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Hypersurfaces with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-Structure of a Product of Two Spheres with Same Dimension

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§ 0. Introduction

Recently, K. Yano ([6]) introduced the so-called (f, g, u, v, λ)-structure on a product of two spheres $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ as a submanifold of codimension 2 of a $(2n+2)$-Euclidean space.

Using this structure, S.-S. Eum, Y. H. Kim and one of the present authors ([2]) investigated real hypersurfaces of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$, and deduced the $(f, g, u, v, w, \lambda, \mu, \nu)$-structure (See § 1) from the (f, g, u, v, λ)-structure defined on the ambient manifold.

In the present paper, we define the normality of the $(f, g, u, v, w, \lambda, \mu, \nu)$-structure and prove some characterizations of the hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

§ 1. Hypersurfaces of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$

Let E^{n+1} be an $(n+1)$-Euclidean space and O the origin of the Cartesian coordinate system in E^{n+1}, and denote by X the position vector of a point in E^{n+1} with respect to the origin.

We consider a sphere $S^n(1/\sqrt{2})$ with center at O and radius $1/\sqrt{2}$ and suppose that $S^n(1/\sqrt{2})$ is covered by a system of coordinate neighborhoods $\{U: x^a\}$, where here and in the sequel the indices a, β, γ and δ run over the range $\{1, 2, \cdots, n\}$.

We next suppose that $S^n(1/\sqrt{2})$ is also covered by a system of coordinate neighborhoods $\{V: y^\kappa\}$ and denote by Y the position vector as above. Here and in the sequel the indices κ, μ, ν and τ running over the range $\{n+1, n+2, \cdots, 2n\}$.

Now we put

(1.1) $\quad X_a = \partial_a X, \qquad Y_\kappa = \partial_\kappa Y,$

where $\partial_a = \partial/\partial x^a$, $\partial_\kappa = \partial/\partial y^\kappa$, the position vector X and Y a point on $S^n(1/\sqrt{2})$

satisfies respectively

(1.2) $\quad X\cdot X=\frac{1}{2}, \qquad Y\cdot Y=\frac{1}{2},$

the dot means the inner product of two vectors in a Euclidean space.

Giving the differential structure to $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ naturally as a product manifold which is covered by a system of coordinate neighborhoods $\{U\times V: (x^a, y^\kappa)\}$, $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ as a submanifold of codimension 2 in a $(2n+2)$-dimensional Euclidean space has a position vector Z of a point in $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ such that

(1.3) $$Z(x^h)=\begin{pmatrix} X(x^a) \\ Y(y^\kappa)\end{pmatrix},$$

where here and in the sequel, the indices h, i, j and k run over the range $\{1, 2, \cdots, n, n+1, \cdots, 2n\}$.

Thus, using (1.2) we see that $Z\cdot Z=1$, which shows that $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ is hypersurface of $S^{2n+1}(1)$ in E^{2n+2}.

Putting $Z_i=\partial_i Z$, $\partial_i=\partial/\partial x^i$ we see that Z_i are $2n$ linearly independent vectors tangent to $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ and the induced Riemannian metric on $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ is given by

(1.4) $\quad g_{ji}=Z_j\cdot Z_i.$

Now putting

(1.5) $$C=\begin{pmatrix} -X(x^a) \\ -Y(y^\kappa)\end{pmatrix}, \qquad D=\begin{pmatrix} -X(x^a) \\ Y(y^\kappa)\end{pmatrix},$$

then we easily see that

(1.6) $\quad Z_i\cdot C=0,\ Z_i\cdot D=0,\ C\cdot D=0,\ C\cdot C=1,\ D\cdot D=1$

and consequently C and D also mutually orthogonal unit normal to $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

Let h_{ji} and k_{ji} be the second fundamental tensor with respect to C and D respectively. Then we have (See [6])

(1.7) $\quad h_{ji}=g_{ji},\quad k_{ji}g^{ji}=0,\quad k_j{}^t h_t{}^i=\delta_j^i,$

where $k_j{}^i=k_{jt}g^{ti}$, $(g^{ji})^{-1}=(g_{ji})$. So that the tensor $k_j{}^i$ defines an almost product structure of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

Denoting by ∇_j the covariant differentiation with respect to g_{ji}, we find ([6])

(1.8) $\quad \nabla_j k_i{}^h=0$

and

(1.9) $\quad \nabla_j Z_i = g_{ji}C + K_{ji}D, \quad \nabla_j C = -Z_j, \quad \nabla_j D = -k_j^h Z_h,$

which mean equations of the Gauss and Weingarten of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ respectively.

