# NMR Chemical Shift for $\mathbf{4 d}^{\boldsymbol{*}}$ System (II). Calculation of the Pseudo Contact Shift for a $\mathbf{4 d}^{1}$ System in a Strong Crystal Field Environment of Octahedral Symmetry 

Sangwoon Ahn'. Se Woong Oh* and Euisuh Park<br>Department of Chemitry. Jeonbug National University, Jeonju 520, Korea (Recemed August 24, 1982)


#### Abstract

NMR shift arising from the electron orbital angular momentum and the electron spin dipolar-nuckar spin angular momentum interactions has been investigated for a $4 \mathrm{~d}^{1}$ system in a strong erystal fiekd of octahedral symmetry. To examine the NMR shift for a $4 \mathrm{~d}^{\prime}$ system in a strong crystal field of octahedral symmetry, we derive a gencral expression for $A B / B$ using a nonmultipole expansion tecinnique. From this expression all the multipolar terms are determined. For the $4 \mathrm{~d}^{\prime}$ system in a strong crystal field of octahedral symmetry the exact solution for NMR shift, $\Delta B$, is compared with the multipolar results. $\Delta \mathrm{B} / \mathrm{B}$ for the $4 \mathrm{~d}^{\mathrm{t}}$ system is also compared with that for the $3 \mathrm{~d}^{1}$ system. It is found that the $1 / \mathbf{R}^{7}$ term contributes dominanily to the NMR shift. However, there is good agreement between the nonnouttipole and multipolar results for $R$ values larger than 0.2 nm for the $4 \mathrm{~d}^{2}$ system but for $R$-values larger than 0.4 nm for the $3 \mathrm{~d}^{1}$ sysiem.


## 1. Introduction

For a decade considerable interest has been focussed on the NMR slifits arising from $3 \mathrm{~d}^{n}$ and $4 f^{n}$ systems ${ }^{1}$ and in the use of tanthanide ions as NMR shift reagents ${ }^{2}$. The NMR shift in $3 \mathrm{~d}^{n}$ and $4 \mathrm{f}^{\pi}$ paramagnetic systems has been interpreted as arising through the Fermi contact interaction between the electron bearing nucleus and the NMR nucleus ${ }^{3}$. For this case a simplified form of the expression for the NMR shift, JB, was derived (4),

$$
\begin{equation*}
\frac{d B}{b}=-\frac{a \mu_{R}}{3 g_{\mathrm{N}} \mu_{\mathrm{N}}}\left\{\frac{2}{\lambda}(g-1)(g-2)+(g-1) g \frac{J(J+1)}{k T}\right\} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& g=1+\{J(J+1)-L(L+1)+S(S+1)\} /[2 J(J+1)] \\
& \lambda= \pm \zeta / 2 S
\end{aligned}
$$

Here $\zeta$ is the spin-orbit coupling constant and only the ${ }^{2 S}+{ }^{11} L$, ground state was considered with no bonding effects.
In the other cases ${ }^{5}$ the NMR shifts have been interpreted as arising dominantly through pseudo contact interactions. The expression for NMR shifts first given by McConnel and Robertson ${ }^{6}$, adopting the dipolar approximation, was extended by Kurland and McGravey ${ }^{7}$ in terms of the magnetic susceptibility components, $\chi_{x a}$,

$$
\begin{align*}
-\frac{\Delta B}{B}= & -\frac{1}{3 R^{3}}\left\{\left\{\chi_{z z}-\frac{1}{2}\left(\chi_{x x}+\chi_{y y}\right)\right\}\left(3 \cos ^{2} \theta \cdots 1\right)\right. \\
& \left.+\frac{3}{2}\left(\chi_{x x}-\chi_{y y}\right) \sin ^{2} \theta \cos 2 \phi\right\} \tag{2}
\end{align*}
$$

This expression has been used extensively in interpreting the pscudo contact shift in paramagnetic $3 \mathrm{~d}^{n}$ and $4 \tilde{1}^{n}$ systems. Using the dipolar approximation expressed in terms of the magnetic susceptibility components, Bleaney derived an equation for the NMR shift ${ }^{N}, \Delta \mathrm{~B}$, which may be written as ${ }^{4}$

$$
\frac{\Delta B}{B}=\frac{N \mu_{B}^{2} g^{2}(2 l+1-4 S)[3 X(X-1)-4 J(J+1)(L+1)}{24 \sqrt{5}(2 l+1)(2 l-1)(2 L-1) \sqrt{\pi R^{3} k^{2} T^{2}}}
$$

