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EXTENSIONS OF TOPOLOGICAL ORDERED SPACES II

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0. Introduction

As the title indicates, this paper is a sequel of our previous paper [3]. In [3], we have shown that any Hausdorff convex ordered space has two extremal extensions, i.e., strict one and simple one. To construct those extensions, we use open bifilters on the space. Furthermore, they behave exactly like the strict and simple extensions of a topological space introduced by Banaschewski [1]. In particular, we have shown that every regular extension of a topological ordered space is the strict extension of the space.

In this paper, we introduce the concept of completely regular bifilters and then show that the Nachbin compactification $\beta_0 X$ of a completely regular ordered space X is given by the strict extension of X with the set of all maximal completely regular bifilters as its trace open bifilters. Using this, we characterize compact ordered spaces as completely regular ordered spaces such that every maximal completely regular bifilter on the spaces is convergent.

Moreover the ordered k-compactification $\beta_0^k x$ of X (see [4]) is also determined by all maximal completely regular bifilters on X with the k-intersection property.

Replacing completely regular bifilters by clopen bifilters, simiar results for 0-dimensional k-compact ordered spaces can be obtained. For the terminology, we refer to [3].

1. Completely regular bifilters

It is known [1] that the Stone-Čech compactification βX of a completely regular space X is precisely the strict extension of X with the set of all maximal completely regular filters on X (see [2] for maximal completely regular filters) as the filter trace. In this section, we introduce completely regular bifilters on a topological ordered space and show their corresponding properties to those of completely regular filters.

In the following, the unit interval [0,1] endowed with the usual order and the usual topology will be denoted by I.

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1.1 DEFINITION Let X be a topological ordered space. An open bifilter $(\mathscr{F}, \mathscr{G})$ is said to be a *completely regular bifilter* if $\mathscr{F}$ ($\mathscr{G}$, resp.) has a base $\mathscr{L}$ (£, resp.) consisting of open increasing (decreasing, resp.) sets such that for any $B \in \mathscr{L}$, there is a $B' \in \mathscr{L}$ and a continuous isotone $f: X \longrightarrow I$ with $f(B')=1$ and $f(X-B)=0$, and dually for any $C \in$ £, there is a $C' \in$ £ and a continuous isotone $g: X \longrightarrow I$ with $g(C')=0$ and $g(X-C)=1$. In this case, $(\mathscr{L}$, £) is called a *completely regular bibase* of $(\mathscr{F}, \mathscr{G})$.

By a *maximal completely regular bifilter* we mean a completely regular bifilter not contained in any other completely regular bifilter.

1.2. REMARK By Zorn's lemma, every completely regular bifilter is contained in a maximal completely regular bifilter.

The following is a characterization of maximal completely regular bifilters.

1.3 PROPOSITION *Let $(\mathscr{F}, \mathscr{G})$ be a completely regular bifilter on a topological ordered space X. Then $(\mathscr{F}, \mathscr{G})$ is a maximal completely regular bifilter iff for any pair A, B of open increasing sets on X such that there is a continuous isotone $f: X \longrightarrow I$ with $f(B)=1$ and $f(X-A)=0$, either $A \in \mathscr{F}$ or there is an $F \in \mathscr{F}$ and a $G \in \mathscr{G}$ with $B \cap F \cap G = \phi$, and dually for any pair C, D of open decreasing sets on X such that there is a continuous isotone $g: X \longrightarrow I$ with $g(D)=0$ and $g(X-C)=1$. either $C \in \mathscr{G}$ or there is an $F \in \mathscr{F}$ and a $G \in \mathscr{G}$ with $D \cap F \cap G = \phi$.*