From (1.9) we can easily derive

(1.10) $\quad K_{kji}{}^h = \delta_k^h g_{ji} - \delta_j^h g_{ki} + k_k^h k_{ji} - k_j^h k_{ki},$

$K_{kji}{}^h$ being the curvature tensor of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

Since E^{2n+2} has a natural Kaehlerian structure F, the transform Z_j, C and D by F are respectively given by

(1.11) $\quad FZ_j = f_j^h Z_h + u_j C + v_j D, \quad FC = -u^h Z_h + \lambda D, \quad FD = -v^h Z_h - \lambda C,$

where f_j^h is a tensor field of type (1, 1), u_i and v_i 1-forms and λ the function on $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$, u^h and v^h are respectively given by $u^h = u_j g^{jh}$, $v^h = v_j g^{jh}$.

Applying F to (1.11) respectively, we obtain the so-called (f, g, u, v, λ)-structure given by (See [6] and [8])

$$(1.12)\quad \begin{cases} f_j^t f_t^h = -\delta_j^h + u_j u^h + v_j v^h, \\ u_t f_j^t = \lambda v_j, \quad f_t^h u^t = -\lambda v^h, \\ v_t f_j^t = -\lambda u_j, \quad f_t^h v^t = \lambda u^h, \\ u_t u^t = v_t v^t = 1-\lambda^2, \quad u_t v^t = 0, \\ g_{ts} f_j^t f_i^s = g_{ji} - u_j u_i - v_j v_i. \end{cases}$$

It is well known that the (f, g, u, v, λ)-structure induced on $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ satisfies the following relationships ([6]):

(1.13) $\quad k_t^h f_j^t + f_t^h k_j^t = 0, \quad k_t^h u^t = -v^h, \quad k_t^h v^t = -u^h.$

Now, differentiating (1.11) covariantly and taking account of (1.9), $\nabla F = 0$ and original equations, we find ([6])

$$(1.14)\quad \begin{cases} \nabla_j f_i^h = -g_{ji}u^h + \delta_j^h u_i - k_{ji}v^h + k_j^h v_i, \\ \nabla_j u_i = f_{ji} - \lambda k_{ji}, \quad \nabla_j v_i = -k_{jt}f_i^t + \lambda g_{ji}, \quad \nabla_j \lambda = -2v_j. \end{cases}$$

Let M be a hypersurface immersed isometrically in $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ by the immersion $i: M \to S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ and suppose that M is covered by a system of coordinate neighborhoods $\{W: \eta^a\}$. Throughout this paper the indices a $b, c, d, \cdots$ run over the range $\{1, 2, \cdots, 2n-1\}$.

Letting $B_c^h=\partial_c x^h$, $(\partial_c=\partial/\partial\eta^c)$ then B_c^h are $2n-1$ linearly independent vectors of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ tangent to M.

We denote by N^h the unit normal to M, then the fundamental metric tensor g_{cb} of M is given by

$$g_{ji}B_c^j B_b^i=g_{cb}$$

since the immersion is isometric.

As to transform of B_c^j and N^j by f_j^h, we can respectively write down

$$f_j^h B_c^j=f_c^a B_a^h+w_c N^h,\quad f_j^h N^j=-w^a B_a^h, \tag{1.15}$$

where f_c^a is the components of a tensor field f of type (1.1), w_c components of 1-form and $w^a=w_c g^{ac}$, g^{bc} being the contravariant components of g_{cb}.

Also, we may put in each coordinate neighborhood as follows :

$$u^h=u^a B_a^h+\mu N^h,\quad v^h=v^a B_a^h+\nu N^h, \tag{1.16}$$

$$k_j^h B_c^j=k_c^a B_a^h+k_c N^h,\quad k_j^h N^j=k^a B_a^h+\theta N^h, \tag{1.17}$$

where k_c^a is the components of a tensor field k of type (1, 1), u^a, v^a and k^a the components of a vector field respectively, μ, ν and θ are certain functions on M, k_c being the associated 1-form with the vector k^c.