[^0]$$
X=J(J+1)+L(L+1)-S(S+1)
$$

In equation (3), $l=3, a_{2}^{n}$ is a crystal field parameter, $N$ is the Avogadro constant. This equation can be applied to $4 f^{\circ}$ systems. As far as we are aware no previous attempt has been nude to determine the exact solution of the NMR shift arising from the 4 d -electron angular momentum and the 4 d -electron spin dipolar-nuclear spin angular momentum interactions and, in addition, to compare the results with the multipolar approximations which may be expressed as ${ }^{9}$

$$
\begin{equation*}
\frac{A B}{B}=\sum_{L-2}^{K} \sum_{M=0}^{L}\left(A_{L M} \cos M \phi+B_{L M} \sin M \phi\right) P_{L}^{M}(\cos \theta) / R^{L+1} \tag{4}
\end{equation*}
$$

where $K=2(2 l+1)$ for a specific $l$-electron, $P_{L}{ }^{M}(\cos \theta)$, the associated Legendre polynomials and the coefficients $A_{L A}$ and $B_{L M}$ measure the anisotropy in the multipolar magnetic susceptibilities of the molecule. The purpose of this work is to develop a method for calculating the NMR shift for a $4 \mathrm{~d}^{1}$ system in a strong crystal field environment of octahedral symmetry, adopting the nonmultipole expansion method ${ }^{10}$ and to compare this exact results with the multipotar NMR results. This enable us 10 explore the limitations of the multipolar expansion method in interpreting the NMR shifts in $4 \mathrm{~d}^{1}$ transition metal complexes. We also compare the NMR shift for a $4 \mathrm{~d}^{1}$ system with that of a $3 \mathrm{~d}^{1}$ system in a strong crystal field environment of octahedral symmetry.

## 2. Theory

The hamiltonian representing the variots interactions we shall consider in this paper may be expressed as

$$
\begin{align*}
\mathscr{M}= & -\frac{\hbar^{2}}{2 m_{c}} \nabla^{2}-\frac{Z e^{2}}{4 \pi e_{0} r}+V(r)+J l \cdot S \\
& +\mu_{B}(l+2 S) \cdot B+\mathscr{H}_{h /} \tag{5}
\end{align*}
$$

where $V(r)$ is a crystal field term and

$$
\begin{equation*}
\mathscr{R}_{h f}=\frac{\mu_{0}}{4 \pi} g_{\mathrm{N}} \mu_{B} \mu_{\mathrm{N}}\left\{\frac{2 I_{\mathrm{N}} \cdot \boldsymbol{I}}{r_{\mathrm{N}}^{3}}+g_{S}\left[\frac{3\left(\boldsymbol{r}_{\mathrm{N}} \cdot \boldsymbol{S}\right)}{r_{\mathrm{N}} \cdot \boldsymbol{r}_{\mathrm{N}} \cdot \boldsymbol{I}}-\frac{\boldsymbol{S} \cdot \boldsymbol{I}}{r_{\mathrm{N}}^{3}}\right]\right\} \tag{6}
\end{equation*}
$$

Here $r$ and $\boldsymbol{r}_{\mathrm{N}}$ are electron radius vector about the electronbearing atom and the nucleus with nuclear spin angular mo-
mentum $I$, respectively. The quantity $B$ is the applied magnetic field and the other terms have their usual meaning. In this paper $g s$, the frec electon $g$ value, is taken to be equal to exactly 2 . Since we are examining the interaction of a 4 d eleictron centered at $O$ in a strong crystal field of octahedral symmetry (refer to 1st series of this work,), we shall express the reqired wave functions as

$$
\begin{align*}
& \left\lvert\, 4 \mathrm{~d}_{y z}>=\left(\frac{\beta^{9}}{21 \pi}\right)^{\frac{1}{2}} y z r \exp (-\beta r)\right. \\
& \left\lvert\, 4 \mathrm{~d}_{x z}>=\left(\frac{\beta^{9}}{21 \pi}\right)^{\frac{1}{2}} x z r \exp (-\beta r)\right. \\
& \left|4 \mathrm{~d}_{x y}\right\rangle=\left(\frac{\beta^{9}}{21 \pi}\right)^{\frac{1}{2}} x y r \exp (-\beta r) \tag{7}
\end{align*}
$$

