

《Original》

Proposed Method to Predict Core Inventory History and Operator Time Margin during Small Break Accident

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소규모의 냉각재 상실 사고시 노심내 냉각재 양의 추정과
운전원 시간마진 예측을 위해 제안된 방법

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Abstract

The blowdown history of the TMI-2 accident up to the isolation of the relief valve associated with a small break LOCA is reviewed briefly. An analysis is made to determine what instruments should be added in the core in order to prevent core damage in the case of the TMI-2 accident. With the added instruments a procedure is presented on how to predict the uncovered level of the core and how to calculate operator time margin. Sample calculations are done for the TMI-2 accident to determine the uncovered level and operator time margin. Finally, the map to show the uncovered level of the core and operator time margin is drawn with measurable parameters by the above methods.

요 약

릴리프 밸브의 차단까지 TMI-2 사고의 blowdown history를 검토하고 TMI-2 사고와 같은 소규모의 냉각재 상실 사고 동안 노심 파괴를 막기 위해 더 가산해야 할 측정 기구에 대하여 논의하였다. 가산된 기구를 이용하여 어떻게 노심의 uncovered level과 operator time margin을 계산하는가를 검토하였으며, TMI-2 사고에 대해 uncovered level과 operator time margin을 결정하기 위한 샘플 계산을 수행하였다. 이 방법을 이용해서 측정되는 변수들의 함수로써 uncovered level과 operator time margin을 보여주는 도표를 작성하였다.

1. Introduction

The blowdown of the TMI-2 accident may be divided into the three phases of the subcoo-

led, two-phase, and superheated blowdown.^(1,2) After the relief valve opened at 2,155 psig and a small break LOCA started, more pressurization occurred due to no scram up to 8 sec after loss of feedwater pumps. Pressurization depends on

the relative increase of specific volume and specific internal energy, and can be predicted by the equation of state with the above two variables. The reactor was tripped on high pressure, 2,355 psig, and depressurization occurred due to a sudden power drop and the decay heat removal mode through the heat exchangers.

Since no flow was injected into the steam generators from the auxiliary feedwater system due to closed valves, the steam generator levels were very low. Then the only mode of decay heat removal was blowdown through the break. Owing to pressurizer level off-scale the HPI pumps were manually tripped. The increase in the pressurizer level is the unique characteristic of steam side break in the top side of pressurizer even though the level of primary system dropped.

After about 20 minutes the system pressure reached the saturation pressure and the two-phase blowdown started. There was either no pressure drop or a more gradual pressure decrease due to nearly isothermal trajectory of coolant. At about 100 minutes all reactor coolant pumps had been tripped by the operator in accordance with the emergency operating procedure to preclude the possibility of damage to the reactor coolant pumps from operation in reactor coolant saturation T-P conditions. The natural convection of two phase mixture, which is the effective decay heat removal mode after pump trip, was not possible due to level drop below the exit of the pressure vessel. The primary system behaved like a boiling pot and there was little flow through the core. The level of the pressure vessel continued dropping and the top of the core began to be uncovered.

The core and steam, respectively, began to heatup and to be superheated in the uncovered region of the core. The system underwent a considerable reduction in pressure, down to as low as 600 psia up to 2.3 hours after the acci-

dent. Generally, the initiation of superheated steam blowdown causes slow decrease in the mass loss rate but rapid increase in the enthalpy of coolant through break. Therefore, the amount of energy expelled through the break increases. The decay heat removal mode by loss of superheated steam through break is more effective than that by saturated steam. A more rapid decrease of pressure in superheated blowdown is expected.

After 2.3 hours the relief valve was isolated by the operator. Thus, level turnaround and sudden repressurization occurred due to lack of ability to remove decay heat. It may be expected that level turnaround with sufficient decay heat removal modes would increase heat transfer from the fuel rod to coolant and the cooldown of the fuel rods would start.

This paper will deal with the core in the interval from core uncover to level turnaround.

2. Main Section

Minimum Condition for Prevention of Core Damage

First of all, the minimum condition for prevention of core damage will be determined. The most significant parameter for fuel failure during LOCA is the clad temperature rather than the fuel centerline temperature. During the uncovered period the temperature difference between the clad temperature and the fuel centerline temperature is small due to low power and redistribution of stored energy. Of the temperature at which significant phenomena occur during core heatup, the selection of 2,200° F clad temperature as the minimum condition is conservative and is required by 10 CFR 50.46 as the design basis accident.⁽³⁾

Instruments to be Added in the Core

Instruments to measure the clad temperature

can indicate directly that the clad temperature approaches 2,200° F. But there are no instruments to measure the clad temperature in typical power reactors. Also it may be expected that installation of new instruments to measure directly the clad temperature causes some safety problems as well as highly technological and economical problems. As the clad temperature is related to the coolant temperature by an appropriate correlation, the clad temperature can be known with a measurable parameter being the coolant temperature. In typical power reactors the instruments for measurement of the coolant temperature have a narrow range; for example, 520°F to 620°F in TMI-2 for the hot leg instrument range. In order to measure the coolant temperature during the heatup process, it is necessary to expand the range of instruments for measurement of the coolant temperature up to 2,200°F. The methods to determine the uncovered level, critical uncovered level, and critical time with the above added instruments will be discussed.

Prediction of the Uncovered Level of Core

The uncovered level of core can be predicted with the measurable parameters which are the coolant temperature and the system pressure.

