x)\, dG_B(X) \quad \text{.......} \quad (24)$$

Thus the replacement cost of item B in $(0, t)$, $C_3(T, \delta)$ can be defined as follows:

$$C_3(T, \delta) = C_B \times (N_B(\delta - r(T-\delta)) + 1) \quad \text{.......} \quad (25)$$

5) Cost function

We can derive the cost function per unit time by inserting (17), (22), and (25) in (4); that is,

$$C(T,\delta) = (C_{A1}\cdot M(T-\delta + r(T-\delta)) + C_{A2}\cdot N_A(T-\delta) + C_B\cdot N_B(\delta - r(T-\delta)) + C_p)/T \quad (26)$$

T^* and δ^* are the optimal values of T and δ to minimize (26). The direct solutions of T^* and δ^* must be generally done by numerical procedures, for we can not find T^* and δ^* by analytical method.

The procedure is as follows as in section 2.4.

(1) : We specify the relations between δ and T.

(2) : We determine the value of T_i to minimize $C(T, \delta)$ in (26) for relation i. Then, let $C^i(T, \delta)$ be the minimal cost for relation i.

(3) : We compare the values of cost $C^i(T, \delta)$ for various relations, then find T^*, δ^* having minimal cost $C^*(T, \delta)$ among $C^i(T, \delta)$'s

2.6 Analysis of special cases

1) Case of $\delta = o$

In this case, we do not consider the replacement ot item B, for δ is zero. Maintenance cost is composed of minimal repair cost and replacement cost of item A.

(1) minimal repair cost

The s-expected number of mode-1 failures of item A in (0, t), M(t) is given by (15) in section 2.5. Mode-1 failures of item A can take place in (0, T), therefore the minimal repair cost can be expressed as follows;

$$C_1(T,\delta) = C_{A1} x\, M(T) \quad (27)$$

where $M(T) = \int_o^t \bar{G}_A(x) h_A(x) dx - G_A(T) + \int_o^t M(T-x) dG_A(x)$

(2) replacement cost

The s-expected number of mode-2 failures of item A in (0, t), $N_A(t)$ is given by (20) in section 2.5. Mode-2 failures of item A can take place in (0, T), therefore the replacement cost of item A can be expressed as follows;

$$C_2(T,\delta) = C_{A2}\cdot N_A(T) \quad (28)$$

where $N_A(T) = G_A(T) + \int_o^t N_A(T-x) dG_A(x)$

(3) cost function

We get cost function, C(T) as following from (27) and (28);

$$C(T) = (C_{A1}\cdot M(T) + C_{A2}\cdot N_A(T) + C_p)/T \quad (29)$$

2) Case of $\bar{p}(x) = 0$

In this case, we do not consider the occurrence of model failure of item A during the system operation. Therefore, the cost function in this case is equal to the cost function of (26) only but $\bar{p}(x) = 0$; that is,

$$C(T,\delta) = (C_{A1}. M(T-\delta + r(T-\delta)) + C_{A2}. N_A(T-\delta) + C_B. N_B(\delta - r(T-\delta)) + C_p)/T \quad (30)$$

$$\text{where } M(T-\delta + r(T-\delta)) = \int_o^{T-\delta + (T-\delta)} \bar{G}_A(x) h_A(x) dx - G_A(T-\delta + r(T-\delta)) + \int_o^{T-\delta + r(T-\delta)} M(T-\delta + r(T-\delta) - x)\, dG_A(x) \quad (31)$$

$$N_A(T-\delta) = G_A(T-\delta) + \int_0^{T-\delta} N_A(T-\delta - x)dG_A(x) \quad \ldots\ldots (32)$$

$$N_B(\delta - r(T-\delta)) = G_B(\delta - r(T-\delta)) + \int_0^{\delta - r(T-\delta)} N_B(\delta - r(T-\delta)-x)dG_B(x) \quad \ldots\ldots (33)$$

$$\bar{G}_A(t) = \mathrm{Exp}(-\int_0^t h_A(x)dx) \quad \ldots\ldots (34)$$

We can find T^* and δ^* to minimize (3) by numerical procedure.

III. NUMERICAL EXAMPLE

In many fields of maintenance policy, Weibull distribution is generally used as the failure distribution of system. We use weibull distribution in numerical example.

Table 1. Values of T*, δ* and C* (T, δ) for each cost condition

C_B/C_{A2} \ C_p/C_{A2}	0.1		0.5		1.0	
	T*	δ*	T*	δ*	T*	δ*
	C*(T, δ)		C* (T, δ)		C* (T, δ)	
0.1	5	2.5	5	3.0	7	2.333
	0.111		0.191		0.288	
0.2	6	3.0	6	3.0	14	1.077
	0.129		0.208		0.299	
0.3	6	3.0	6	3.0	14	1.077
	0.145		0.224		0.307	
0.4	7	2.333	6	3.0	14	1.077
	0.193		0.241		0.314	
0.5	8	2.667	6	3.0	14	1.077
	0.207		0.258		0.321	
0.6	14	1.4	6	3.0	15	0.938
	0.256		0.274		0.328	
0.7	15	1.071	6	3.0	15	0.938
	0.268		0.291		0.335	
0.8	15	0.938	14	1.4	15	0.938
	0.335		0.303		0.341	
0.9	15	0.938	14	1.4	15	0.938
	0.341		0.310		0.348	
1.0	15	0.938	14	1.4	15	0.938
	0.348		0.318		0.355	

We assume that

$$h_A(x) = \lambda_A \cdot \beta \cdot X^{\beta-1}$$

where

$$\lambda_A = 0.1, \beta=2.0$$

$$h_B(x) = \lambda_B \cdot \beta \cdot X^{\beta-1}$$

where

$$\lambda_B = 0.8, \beta=2.0$$

$$p(x) = 0.4, C_{A1} = 0.1, C_{A2} = 1.0$$

Table 1. presents values of T^*, δ^* and $C^*(T, \delta)$ for each cost condition.

IV. CONCLUSIONS

A block replacement policy has been analyzed for the purpose of determining the preventive replacement interval T^* and item B replacement interval δ^*

In this analysis, we found solutions by numerical procedure after specifying the relation T/δ.

The planned preventive replacement interval has a large value as the ratio C_p/C_{A_2} increases. The item B replacement interval has a small value as the ratio C_p/C_{A_2} increases.

Factors such as reliability and availability are not considered with the system model. If we would consider such factors, we could have more effective maintenance policy. A maintenance policy consider ing such factors could be studied as an extension of this study.

ACKNOWLEDGEMENT

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