where $\beta=2.848 \mathrm{I} / \mathrm{a}_{10}{ }^{11}$.
To determine the NMR shift of a $4 \mathrm{~d}^{1}$ system in liquid solution, the principal values $\sigma_{x x}, \sigma_{y y}$ and $\sigma_{z x}$ of the NMR screening tensor, $\sigma$, are determined by considering the magnetic field, $B$, interaction as paraliet to the $x, y$ and $z$ axes and averaged assuming a Boltzmann distribution. The contribution to the NMR shift, $A B$, is given by

$$
\begin{equation*}
S B-{ }_{3}^{1} B\left(\sigma_{x x}+\sigma_{y y}+\sigma_{z z}\right) \tag{8}
\end{equation*}
$$

where

$$
\begin{align*}
\sigma_{a \alpha} & =\left(\frac{\partial^{2}<\mu_{k f} \geq}{\partial \mu_{r} \partial B_{a}}\right)_{H-B \cdot v}  \tag{9}\\
\mu & =g_{\mathrm{N} / \ell_{\mathrm{N}} I}
\end{align*}
$$

The term $\left\langle\mathcal{X}_{h f}\right\rangle$ refers to the Boltzmann average of the hyperfine interaction represented by equation (6),

$$
\begin{equation*}
<\mathscr{x}_{h f}>=\frac{\sum_{i}<\phi_{i}\left|\mathscr{R}_{h f}\right| \phi_{i}>\exp \left(-E_{i} / k T\right)}{\sum_{i} \exp \left(-E_{i} / k T\right)} \tag{10}
\end{equation*}
$$

where $s^{t}$ are the eigenfunctions of the spin-orbit coupling and magnetic ficid interactions and $E_{i}$ the corresponding eigenvalues. To evaluate the hyperfine interaction matrix elements, $\left\langle\phi_{j}^{\prime}\right| \mathcal{X}_{h f}\left|\psi_{i}\right\rangle$, we adopt the calculated hyperfine integrals for 4 d orbitals ${ }^{12}$. In a strong crystal field of octahedral symmetry, the NMR shift for a $4 d^{1}$ system is given by

$$
\begin{equation*}
\frac{A B}{13}=-\frac{\mu_{0}}{4 \pi} \frac{\mu_{\mathrm{B}}^{2}}{k T}\left\{\frac{d(R)+[1-\exp (3 \zeta / 2 k T)(k T / \zeta) S(R)]}{1+2 \exp (3 \zeta / 2 k T)}\right\} \tag{11}
\end{equation*}
$$

where

$$
\begin{aligned}
d(\boldsymbol{R})= & \frac{8 \sqrt{\pi}}{147} Y_{00}(\Theta, \Phi) E(t) \\
& +\frac{11520}{t^{5}} \sqrt{\frac{\pi}{21}}\left[\frac{1}{2} \sqrt{\frac{7}{3}} Y_{40}(\Theta, \Phi)\right. \\
& +\frac{1}{2} \sqrt{\frac{5}{6}}\left(Y_{4-4}(\theta, \Phi)+Y_{44}(\Theta, \Phi)\right] H(t) \\
& +\frac{15206400}{7 t^{7}} / \frac{\pi}{26}\left[\frac{1}{2 \sqrt{2}} Y_{60}(\Theta, \Phi)\right. \\
& -\sqrt{\frac{7}{4}}\left(Y_{6-4}(\theta, \Phi)+Y_{64}(\theta, \Phi)\right] J(t)
\end{aligned}
$$