Applying f_k^h to (1. 15) and (1. 16) respectively, and making use of (1. 11) and these equations, we find the so-called $(f, g, u, v, w, \lambda, \mu, \nu)$-structure as follows ([2], [3], [7]) :

$$\left\{\begin{array}{ll}
f_c^e f_e^a=-\delta_c^a+u_c u^a+v_c v^a+w_c w^a, & (1.18)\\
f_e^a u^e=-\lambda v^a+\mu w^a,\quad u_e f_c^e=\lambda v_c-\mu w_c, & \\
f_e^a v^e=\lambda u^a+\nu w^a,\quad v_e f_c^e=-\lambda u_c-\nu w_c, & (1.19)\\
f_e^a w^e=-\mu u^a-\nu v^a,\quad w_e f_c^e=\mu u_c+\nu v_c, &
\end{array}\right.$$

$$\left\{\begin{array}{l}
u_e u^e=1-\lambda^2-\mu^2,\quad u_e v^e=-\mu\nu,\quad u_e w^e=-\lambda\nu,\\
v_e v^e=1-\lambda^2-\nu^2,\quad v_e w^e=\lambda u,\\
w_e w^e=1-\mu^2-\nu^2,
\end{array}\right. \tag{1.20}$$

$$f_c^e f_b^d g_{ed}=g_{cb}-u_c u_b-v_c v_b-w_c w_b, \tag{1.21}$$

where u_c and v_c are 1-forms associated with u^a and v^a respectively. The last expression follows from (1. 12), (1. 15) and (1. 16).

If we apply k_i^h to (1. 17) and taking account of (1. 7) and original equations, we obtain

(1. 22) $k_c{}^e k_e{}^a = \delta_c{}^a - k_c k^a,$

(1. 23) $k_{ce} k^e = -\theta k_c, \quad k_e k^e = 1 - \theta^2.$

Transforming (1. 17) by f_i^h and remembering (1. 13), (1. 15) and (1. 17), we get

(1. 24) $k_c{}^e f_e{}^a + f_c{}^e k_e{}^a = k_c w^a - w_c k^a,$

(1. 25) $k_{ce} w^e + f_{ce} k^e = -\theta w_c.$

If we transvect (1. 17) with u^h and v^h successively and take account of (1. 13) (1. 16) and (1. 17) itself, then we have respectively

(1. 26) $k_{ce} u^e = -v_c - \mu k_c, \quad k_{ce} v^e = -u_c - \nu k_c,$

(1. 27) $k_e u^e = -(\nu + \theta\mu), \quad k_e v^e = -(\mu + \theta\nu).$

Putting $f_{cb} = f_c{}^a g_{ab}$, $k_{cb} = k_c{}^a g_{ab}$, we can easily verify that f_{cb} is skewsymmetric and k_{cb} is symmetric.

We denote ∇_c by the operator of the van der Waerden-Bortolotti covariant differentiation, we can write down the equations of Gauss and Weingarten respectively

(1. 28) $\nabla_c B_b{}^h = l_{cb} N^h, \quad \nabla_c N^h = -l_c{}^a B_a{}^h,$

where $l_c{}^a = l_{cb} g^{ba}$, l_{cb} being the components of the second fundamental form l with respect to the unit normal N^h.

Thus, the equations of Gauss and Codazzi are respectively given by

(1. 29) $k_{dcb}{}^a = \delta_d{}^a g_{cb} - \delta_c{}^a g_{db} + k_d{}^a k_{cb} - k_c^a k_{db} + l_d{}^a l_{cb} - l_c{}^a l_{db},$

(1. 30) $\nabla_d l_{cb} - \nabla_c l_{db} = k_d k_{cb} - k_c k_{db}$

because of (1. 10) and (1. 28), where $K_{dcb}{}^a$ being the components of curvature tensor of M.

Differentiating (1. 15) ~ (1. 17) covariantly along M and taking account of (1.8), (1. 14), (1. 28) and original equations, we have respectively ([2])

(1. 31) $\nabla_c f_b{}^a = -g_{cb} u^a + \delta_c{}^a u_b - k_{cb} v^a + k_c{}^a v_b - l_{cb} w^a + l_c{}^a w_b,$

(1. 32) $\nabla_c u_b = -\lambda k_{cb} + \mu l_{cb} + f_{cb},$

(1. 33) $\nabla_c v_b = -k_{ce} f_b{}^e - k_c w_b + \nu l_{cb} + \lambda g_{cb},$

(1. 34) $\nabla_c w_b = -\mu g_{cb} - \nu k_{cb} + k_c v_b - l_{ce} f_b{}^e,$

(1. 35) $\nabla_c \lambda = -2 v_c, \quad \nabla_c \mu = w_c - \lambda k_c - l_{ce} u^e, \quad \nabla_c \nu = k_{ce} w^e - l_{ce} v^e,$

(1. 36) $\nabla_c k_b{}^a = l_{cb} k^a + l_c{}^a k_b,$

(1. 37) $\nabla_c k_b = -k_{be} l_c{}^e + \theta l_{cb}$,

(1. 38) $\nabla_c \theta = -2\, l_{ce} k^e$.