PROOF: ($\Longrightarrow$) Let A, B be open increasing sets with the given conditions. Suppose $A \notin \mathscr{F}$ and $B \cap F \cap G \neq \phi$ for all $F \in \mathscr{F}$ and all $G \in \mathscr{G}$. Since $f(B)=1$, for any r with $0<r<1$, $f^{-1}(]r, 1])$ contains B, so that $f^{-1}(]r,1]) \cap F \cap G \neq \phi$ $(F \in \mathscr{F}.\ G \in \mathscr{G})$. Let $\mathscr{H}$ be the filter generated by $\{f^{-1}(]r, 1]) | 0<r<1\} \cup \mathscr{F}$. Then $(\mathscr{H}, \mathscr{G})$ is clearly a bifilter containing $(\mathscr{F}, \mathscr{G})$. Furthermore. $(\mathscr{H}. \mathscr{G})$ is a completely regular bifilter. Indeed, let $(\mathscr{L}, \mathscr{C})$ be a completely regular bibase for $(\mathscr{F}. \mathscr{G})$. Take any $U \in \mathscr{L}$ and any r with $0<r<1$. Then there is a $U' \in \mathscr{L}$ such that there is a continous isotone $g: X \longrightarrow I$ with $g(U')=1$ and $g(X-U)=0$. Pick a real number s with $r<s<1$, and let $k: I \longrightarrow I$ be the continuous isotone such that $k([0,r])=0$, $k([s,1])=1$ and k is linear on $[r, s]$ onto $[0.1]$. Let $h=g \wedge (k \circ f): X \longrightarrow I$, which is clearly a continuous isotone. Now it is immediate that $h(U' \cap f^{-1}(]s, 1]))=1$ and $h(X-(U \cap f^{-1}(]r, 1])))=0$. Thus $(\mathscr{H}, \mathscr{G})$ has as a completely regular bibase $(\{U \cap f^{-1}(]r, 1]) | 0<r<1, U \in \mathscr{L}\}$, £); hence $(\mathscr{H}, \mathscr{G})$ is a completely regular bifilter containing $(\mathscr{F}, \mathscr{G})$. Since

$(\mathscr{F}, \mathscr{G})$ is maximal, for any $r>0$, $f^{-1}(]r, 1])\in\mathscr{F}$. Since A contains $f^{-1}(]r, 1])$, one has a contradition. Dually, one can show that $(\mathscr{F}, \mathscr{G})$ satisfies the remaining half of the condition.

$(\Longleftarrow)$ Suppose $(\mathscr{F}, \mathscr{G})$ is not a maximal completely regular bifilter, then by Remark 1.2, $(\mathscr{F}, \mathscr{G})$ is properly contained in a completely regular bifilter $(\mathscr{H}, \mathscr{K})$. Then we have $\mathscr{F}\subsetneq\mathscr{H}$ or $\mathscr{G}\subsetneq\mathscr{K}$. Suppose $\mathscr{F}\subsetneq\mathscr{H}$, and pick $H\in\mathscr{H}-\mathscr{F}$. Then there is a $K\in\mathscr{H}$ and a continuous isotone $f: X\longrightarrow I$ with $f(K)=1$ and $f(X-H)=0$. Since $H\notin\mathscr{F}$, there is an $F\in\mathscr{F}$ and $G\in\mathscr{G}$ with $F\cap G\cap H=\phi$, but $F\cap H\in\mathscr{H}$ and $G\in\mathscr{K}$. Thus we have a contradiction. Similarly one has a contradiction for the case of $\mathscr{G}\subsetneq\mathscr{K}$. This completes the proof.

1.4 REMARK Let $(\mathscr{F}, \mathscr{G})$ and $(\mathscr{H}, \mathscr{K})$ be completely regular bifilters on a topological ordered space. Using the argument in the proof of the above theorem, $(\mathscr{F}\vee\mathscr{H}, \mathscr{G}\vee\mathscr{K})$ is again a completely regular bifilter, provided each member of $\mathscr{F}\vee\mathscr{H}$ meets any member of $\mathscr{G}\vee\mathscr{K}$.

We recall [2] that a maximal completely regular filter $\mathscr{F}$ on a topological space X is a Cauchy filter on X endowed with the initial uniform structure with respect to $C(X, I)$, i.e., for any continuous map $f: X\longrightarrow I, f(\mathscr{F})$ is convergent. The following theorem is a counterpart to the above fact for maximal completely regular bifilters on a topological ordered space.

1.5 THEOREM *If $(\mathscr{F}, \mathscr{G})$ is a maximal completely regular bifilter on a topological ordered space X, then for any continuous isotone $f: X\longrightarrow I$, $f(\mathscr{F}\vee\mathscr{G})$ is convergent.*