$$
\begin{align*}
S(\boldsymbol{R})= & -\frac{40 \sqrt{\pi}}{441} Y_{00}(\theta, \Phi) M(t) \\
& -\frac{5120}{t^{5}} \sqrt{\frac{\pi}{21}}\left[\frac{1}{2} \sqrt{7} Y^{-}(\theta, \Phi)\right. \\
& \left.\left.+\frac{1}{2} \sqrt{6} Y_{4-1}(\theta, \Phi)+Y_{44}(\theta, \Phi)\right)\right] N(t) \\
& -\frac{10137600}{7 t^{7}} \sqrt{\frac{\pi}{26}}\left[\frac{1}{2 \sqrt{2}} Y_{60}(\theta, \Phi)\right. \\
& \left.-\sqrt{7}\left(Y_{6-4}(\Theta, \Phi)+Y_{61}(\theta, \Phi)\right)\right] J(t)  \tag{12}\\
& E(t)=\beta^{3} e^{-t}\left(\frac{1}{3} \frac{t^{6}}{6!}+\sum_{n 0}^{5} \frac{t^{n}}{n!}\right) \\
& H(t)=\beta^{3}\left\{1-e^{-t}\left(\frac{8}{9} \frac{t^{11}}{11!}+\sum_{n=0}^{10} \frac{t^{n}}{n!}\right)\right\} \\
& J(t)=\beta^{3}\left\{1-e^{-t}\left(\frac{13}{11} \frac{t^{13}}{13!}+\sum_{n=0}^{12} \frac{t^{n}}{n!}\right)\right\} \\
& M(t)=\beta^{3} e^{-t}\left(-\frac{2}{15} \frac{t^{6}}{6!}+\sum_{n=0}^{5}-\frac{t^{n}}{n!}\right) \\
& N(t)=\beta^{3} e^{-t}\left(\frac{4}{3} \frac{t^{11}}{11!}\right)
\end{align*}
$$

Since this expression for $A B / B$ is applicable for all values of $R$ we may determine from equation (11) the case when $R \rightarrow \mathrm{O}$, namely,

When $R$ is large, the terms in equation (11) involving the polar coordinates of the NMR nucleus may be expressed in terms of $1 / R^{5}$ and $1 / R^{7}$,

$$
\begin{align*}
d(R)= & \frac{1}{R^{5}} \frac{360}{\beta^{2}} \sqrt{\frac{\pi}{21}}\left\{\frac{1}{2} \sqrt{\frac{7}{3}} Y_{40}(\Theta, \Phi)\right. \\
& \left.+\frac{1}{2} \sqrt{6}\left[Y_{4-4}(\Theta, \Phi)+Y_{4}(\Theta, \Phi)\right]\right\} \\
& +\frac{1}{R^{7}} \frac{118800}{7 \beta^{4}} \sqrt{\frac{\pi}{26}}\left\{\frac{1}{2 \sqrt{2}} Y_{60}(\Theta, \Phi)\right. \\
& \left.-\frac{\sqrt{7}}{4}\left[Y_{6-4}(\Theta, \Phi)+Y_{64}(\Theta, \Phi)\right]\right\} \\
S(R)= & -\frac{1}{R^{7}} \frac{79200}{7 \beta^{4}} \sqrt{\frac{\pi}{26}}\left\{\frac{1}{2 \sqrt{2}}-Y_{60}(\Theta, \phi)\right. \\
& \left.-\frac{\sqrt{7}}{4}\left[Y_{6-4}(\theta, \Phi)+Y_{64}(\Theta, \Phi)\right]\right\} \tag{14}
\end{align*}
$$

In equation (12), (13) and (14), the symmetry of the system is reflected in the analytical form for $\Delta B / B$, namely, only those combinations of the spherical harmonics $Y_{l m}(\theta, \Phi)$ that transform as the irreducible representation $A_{1} a_{1}$ of the octahedral group occur.

## 3. Results and Discussion

The calculated NMR shifts along the $x, y$ and $z$ axes for a $4 d^{7}$ system in a strong crystal ficld environment of octahedral symmetry are listed in Table 1. Here we choose $\beta=$ $2.8481 / a_{0}$, the spin-orbit coupling constant as $1030 \mathrm{~cm}^{-1}$. These parameter values are similar to those for $\mathrm{Mo}^{5+}$ ion. The temperature is taken as $T=300 \mathrm{~K}$. As shown in Table 1 , $\Delta B / B$ for specific $R$-values along the $\langle 100\rangle,\langle 010\rangle$ and $\langle 001\rangle$ axes for a $4 \mathrm{~d}^{1}$ system in a strong crystal field environments of octahedral symmetry are consistent with each other. This result represents an isotropic NMR shielding for a $4 \mathrm{~d}^{\text { }}$ system