Now, we introduce the following theorems for later use.

Theorem A ([2]). *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n>1)$ with $(f, g, u, v, w, \lambda, \mu, \nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2=1$, then we have $\mu=0$. Moreover, if $\nu^2=1$, then $(f_c{}^a, g_{cb}, k_c)$ defines a Sasakian structure and M is a minimal C-Einstein manifold.*

Theorem B ([2]. *Under the same assumptions as those stated in Theorem A, M as a submanifold of codimension 3 of a $(2n+2)$-Euclidean space is an intersection a complex cone with generator C and $(2n+1)$-unit sphere.*

Finally we prepare a useful lemma.

Lemma 1.3 *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n>1)$. Then the 1-form u_c is nonzero on an open set in M, and if the function $\lambda=0$ on an open of M, then $\nu^2=1$, $\mu=0$ and $\theta=0$ on the set.*

Proof. Suppose that the 1-form $u_c=0$ on an open set M_0 in M. Then the first equation (1. 20) and (1. 32) becomes respectively

(1. 39) $1-\mu^2-\lambda^2=0$,

(1. 40) $-\lambda k_{cb}+\mu l_{cb}+f_{cb}=0$

on M_0. If we take the skew-symmetric part of (1. 40), we obtain $f_{cb}=0$ on M_0 because k_{cb} and l_{cb} are symmetric and f_{cb} is skew-symmetric. Thus we see from (1. 18) that

$$-\delta_c{}^a + v_c v^a + w_c w^a = 0$$

on M_0. Contracting this a and c and taking account of (1. 20) and (1. 39), we find $(n-1)+\nu^2=0$ on M_0 and hence $n<1$. It contradicts the fact that $n>1$. So that $u_c \neq 0$ on M_0.

In the next plase, if the function λ vanishes on an open set M_1 of M, then the first equation of (1. 35) is turned out to be $v_c=0$ on M_1 and consequently $\nu^2=1$ because of the fact that $0=v_e v^e=1-\lambda^2-\nu^2$.

So the last equation of (1. 20) implies $\mu=0$ on M_1.

Consequently we have $\theta\nu=0$ on M_1 because of the second equation of (1. 27) and hence $\theta=0$ since $\nu^2=1$. Thus the proof of this lemma is completed.

§ 2. Normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure on the hypersurface.

We now define a tensor field S of type (1, 2) on the hypersurface M of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ as follows :

$$S_{cb}{}^a=f_c{}^e\nabla_e f_b{}^a-f_b{}^e\nabla_e f_c{}^a-(\nabla_c f_b{}^e-\nabla_b f_c{}^e)f_e{}^a+(\nabla_c u_b-\nabla_b u_c)u^a$$
$$+(\nabla_c v_b-\nabla_b v_c)v^a+(\nabla_c w_b-\nabla_b w_c)w^a.$$

When S vanishes identically, the $(f, g, u, v, w, \lambda, \mu, \nu)$-structure is said to be normal ([7]).

We suppose, in the sequel, that the induced structure on M is normal. Then, by substituting (1. 24) and (1. 31)~(1. 34) into the last equation, we find

$$R_{ac}w_b-R_{ab}w_c=(k_{ae}f_b{}^e+k_{be}f_a{}^e)v_c-(k_{ae}f_c{}^e+k_{ce}f_a{}^e)v_b$$
$$+(k_{be}f_c{}^e-k_{ce}f_b{}^e)v_a+(k_b v_c-k_c v_b)w_a, \tag{2. 1}$$

where we have put

$$R_{cb}=l_{ce}f_b{}^e+l_{be}f_c{}^e, \tag{2. 2}$$

Contracting a and b in (2. 1), we obtain

$$R_{ce}w^e=(\tau+2\lambda)v_c+\lambda\mu k_c-2\nu k_{ce}w^e, \tag{2. 3}$$

where we have used (1. 19), (1. 20), (1. 26) and put

$$\tau=k_e w^e. \tag{2. 4}$$

If we transvect (2. 1) with w^b and make use of (1. 19), (1. 20) and (2. 3), we get