PROOF: Take any continuous isotone $f: X\longrightarrow I$. Since $f(\mathscr{F}\vee\mathscr{G})$ is a filter base on the compact space I, $f(\mathscr{F}\vee\mathscr{G})$ has a cluster point, say $p\in I$. Suppose q is another cluster point of $f(\mathscr{F}\vee\mathscr{G})$ on I, and we may assume $p<q$. We can claim that for any r with $p<r<q$, $f^{-1}(]r,1])$ belongs to $\mathscr{F}$. Indeed, take any s with $r<s<q$, and let $k: I\longrightarrow I$ be the continuous isotone constructed in the proof of Proposition 1.3. Let $g=k\circ f: X\longrightarrow I$. It is obvious that $g(f^{-1}(]s, 1]))=1$ and $g(X-(f^{-1}(]r, 1]))=0$. Thus by Proposition 1.3, one has either $f^{-1}(]r, 1])\in\mathscr{F}$ or there is an $F\in\mathscr{F}$ and a $G\in\mathscr{G}$ with $F\cap G\cap f^{-1}(]s, 1])=\phi$. For the latter case, $]s,1]$ is a neighborhood of q. Since q is a cluster point of $f(\mathscr{F}\vee\mathscr{G})$, $f(F\cap G)\cap]s, 1]\neq\phi$; therefore $F\cap G\cap f^{-1}(]s, 1])\neq\phi$, which is a contradiction. Thus $f^{-1}(]r, 1])\in\mathscr{F}$ for $p<r<q$. Since $[0,r[$ is a neighborhood of p and $f^{-1}(]r, 1])\in\mathscr{F}$, p is not a cluster point of $f(\mathscr{F}\vee\mathscr{G})$, which is again a

contradiction. Thus p is the unique cluster point of $f(\mathscr{F} \vee \mathscr{G})$ on the compact space I, so that p is in fact the limit of $f(\mathscr{F} \vee \mathscr{G})$.

2. Nachbin compactifications

Let *CROS* denote the category of completely regular ordered spaces and continuous isotones and let *COS* denote the category of compact ordered spaces and continuous isotones.

It is known [8] that *COS* is an epireflective subcategory of *CROS* and that for any $X \in CROS$, the *COS*-reflection of X is given by the Nachbin compactification $\beta_0 : X \longrightarrow \beta_0 X$. Moreover, $\beta_0 X$ is an extension of X. Since $\beta_0 X$ is a regular ordered space, $\beta_0 X$ is a strict extension of X by Theorem 2.10 in [3]. Thus $\beta_0 X$ is completely determined by its trace open bifilters.

As expected, we will show that the trace open bifilters of $\beta_0 X$ on X are precisely maximal completely regular bifilters. Using this, we show that a completely regular ordered space is compact iff every completely regular bifilter on the space has a cluster point.

2.1 PROPOSITION *For a completely regular ordered space X, let $\beta_0 : X \longrightarrow \beta_0 X$ be the Nachbin compactification of X. If for $t \in \beta_0 X$, $(\mathscr{F}(t), \mathscr{G}(t))$ is the trace bifilter of X, then $(\mathscr{F}(t), \mathscr{G}(t))$ is a maximal completely regular bifilter on X.*

PROOF: For any $U \in \mathscr{F}(t)$, there is a $B \in \vartheta_{\beta_0 X}(t)$ with $B \cap X = U$. Then there is a continuous isotone $g : \beta_0 X \longrightarrow I$ with $g(t)=1$ and $g(\beta_0 X - B)=0$. Clearly $g^{-1}\left(]\frac{1}{2}, 1]\right)$ is an increasing open neighborhood of t. Let $V = g^{-1}\left([\frac{1}{2}, 1]\right) \cap X$, then $V \in \mathscr{F}(t)$. Let $h : I \longrightarrow I$ be the continuous isotone such that $h\left([\frac{1}{2}, 1]\right) = 1$ and h is linear on $\left[0, \frac{1}{2}\right]$ onto $[0, 1]$. Let $f = h \circ (g|X : X \longrightarrow I$, then f is a continuous isotone with $f(V)=1$ and $f(X-U)=0$. Using the dual arguments to those in the above, we can claim that $(\mathscr{F}(t), \mathscr{G}(t))$ is a completely regular bifilter on X. Take any pair A, B of increasing open sets such that there is a continuous isotone $k : X \longrightarrow I$ with $k(A)=1$ and $k(X-B)=0$. Let $\bar{k} : \beta_0 X \longrightarrow I$ be the extension of k, and $\bar{k}(t)=p$. If $p=1$, then $\bar{k}^{-1}(]0, 1])$ is an open increasing neighborhood of t. Let $U = \bar{k}^{-1}(]0, 1]) \cap X$, which is a member of $\mathscr{F}(t)$ and $U \subseteq B$. Thus B is also a member of $\mathscr{F}(t)$. If $p<1$, then take r with $p<r<1$. Since $\bar{k}^{-1}([0, r[)$ is a decreasing open neighborhood of t, $\bar{k}^{-1}([0, r[) \cap X$ is a member of $\mathscr{G}(t)$ and $\bar{k}^{-1}([0, r[) \cap X \cap A = \phi$. Using the dual argument, one can show that $(\mathscr{F}(t), \mathscr{G}(t))$ satisfies the conditions in Proposition 1.3. Hence $(\mathscr{F}(t), \mathscr{G}(t))$ is a maximal completely regular bifilter on X.