TABLE 1: $\Delta \mathrm{B} / \mathrm{B}(\mathrm{ppm})$ for Specific $R$-values for a 4d' System along the $x, y$ and $z$ axes in a Sirong Crystal Field Environment of Octahedral Symmetry

| $R$ (nm) | $\Delta B / B$ (ppni) |  |  |
| :---: | :---: | :---: | :---: |
|  | <100> axis | $\langle 010\rangle$ axis | <001> axis |
| 0.05 | -92044.3724 | -92044.3724 | -92044.3724 |
| 0.1 | -879.2927 | -879.2927 | -879.2927 |
| 0.15 | -60.9222 | -60.9233 | -60.9233 |
| 0.2 | $-8.4320$ | -8.4320 | -8.4320 |
| 0.25 | $-1.7866$ | $-1.7866$ | $-1.7866$ |
| 0.3 | -0.5036 | -0.5036 | $-0.5036$ |
| 0.35 | -0.1732 | -0.1732 | $-0.1732$ |
| 0.4 | $-0.0689$ | -0.0689 | -0.0689 |
| 0.45 | $-0.0307$ | -0.0307 | $-0.0307$ |
| 0.5 | $-0.0149$ | -0.0149 | -0.0149 |

TABLE 2: $\Delta B / B$ (ppm) for Specific $R$-Values for a $4^{1}{ }^{1}$ System in a Strong Field Environment of Octahedral Symmetry

| R (nm) | $\Delta B / B$ (ppm) |  |  |
| :---: | :---: | :---: | :---: |
|  | $\langle 100\rangle$ axis | $\langle 110\rangle$ axis | <111> axis |
| 0.05 | -992044.3724 | 147206.3179 | $-162486.2417$ |
| 0.1 | -879.2927 | 1216.9444 | -1401.0489 |
| 0.15 | $-60.9233$ | 91.3902 | -100.1516 |
| 0.2 | -8.4320 | 13.3953 | $-14.5252$ |
| 0.25 | $-1.7869$ | 2.8466 | $-3.0761$ |
| 0.3 | $-0.5036$ | 0.7963 | $-0.8561$ |
| 0.35 | $-0.1732$ | 0.2712 | -0.2897 |
| 0.4 | $-0.0689$ | 0.1069 | $-0.1132$ |
| 0.45 | $-0.0307$ | 0.0469 | -0.0493 |
| 0.5 | $-0.0149$ | 0.0225 | -0.02.34 |

Parameters used: $\zeta=1010 \mathrm{~cm}^{-1}, \beta=2,8481 / a_{0}$ and $T=300 \mathrm{~K}$.

TABLE 3: A Comparison of the Fxact values of $\Delta B / B$ (ppm) using Equation (t1) with the Multipolar Terms for Specific $R$-values along <100〉 Axis

| $R(\mathrm{~nm})$ | $\Delta B / B(\mathrm{ppm})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $1 / R^{5}$ | $1 / R^{7}$ | sum of all multipolar terms | exact from eq. (11). |
| 0.05 | $-219.5464$ | -91038.5742 | -91258.1206 | -92044.3724 |
| 0.1 | -48.7723 | -775.3416 | -824.1140 | -879.2927 |
| 0.15 | -2.8949 | -56.6456 | -59.4506 | -60.9233 |
| 0.2 | -0.1837 | -8.2277 | -8.4114 | -.8.4320 |
| 0.25 | -0.0410 | $-1.7456$ | -1.7866 | $-1.7869$ |
| 0.3 | -0.0161 | -0.4876 | -0.5036 | -0.5036 |
| 0.35 | -0.0074 | -0.1657 | -0.1732 | -0.1732 |
| 0.4 | -0.0038 | -0.0.65t | -0.0689 | -0.0689 |
| 0.45 | -0.0021 | -0.0285 | -0.0307 | -0.0307 |
| 0.5 | -0.0012 | $-0.0136$ | -0.0149 | -0.0149 |

along the $x, y$ and $z$ axes in a strong crystal field of octahedral symmetry. The calculated NMR shifts, $\Delta B / B$, along the $\langle 100\rangle,\langle 110\rangle$ and $\langle 111\rangle$ axes for specific $R$-values are tabulated in Table 2. As expected, the NMR shift, $\Delta B / B$, decreases in magnitude rapidly as $R$ increases. Along the $\langle 100\rangle$ and $\langle 111\rangle$ axes, $\Delta B / B$ is negative for all values of $R$, while along the $\langle 110\rangle$ axis, $\Delta B / B$ is positive. The NMR results for a $4 d^{1}$ system using equation (11) and the corresponding multipolar terms, $1 / R^{5}$ and $1 / R^{7}$, given by

TABLE 4: A Comparison of $\Delta \mathrm{B} / \mathrm{B}(\mathrm{ppm})$ using Equation( 1 1) with the Multipolar Terms for Specific $\boldsymbol{R}$-values Alung the $\langle\boldsymbol{I t} 0\rangle \boldsymbol{A x}$.