$$\begin{cases}(1-\mu^2-\nu^2)R_{ac}+\lambda\mu(k_{ae}f_c{}^e+k_{ce}f_a{}^e)-k_{be}w^b(f_a{}^e v_c+f_c{}^e v_a)\\ =\tau(v_a w_c+v_c w_a)+2\lambda v_a w_c+\lambda\mu(k_a w_c-k_c w_a)-2\nu(k_{ae}w^e)w_c\\ +\nu(u_a v_c-u_c v_a)+(\mu^2+\nu^2)(k_a v_c-k_c v_a),\end{cases} \tag{2. 5}$$

from which, taking the skew-symmetric part with respect to indices a and c,

$$\nu(k_{ae}w^e)w_c-\nu(k_{ce}w^e)w_a=\lambda(v_a w_c-v_c w_a)+\lambda\mu(k_a w_c-k_c w_a)$$
$$+\nu(u_a v_c-u_c v_a)+(\mu^2+\nu^2)(k_a v_c-k_c v_a). \tag{2. 6}$$

On the other hand, transvecting (1. 24) with w^b and remembering (1. 19), (1. 20) and (1. 27), we get

$$k_{cb}w^c w^b=-\theta-2\mu\nu. \tag{2. 7}$$

Transvecting (2.6) with w^c and using (1.20), (2.4) and (2.7), we have

$$(2.8)\quad \begin{cases} \nu(1-\mu^2-\nu^2)\, k_{ce}w^e = \lambda\mu\nu u_c + \lambda\mu k_c + \{\lambda(1-\mu^2) - \tau(\mu^2+\nu^2)\}\, v_c \\ \qquad\qquad - \{\theta\nu + 2\mu\nu^2 + \lambda^2\mu + \lambda\mu\tau\}\, w_c. \end{cases}$$

If we transvect u^c to (2.8) and make use of (1.20), (1.26) and (2.4), then we obtain

$$(2.9)\quad \mu\nu\tau(1+\lambda^2) = \lambda(\theta+\mu\nu)(\mu^2-\nu^2).$$

Applying also v^c and k^c to (2.8) successively, we get respectively

$$(2.10)\quad \tau(\lambda^2\nu^2-\mu^2) = \lambda(-1+\lambda^2+2\mu^2+2\nu^2+2\theta\mu\nu-\nu^4+\mu^2\nu^2).$$

$$(2.11)\quad \mu\tau(\lambda^2+\nu^2-\mu^2+\lambda\tau) = \lambda(\mu^3-\theta\nu-\mu\nu^2-\mu\theta^2),$$

where we have used (1.20), (1.23) and (1.27).

First of all we prove

Lemma 2.1. *Let M be a hypersurface with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ $(n>1)$. Then the function μ vanishes identically if and only if the function θ so does.*

Proof. In the first place we suppose that the function μ vanishes identically, then (2.11) is turned out to be $\lambda\theta\nu=0$, which together with (2.10) gives $\lambda\theta(1-\lambda^2)=0$.

But, $\lambda(1-\lambda^2)$ does not vanish because of Lemma 1.3. Thus it follows that $\theta=0$ on M.

Conversely if the function θ vanishes on M, then the equations (2.9)~(2.11) reduce to

$$(2.12)\quad \mu\nu\tau(1+\lambda^2) = \lambda\mu\nu(\mu^2-\nu^2),$$

$$(2.13)\quad \tau(\lambda^2\nu^2-\mu^2) = \lambda(-1+\lambda^2+2\mu^2+2\nu^2-\nu^4+\mu^2\nu^2),$$

$$(2.14)\quad \mu\tau(\lambda^2+\nu^2-\mu^2+\lambda\tau) = \lambda\mu(\mu^2-\nu^2).$$

We also have from (1.38)

$$(2.15)\quad l_{ce}k^e = 0.$$

On the other hand, we have

$$(2.16)\quad (k_{de}w^d f_c{}^e v_b)\, k^c k^b = -\mu(k_{de}w^d)(k_c{}^e w^c) = -\mu(1-\mu^2-\nu^2-\tau^2)$$

with the aid of (1.22), (1.25), (1.27), (2.4), (2.7) and the fact that $\theta=0$.

If we transvect (2.5) with $k^a k^c$ and make use of (1.23), (1.27), (2.2), (2.

4), (2.15) and (2.16), we find

(2.17) $\mu(1-\mu^2-\nu^2+\lambda\tau)=0$,

where we have used the fact that $\theta=0$.

Combining (2.12)~(2.14) and (2.17) and taking account of Lemma 1.3, we can easily prove that μ vanishes on M. This completes the proof of the lemma.