2.2 COROLLARY *Let be a completely regular ordered space. Then one has,*
a) for each $x\in X$, $(\vartheta(x), \mathscr{D}(x))$ is a maximal completely regular bifilter on X,
b) a maximal completely regular bifilter $(\mathscr{F}, \mathscr{G})$ converges to x on X iff $(\mathscr{F},)\mathscr{G}=(\vartheta(x), \mathscr{D}(x))$.

PROOF: a) It is immediate from the above proposition, for $(\vartheta(x), \mathscr{D}(x))$ is the trace bifilter of x on X.

b) If $(\mathscr{F}, \mathscr{G})$ converges to x, then $(\mathscr{F}\vee\vartheta(x), \mathscr{G}\vee\mathscr{D}(x))$ is by Remark 1.4, a completely regular bifilter. Thus the above two bifilters are identical with $(\vartheta(x), \mathscr{D}(x))$, for $(\mathscr{F}, \mathscr{G})$ and $(\vartheta(x), \mathscr{D}(x))$ are both maximal completely regular bifilters on X. The converse is trivial.

2.3 THEOREM *Let X be a completely regular ordered space. A bifilter on X is a maximal completely regular bifilter iff it is a trace bifilter of some point of β_0X on X.*

PROOF: Suppose $(\mathscr{F}, \mathscr{G})$ is a maximal completely regular bifilter on X. If $(\mathscr{F}, \mathscr{G})$ is convergent to x in X, then $(\mathscr{F}, \mathscr{G})=(\vartheta(x), \mathscr{D}(x))$ and hence it is the trace bifilter of x on X. Suppose $(\mathscr{F}, \mathscr{G})$ is not convergent on X. Since β_0X is compact, $(\mathscr{F}, \mathscr{G})$ has a clutster point in β_0X, say t. In fact, $(\mathscr{F}, \mathscr{G})$ converges to t on β_0X. To prove this, it is enough to show that t is the unique cluster point of $\mathscr{F}\vee\mathscr{G}$. Suppose s is a cluster point of $\mathscr{F}\vee\mathscr{G}$ and $s\neq t$, then we may assume $s\not\leq t$. Since β_0X is a compact ordered space, there is a continuous isotone $g: \beta_0X\longrightarrow I$ with $g(t)=0$ and $g(s)=1$. Let $A=g^{-1}\left(]\frac{1}{2}, 1]\right)\cap X$, $B=g^{-1}\left(]\frac{1}{4}, 1]\right)\cap X$, and $h: I\longrightarrow I$ be the continuous isotone such that $h\left([0, \frac{1}{4}]\right)=0$, $h\left([\frac{1}{2}, 1]\right)=1$ and h is linear on $\left[[\frac{1}{4}, \frac{1}{2}]\right]$ onto $[0, 1]$. If we put $f=h\circ(g|X)$, then we have $f(A)=1$ and $f(X-B)=0$. Since $(\mathscr{F}, \mathscr{G})$ is maximal, by Proposition 1.3, we have either $B\in\mathscr{F}$ or $A\cap F\cap G=\phi$ for some $F\in\mathscr{F}$ and some $G\in\mathscr{G}$. If $B\in\mathscr{F}$, then $g^{-1}\left([0, \frac{1}{4}[\right)$ is a neighborhood of t and $B\cap g^{-1}\left([0, \frac{1}{4}[\right)=\phi$, which is a contradiction to the fact that t is a cluster point of $\mathscr{F}\vee\mathscr{G}$. If $A\cap F\cap G=\phi$ for some $F\in\mathscr{F}$ and $G\in\mathscr{G}$, then $g^{-1}\left(]\frac{1}{2}, 1]\right)$ is a neighborhood of s and $g^{-1}\left(]\frac{1}{2}, 1]\right)\cap F\cap G=A\cap F\cap G=\phi$, which is again a contradiction to the fact that s is a cluster point of $\mathscr{F}\vee\mathscr{G}$. In all, t is the unique cluster point of $\mathscr{F}\vee\mathscr{G}$. Since $\mathscr{F}\vee\mathscr{G}\longrightarrow t$ on β_0X, for any increasing open neighborhood U of t in β_0X and any decreasing open neighborhood V of t in β_0X, $U\cap V$ contains $F\cap G$ for some $F\in\mathscr{F}$ and $G\in\mathscr{G}$, so that $U\cap V\cap X$ meets every member of $\mathscr{F}\vee\mathscr{G}$. Thus by Remark 1.4, $(\mathscr{F}\vee\mathscr{F}(t),\ \mathscr{G}\vee\mathscr{G}(t))$ is again a completely regular bifilter containing the maximal completely regular bifilters $(\mathscr{F},\ \mathscr{G})$ and $(\mathscr{F}(t),\ \mathscr{G}(t))$, so that $(\mathscr{F},\ \mathscr{G})=\mathscr{F}(t),\ \mathscr{G}(t))$. The converse is simply Proposition 2.1.