| $R(\mathrm{~nm})$ | $\Delta \mathrm{B} / \mathrm{B}$ (ppm) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $/ / R^{5}$ | $1 / R^{7}$ | sum of ali mul. lipolar terms | exact from eq. (I). |
| 0.05 | \$4.8866 | 147937.6831 | 147992.5697 | 147206.3179 |
| 0.1 | 12.1931 | 1259.9301 | 1272.1232 | 1216.9444 |
| 0.15 | 0.7237 | 92.0492 | 92.7729 | 91.3902 |
| 0.2 | 0.0459 | 13.3700 | 13.4159 | 13.3953 |
| 0.25 | 0.0103 | 2.8366 | 2.8469 | 2.8466 |
| 0.3 | 0.0040 | 0.7923 | 0.7963 | 0.7963 |
| 0.35 | 0.0019 | 0.2693 | 0.2712 | 0.2712 |
| 0.4 | 0.0010 | 0.1058 | 0.1067 | 0.1067 |
| 0.45 | 0.0005 | 0.0464 | 0.0469 | 0.0469 |
| 0.5 | 0.0003 | 0.0222 | 0.0225 | 0.0225 |

Parameters used; $\zeta=1030 \mathrm{~cm}^{-1}, \beta=2.848 \mathrm{l} / \mathrm{a}_{0}$ and $T=300 \mathrm{~K}$.

TABLE 5: $\Lambda$ Comparison of $\Delta B / B$ (ppm) using Equation (11) with the Multioplar Terms for Specific $\boldsymbol{R}$-values along the $\langle 111\rangle$ Axis

| $R(\mathrm{~nm})$ | $\Delta B / B$ ( ppm ) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $1 / R^{5}$ | $l / R^{7}$ | sum of afl mulltipolar terms | exact from eq. (1) |
| 0.05 | 146.3643 | -161846.3342 | -161699.9899 | -162486.2417 |
| 0.1 | 32.5149 | -1378.3851 | -1345.8702 | -1401.0\$89 |
| 0.15 | 1.9300 | -100.7034 | -98.7734 | -100.1561 |
| 0.2 | 0.1224 | - 14.6270 | --14.5046 | - 14.5252 |
| 0.25 | 0.0274 | -3.1033 | -3.0759 | -3.0761 |
| 0.3 | 0.0107 | -0.8668 | -0.8561 | -0.8561 |
| 0.35 | 0.0050 | -0.2946 | $-0.2897$ | -0.2897 |
| 0.4 | 0.0025 | -0.1157 | -0.1132 | -0.1132 |
| 0.45 | 0.0014 | -0.0507 | $-0.0493$ | -0.0493 |
| 0.5 | 0.0008 | -0.0243 | -0.0234 | -0.0234 |

Parameters used: $\zeta=1030 \mathrm{~cm}^{-1}, \beta=2.8481 / \mathrm{a}_{0}$ and $T=300 \mathrm{~K}$.

TABLE 6: $\Delta \mathrm{B} / \mathrm{B}$ (ppm) for Specific R-values for a 3d' System in a Strong Crystal Field Environment of Octahedral Symmetry

| $R(\mathrm{~nm})$ | $A B / B$ (ppm) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | <100〉 axis | $\langle 001\rangle$ axis | $\langle 10\rangle$ axis | (111) axis |
| 0.05 | -783171.9483 | -783171.9483 | 1154326.66 | -1187769.763 |
| 0.1 | -8824.8011 | -8824.8011 | 9118.2366 | -8378.6768 |
| 0.15 | -935.3065 | -935.3065 | 572.9484 | -403.5502 |
| 0.2 | -215.3654 | -215.3654 | 107.8626 | -23.6982 |
| 0.25 | -67.9963 | -67.9963 | 33.4199 | 5.8437 |
| 0.3 | -26.0932 | $-26.0932$ | 12.3354 | 5.6731 |
| 0.35 | $-11.5494$ | $-11.5493$ | 5.0923 | 3.6171 |
| 0.4 | -5.7325 | -5.7325 | 2.3340 | 2.2015 |
| 0.45 | -3.1050 | -3.1050 | 1.1761 | 1.3570 |
| 0.5 | -1.8008 | -1.8008 | 0.6421 | 0.8592 |
| Parameters used: $\zeta=154 \mathrm{~cm}^{-1}, \beta=4 / 3 \mathrm{a}_{0}$ and $T=300 \mathrm{~K}$. |  |  |  |  |