Now, we suppose that the hypersurface M with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure has the following condition:

(2.18) $k_c{}^e l_e{}^a + l_c{}^e k_e{}^a = 0$.

Then, by transvecting $k^a k^c$ and using the first equation of (1.23), we obtain $\theta l_{cb} k^c k^b = 0$, which together with (1.38) gives

(2.19) $l_{cb} k^c k^b = 0$.

Transforming (2.18) by $k_d{}^c$ and remembering (1.22), we get

$$l_{db} - (l_{be} k^e) k_d + l_c{}^e k_d{}^c k_{be} = 0,$$

from which, taking the skew-symmetric part with respect to indices d and b,

$$(l_{be} k^e) k_d - (l_{de} k^e) k_b = 0.$$

If we transvect this with k^d and take account of (1.23) and (2.19), we get

$$(1-\theta^2) l_{be} k^e = 0.$$

Hence, it follows that θ is constant because of (1.38). Thus (2.15) is valid. Now, transforming (2.18) with $k_a{}^c$, we find

$$k^{cb} l_{ce} k_b{}^e = 0,$$

which together with (1.22) and (2.19) gives

(2.20) $l_e{}^e = 0$,

which shows that M is minimal.

On the other hand, transvecting the first equation of (1.17) with $B^b{}_h = B_c{}^j g^{bc} g_{jh}$ and taking account of the second equations of (1.7) and (1.17), we find

(2.21) $k_e{}^e = -\theta$.

If we transform (1.30) by g^{ab} and use (2.20) and (2.21), we obtain

(2.22) $\nabla^e l_{ce} = 0$.

Applying ∇^c to (2.15) and making use of (1.37) and (2.22), we get

$$l^{cb}(-k_{be} l_c{}^e + \theta l_{cb}) = 0$$

and consequently

(2.23) $\theta l_{cb} l^{cb} = 0$

because $k_{be} l_c^{\ e}$ is skew-symmetric.

We now suppose that the hypersurface M is totally geodesic, then (1.30) leads to

$$k_a k_{cb} - k_c k_{ab} = 0.$$

Transvecting the last equation with $k^a k^{cb}$ and remembering (1.22) and (1.23), we see that $1-\theta^2=0$, that is, $k_c=0$. So (1.25) becomes

$$k_{ce} w^e = -\theta w_c.$$

So that (2.6) leads to

$$\lambda(v_a w_c - v_c w_a) + \nu(u_a v_c - u_c v_a) = 0.$$

If we transvect $v^a w^c$ to this and make use of (1.20), we find

$$\lambda(1-\lambda^2-\mu^2-\nu^2) = 0.$$

But, in a consequence of Lemma 1.3, the function λ cannot be vanish on M. Consequently we get

$$\lambda^2+\mu^2+\nu^2 = 1.$$

From this fact and Theorem A in § 1, we see that μ vanishes on M. So we should have $\theta=0$ because of Lemma 2.1. This contradicts the fact that $\theta^2=1$.

Thus, it follows from (2.23) that the constant θ vanishes on M and hence μ so does because of Lemma 2.1. Therefore, by transvecting the second equation of (1.35) with k^c and remembering (2.4) and (2.15) gives the fact that $\tau=\lambda$.

Using these facts obtained above, the equation (2.10) is turned out to be

$$\lambda^3\nu^2 = \lambda(-1+\lambda^2+2\nu^2-\nu^4),$$

or equivalently $\lambda(1-\lambda^2)(1-\nu^2)=0$.

According to Lemma 1.3 and the last equation, we have $\nu^2=1$ on M.

Thus, due to Theorem A and Theorem B in § 1, we have

Theorem 2.2. *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n>1)$ with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure. If $k \cdot l + l \cdot k = 0$ holds at every point of M, then M is a minimal Sasakian C-Einstein space. Moreover, M as a submanifold of codimension 3 of a $(2n+2)$-Euclidean space is an intersection of a complex cone with generator C and $(2n+1)$-unit sphere.*

Finally we prove

Theorem 2. 3. *Let M be a compact hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure $(n>1)$. If the function μ has definite sign, then M is the same type of Theorem 2. 2.*

Proof. We have from (1. 34)

$$\nabla_e w^e = -2n\mu$$

with the aid of (1. 27) and (2. 21). Since the function μ has definite sign, apply the Green theorem, we have $\mu=0$ on M because the hypersurface is compact.

Therefore, the function θ vanishes identically by virtue of Lemma 2. 1. Thus, according to Theorem 2. 2, our assertion is true.

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