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Isotone Map in Boolean Rings

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In [1], K. Iseki proved that a homomorphism of BCK-Algebras is isotone. In this paper, we apply this conception—isotone map—to Boolean ring. An element x of a ring is idempotent if $x^2=x$. A Boolean ring is a ring with a unit in which every element is idempotent.

Example. Let R be the set of subsets of a set S. we define addition and multiplication in R as follows:

$X+Y=(X\cup Y)/(X\cap Y)=$all elements in $X\cup Y$ but not in $X\cap Y$. (This is called the symmetric difference of X and Y)

$X\cdot Y=X\cap Y$ for any $X, Y\in R$.

Then R is a Boolean ring.

A Boolean ring R has the following properties: (1) R has characteristic 2 (that is, $r+r=0$ for every r in R and (2) a Boolean ring R is commutative.

Let R be a Boolean ring with identity and M a unitary R-module. Define a relation $\leq$ in M by $x\leq y$ iff there exist $r\in R$ such that $rx=y$ for all $x, y\in M$.

The first purpose of this note is to show that the following theorem holds:

Theorem 1. *The relation $\leq$ in M is a partial order in M.*

For $x\in M$, $Rx=\{rx|r\in R\}$ is an R-module. Then the proof of the following theorem 2 is similiar to the proof of the corresponding theorem in ordinary ring theory and will be omitted.

Theorem 2. *The map $R\rightarrow Rx$ given by $r\rightarrow rx$ is an R-module epimorphism.*

The second purpose is to prove that the following holds:

Theorem 3. *The map in theorem 2 is isotone.*

The proof of theorem 1. Since R has an identity, $1x=x$ for all $x\in M$. So $x\leq x$. Let x and y be elements of M satisfying $x\leq y$ and $y\leq x$. Then there exist $r_1, r_2\in R$ such that $r_1x=y$, $r_2y=x$ and so $r_1r_2y=r_1x=y$. Also $r_1r_2y=r_1r_2x=r_2r_1=r_2y=x$. Hence $x=y$. Suppose $x\leq y$, $y\leq z$ for all $x, y, z\in M$. Then there exist $r_1, r_2\in R$ such that $r_1x=y$, $r_2y=z$. and so $r_2r_1x=z$. Since $r_2r_1\in R$, $x\leq z$.

The proof of theorem 3. Let $h: R\rightarrow Rx$ defined by $h(r)=rx$ for all $r\in R$ and let r_1 and r_2

in R satisfying $r_1 \leq r_2$. Then $r_1 r_2 = r_2$, and so $h(r_2) = r_2 x = (r_1 r_2) x = (r_2 r_1) x = r_2 (r_1 x) = r_2 h(r_1)$. Therefore $h(r_1) \leq h(r_2)$.

References

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