equation (4) are shown in Table 3, 4 and 5. A comparison of the multipolar terms with the exact solution given by equation (II) show that the first multipolar term, $1 / R^{5}$ in this case, is inadequate to describe the NMR shift for all values of $R$. When the $1 / R^{7}$ term is included, there is good agreement between the nonmultipole and multipolar results for $R$ values larger than 0.2 nm . It is interesting to note that along the $\langle 111\rangle$ axis, the $1 / R^{5}$ term gives values opposite in sign to that of the $1 / R^{7}$ term. In addition, the results show that the NMR

TABLE 7: A Comparison of the Exact Values of $A B / B$ (ppm) with the Multipolar Terms for Specific $\mathbf{R}^{2}$-values for a $\mathbf{3 d}^{1}$ System in a strong Crystal field of $O_{6}$ Symmelry.
(a) Along $\langle 100\rangle$ axis

| $R(\mathrm{~nm})$ | $\Delta B / B(\mathrm{ppm})$ |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | :---: |
|  | $1 / R^{5}$ | $1 / R^{7}$ | sum of all mul- <br> tipolar terms |  |  |
| 0.05 | -698476.9775 | -83197.2342 | -781674.2117 | -783171.9483 |  |
| 0.1 | -5488.3463 | -2828.9029 | -8317.2492 | -8824.8011 |  |
| 0.15 | -345.7988 | -480.4265 | -826.2253 | -935.3065 |  |
| 0.2 | -55.9411 | -141.1062 | -197.0472 | -215.3654 |  |
| 0.25 | -14.3482 | -50.9938 | -65.3420 | -67.9863 |  |
| 0.3 | -4.5484 | -21.1516 | -25.7045 | -26.0493 |  |
| 0.35 | -1.6416 | -9.8659 | -11.5076 | -11.5494 |  |
| 0.4 | -0.6595 | -5.0682 | -5.7277 | -5.7325 |  |
| 0.45 | -0.2913 | -2.8131 | -3.1044 | -3.1050 |  |
| 0.5 | -0.1396 | -1.6612 | -1.8008 | -1.8008 |  |

(b) Along the $\langle 110\rangle$ axis

| $R(\mathrm{~nm})$ | $\Delta B / B(\mathrm{ppm})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $1 / R^{5}$ | $1 / R^{7}$ | Sum of all multipolar terms | Exact values |
| 0.05 | 1135025.088 | 20799.3085 | 1155824.397 | 1154326.66 |
| 0.1 | 8918.5628 | 707.2257 | 9625.7885 | 9118.2366 |
| 0.15 | 561.9230 | 120.1066 | 682.0296 | 572.9484 |
| 0.2 | 90.9042 | 35.2765 | 126.1807 | 107.8626 |
| 0.25 | 23.3158 | 12.7484 | 36.0642 | 33.4199 |
| 0.3 | 7.3912 | 5.2890 | 12.6804 | 12.3354 |
| 0.35 | 2.6676 | 2.4665 | 5.1341 | 5.0923 |
| 0.4 | 1.0717 | 1.2670 | 2.3388 | 2.3340 |
| 0.45 | 0.4733 | 0.7033 | 1.1766 | 1.1761 |
| 0.5 | 0.2269 | 0.4153 | 0.6421 | 0,6421 |

(c) Along the $\langle 1[1\rangle$ axis

| $R(\mathrm{~nm})$ | $A B / B(\mathrm{ppm})$ |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: |
|  | $1 / R^{5}$ | $1 / R^{7}$ |  |  |  |  | Sum of all mul- <br> tipolar terms | Exact values |
| 0.05 | -1241736.849 | 55464.8228 | -1186272.026 | -1187769.763 |  |  |  |  |
| 0.1 | -9757.0602 | 1885.9353 | -7871.1249 | -8378.6768 |  |  |  |  |
| 0.15 | -614.7534 | 320.2843 | -294.4690 | -403.5502 |  |  |  |  |
| 0.2 | -99.4508 | 94.0708 | -5.3799 | -23.6982 |  |  |  |  |
| 0.25 | -25.5079 | 33.9959 | -8.4880 | 5.8437 |  |  |  |  |
| 0.3 | -8.0861 | 14.1040 | 6.0179 | 5.6731 |  |  |  |  |
| 0.35 | -2.9184 | 6.5773 | 3.6589 | 3.6171 |  |  |  |  |
| 0.4 | -1.1725 | 3.3788 | 2.2063 | 2.2015 |  |  |  |  |
| 0.45 | -0.5179 | 1.8754 | 1.3576 | 1.3570 |  |  |  |  |
| 0.5 | -0.2482 | 1.1074 | 0.8593 | 0.8592 |  |  |  |  |