2.4 REMARK In [4], β_0X is characterized by the strict extension of X in the sense of [1] with the set of all maximal completely regular filters on X as the filter trace. But the order on β_0X in the above characterization is given as follows: for maximal completely regular filters $\mathscr{F}$ and $\mathscr{G}$ on X, $\mathscr{F}\leq\mathscr{G}$ iff for any continuous isotone $f: X\longrightarrow I$, $\lim f(\mathscr{F})\leq\lim f(\mathscr{G})$. Using Thoerem2.3, β_0X is characterized by the strict extension of X in our sense [2] with the set of all maximal completely regular bifilters as its trace open bifilters. In the latter case, the order of β_0X is more easily determined (see [3]).

2.5 COROLLARY *Let X be a completely regular orderd space. Then the following are equiealent:*

a) X is compact.

b) Every completely regular bifilter on X has a cluster point.

c) Every maximal completely regular bifilter on X is coovergent.

PROOF: a)$\Longrightarrow$b) It is immediate.

b)$\Longrightarrow$c) Let $(\mathscr{F},\ \mathscr{G})$ be a maximal completely regular bifilter and x a cluster point of $(\mathscr{F},\ \mathscr{G})$. Then by Remark 1.4, $(\mathscr{F}\vee\vartheta(x),\ \mathscr{G}\vee\mathscr{D}(x))$ is a completely regular bifilter containing $(\mathscr{F},\ \mathscr{G})$ and $(\vartheta(x),\ \mathscr{D}(x))$. Thus $(\mathscr{F},\ \mathscr{G})=(\vartheta(x),\ \mathscr{D}(x))$, so that $(\mathscr{F},\ \mathscr{G})$ converges to x.

c)$\Longrightarrow$a) For any $t\in\beta_0X$, let $(\mathscr{F}(t),\ \mathscr{G}(t))$ be the trace bifilter of t on X. By the above theorem, it is a maximal completely regular bifilter. Hence $(\mathscr{F}(t),\ \mathscr{G}(t))$ converges to some $x\in X$. Since β_0X is a Hausdorff space, we have $t=x\in X$. Thus $X=\beta_0X$ is compact.

2.6 REMARK Let T be a completely regular ordered space and X a dense subspace of T. Then the inclusion map $j: X\hookrightarrow T$ is I-extendable, or equivalently COS-extendable iff T is a sub-space of β_0X, or equivalently $\beta_0X=\beta_0T$. Thus if $j: X\hookrightarrow T$ is I-extendable, then each point of T should be the limit of a unique maximal completely regular bifilter on X.

3. k-Compact ordered spaces

In what follows, k will denote a regular infinite cardinal. Using the k-closure [7], the concept of k-compact ordered spaces has been introduced in [4], namely a completely regular ordered space is said to be k-compact if it is k-closed in its Nachbin compactification. It is known [4] that the category $kCOS$ of k-compact ordered spaces is epireflective in $CROS$ and that for any $X \in CROS$, the $kCOS$-reflection $\beta_0^k X$ of X is given by the k-closure of X in $\beta_0 X$. We note that $\beta_0^k X$ is again a completely regular ordered extension of X and hence it is given by the strict extension of X.