Parameters used ; $\zeta=154 \mathrm{~cm}^{-1}, \beta=4 / 3 a_{0}$ and $T=300 \mathrm{~K}$.
shift arising from the electron angular momentum, and the electron spin dipolar-nuclear spin angular momentum interactions in a $4 \mathrm{~d}^{2}$ system in a strong crystal field environment of octahedral symmetry is large for significant distances between the NMR nucleus and 4d-electron bearing atom. For distances less than 0.35 nm it should not be neglected.

The calculated NMR shifts for a $3 \mathrm{~d}^{1}$ system in a strong crystal field environment of octathe:Iral symmetry, using the equation derived by Golding and Stubbs (I3), are listed in Table 6 and 7. Here we choose $\beta=4 / 3 a_{0}$. the spin -orbit coupling constant as $154 \mathrm{~cm}^{-1}$. These parameter values correspond to those for $\mathrm{Ti}^{3+}$ ion. The temperature is chosen as $T:=$ 300 K . As shown in Table 6-7, a simitlar trend for the NMR results with the $4 \mathrm{~d}^{\prime}$ system appears in a $3 \mathrm{~d}^{\prime}$ system in a strong crystal field of octahedral symmetry. However, the NMR shifts in magnitude increase significantly in the $3 \mathrm{~d}^{1}$ system in a strong crystal field environment of octahedral symmetry and along the $\langle 111\rangle$ axis, the NMR shift changes sign for $R$ values larger than 0.25 nm . As shown in Table 7(c). the $1 / \mathrm{R}^{5}$ term gives the negative shift while the $1 / R^{7}$ term gives the positive shift. The contribution of $1 / R^{7}$ term to $\Delta B / B$ is bigger than that of $1 / R^{5}$ term for $R$-values Jarger than 0.25 nm . From Table 7, we can find that for a $3 \mathrm{~d}^{1}$ system in a strong crystal field of octahedral symmetry, there is good agreement between the nonmultipole and multipolar results for $R$ values larger than 0.4 nm . These are different from the NMR results for a $4 \mathrm{~d}^{1}$ system. This calculation can be applied to investigate the NMR shift for $4 d^{n}$ system. However, $n$ n experimental value of $A B / B$ has been reported for $4 \mathrm{~d}^{1}$ and 3d' systems.

## References

(1) G. A. Webb. Ann. Rept. on NMR Spensroscopy, 6A. 1 (1975),
(2) A. F. Cockrill, G. L. O. Davies, R. C. Harden and D. M. Hackham, Chem. Revs, 73, 553 (1973).
(3) W. B. Lewis. J. A. Jackson and R. J. Taube. J. Chem. Phys., 36, 694 (1962).
(4) R. M. Golding. R. O. Pascual and S. Ahn. J. Magn. Res, 46. 406 (1982).
(5) C. C. Hinckley. J. Amer. Chem. Soc., 91, 5160 (1969): B Bleanev, C. M. Dobson. B. A. Levine. B. B. Martin. R. J. P. Williams and A. V. Xavier, Chem. Commur., 791 (1972).
(6) H. M. McConnell and R. E. mobertson., J. Chem. Phys., 29. 1361 (1958).
(7) R. J. Kurland and B. R. ReGravey. J. Magn. Res., 2, 286 (1970).
(8) B. Bleaney, J. Magn. Res., 8, 91 (1972).
(9) P. J. Stiles, Nol. Phys., 29, 1271 (1975).
(10) R. M. Golding and L. C. Stubbs. Proc. Roy. Sos., A354. 223 (1977).
(11) E. Clementi. D. L. Raimondi and W. P. Reinhart. J. Chem. Phys., 47, 1300 (1968).
(12) S. Ahn and H. C. Suh. Submitted to Bull. Korean Chem. Soc. for publication.
(13) L. C. Stubbs. Unpublished Thesis (PhD. The University of New South Wales, 1978).


[^0]:    * Department of chemistry, Mock Po National College, Mock Po 580, Korea