3.1. DEFINITION Let $(\mathscr{F}, \mathscr{G})$ be a bifilter on an ordered set. Then $(\mathscr{F}, \mathscr{G})$ is said to have the *k-intersection property* if for any subfamily $(U_i)_{i\in\Lambda}$ of $\mathscr{F}$ and $(V_j)_{j\in\Gamma}$ with $|\Lambda| < k$ and $|\Gamma| < k$, we have $(\bigcap\{U_i | i\in\Lambda\})\cap(\bigcap\{V_j | j\in\Gamma\}) \neq \phi$.

Now we will show that for any $X \in CROS$, the trace open bifilters of $\beta_0^k X$ on X are exactly maximal completely regular bifilters with the k-intersection property.

3.2 THEOREM *Let X be a completely regular ordered space. A bifilter $(\mathscr{F}, \mathscr{G})$ on X is a trace open bifilter of some point of $\beta_0^k X$ on X iff it is a maximal completely regular bifilter with the k-intersection property.*

PROOF: Let $(\mathscr{F}, \mathscr{G})$ be a trace open bifilter of $t \in \beta_0^k X$ on X. Since $\beta_0^k X$ is subspace of $\beta_0 X$ with the k-closure of X as its underlying set, $(\mathscr{F}, \mathscr{G})$ is also the trace open bifilter of $t \in \beta_0 X$ on X. Hence by Theorem 2.3, $(\mathscr{F}, \mathscr{G})$ is a maximal completely regular bifilter on X. Take any family $(U_i)_{i\in\Lambda}$ in $\mathscr{F}$ ($(V_j)_{j\in\Gamma}$ in $\mathscr{G}$, resp.) with $|\Lambda| < k$($|\Gamma| < k$, resp.). Then one has a subfamily $(A_i)_{i\in\Lambda}$ of $\mathscr{I}_{\beta_0 X}(t)$ and a subfamily $(B_j)_{j\in\Gamma}$ of $\mathscr{D}_{\beta_0 X}(t)$ such that for each i and j, $A_i \cap X = U_i$ and $B_j \cap X = V_j$. Since $(\bigcap A_i)\cap(\bigcap B_j)$ is a k-neighborhood of t in $\beta_0 X$, $(\bigcap A_i)\cap(\bigcap B_j)\cap X \neq \phi$, i.e., $(\bigcap U_i)\cap(\bigcap V_j) \neq \phi$. Thus $(\mathscr{F}, \mathscr{G})$ is a maximal completely regular bifilter with the k-intersection property. Conversely, let $(\mathscr{F}, \mathscr{G})$ be a maximal completely regular bifilter with the k-intersection property, Then there is a $t \in \beta_0 X$ such that $(\mathscr{F}, \mathscr{G})$ is the trace open bifilter of t on X. Take any subfamily $(A_i)_{i\in\Lambda}$ of $\mathscr{I}_{\beta_0 X}(t)$ and a subfamily $(B_i)_{i\in\Lambda}$ of $\mathscr{D}_{\beta_0 X}(t)$ with $|\Lambda| < k$. Since $A_i \cap X \in \mathscr{F}$ and $B_i \cap X \in \mathscr{G}$ for all $i \in \Lambda$, and $(\mathscr{F}, \mathscr{G})$ has the k-intersection property, one has $\bigcap\{A_i \cap B_i \cap X | i \in \Lambda\} \neq \phi$. Thus t

belongs to the k-closure of X in $\beta_0 X$, i.e., $t \in \beta_0^k X$ so that $(\mathscr{F}, \mathscr{G})$ is the trace open bifilter of $t \in \beta_0^k X$ on X.

Using the same argument as that in Corollary 2.5, one has the following.

3.3 COROLLARY *A completely regular ordered space X is a k-compact ordered space iff every maximal completely regular bifilter on X with the k-intersection property is convergent.*

3.4 REMARK Let ZCO be the category of O-dimensional compact ordered spaces and continuous isotones. For a O-dimensional ordered space X, the ZCO-reflection $\zeta_0 X$ of X is given by the strict extension of X with the set of all maximal clopen bifilters on X as the trace open bifilters (see [6]). Using the exactly same argument as that in the above theorem, the $kZCO$-reflection $\zeta_0^k X$ of X is given by the strict extension of X with the set of maximal clopen bifilters with the k-intersection property on X as the trace open bifilters, where $kZCO$ is the category of O-dimensional k-compact ordered spaces. We omit the detail of the proof.

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