By energy consideration;

$$\int_0^z q'(z) dz = W_{rod}(h_{top} - h_g) \quad \text{Eq(1)}$$

Defining $q^*(z)$ as the average linear power over the submerged fuel pin length,

$$q^*(z) = \frac{\int_0^z q'(z) dz}{\int_0^z dz} = \frac{1}{z} \int_0^z q'(z) dz \quad \text{Eq(2)}$$

Equation (1) transforms to

$$z = \frac{W_{rod}}{q^*(z)} (h_{top} - h_g) \quad \text{Eq(3)}$$

where z : unconvered length from the top of core to the surface,

W_{rod} : mass flow rate of a channel,

h_{top} : specific enthalpy at the top of core,

h_g : specific enthalpy of saturation vapor in the system pressure, which is equivalent to steam enthalpy at the surface of two-phase mixture,

$q^*(z)$: average linear heat generation rate of a fuel rod from the top to the surface.

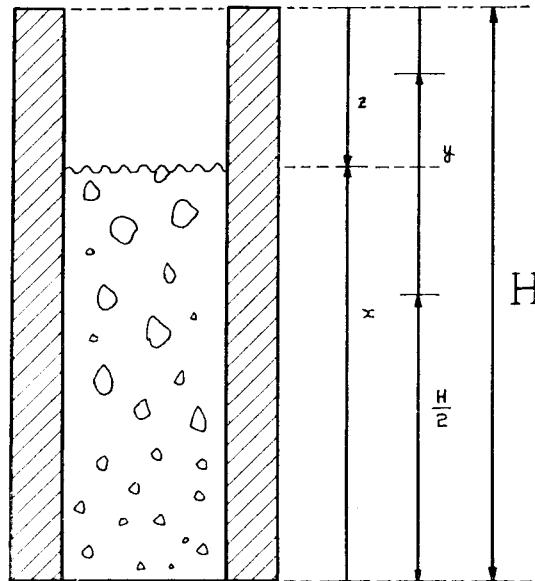


Fig. 1. Uncovered Channel in the Core

Actually, the linear heat generation rate, q' is a function of the uncovered length z and time t . However, since it takes a relatively long time to uncover the top of core after reactor trip (see page 6) for a small break LOCA, the amount of decay heat released in the core is nearly independent of time. Therefore, $q^*(z)$ can be assumed to be independent of time.

The calculation of the uncovered level of core requires the prediction or measurement of the velocity of coolant in order to know the mass flow rate of a channel in Eq(3). Also this velocity can be used to obtain the heat transfer coefficient at the top of core. From energy and mass considerations,

$$\frac{dM_v}{dt} = \frac{q^*(x) \cdot x}{h_{fg}(p)} \quad \text{Eq(4)}$$

$$\frac{dM_v}{dt} = V_s \rho_g A_f \quad \text{Eq(5)}$$

$$V_g \rho_g A_f = V_{\text{top}} \rho_{\text{top}} A_f = W_{\text{rod}} \quad \text{Eq(6)}$$

where $\frac{dM_v}{dt}$: the amount of vapor to be formed per second in the two-phase mixture,

$q^*(x)$: average linear heat generation rate from the bottom of the core to the surface of two-phase mixture,

A_f : channel area,

V_g and V_{top} : velocity of saturation vapor at the surface of two-phase mixture and superheated steam at the top, respectively,

ρ_g and ρ_{top} : density of saturation vapor in system pressure at the surface and superheated steam at the top of core, respectively,

h_{fg} ; enthalpy change by evaporation.

In Eq(1) it is assumed that the coolant in the core is in the saturated condition, because after the uncovering of the pressure vessel, natural convection stops and pot boiling starts.⁽⁴⁾

By Eqs (3), (4), (5), and (6), we can predict the uncovered length and the steam velocity during the heatup process with the instruments which can measure the coolant temperature up to 2200°F.

Prediction of the Critical Uncovered and Critical Time

The critical uncovered level is the uncovered level when the peak clad temperature reaches 2,200°F and the critical time is the interval in which the uncovered level at time t decrease to the critical uncovered level.

The critical uncovered level, z_c can be calculated with Eq(3);

$$z_c = \frac{W_{\text{rod}}}{q^*(z_c)} [h_c^{\text{top}} - h_g(p)],$$

where c : denotes a critical value, and

h_c^{top} : is the enthalpy of superheated steam at the top of the core when the maximum clad temperature is 2,200°F.

Here, h_c^{top} can be determined from steam table with the critical coolant temperature at the top, $T_{\text{cool}}^{\text{top}}$ to be obtained by Eq (27) in Appendix A.

Next, the critical time, t_c will be derived by energy and mass conservation. From Eq(4);

$$\begin{aligned} \frac{q^*(x) \cdot x}{h_{fg}(p)} &= \frac{dM_v}{dt} = -\frac{dM_l}{dt} \\ \frac{dM_l}{dt} &= \rho_f A_f \frac{dx}{dt} \\ \frac{q^*(x) \cdot x}{h_{fg}(p)} &= -\rho_f A_f \frac{dx}{dt} \\ \int_0^{t_c} dt &= -\rho_f A_f h_{fg} \int_x^{x_c} \frac{dx}{q^*(x) \cdot x} \\ t_c &= \rho_f A_f h_{fg} \int_{x_c}^x \frac{dx}{q^*(x) \cdot x} \end{aligned} \quad \text{Eq(7)}$$

Here, $(-dM_l)$ is the amount of saturated liquid evaporated in the interval, dt at time t .

Actually, we can expect that the boiling-off rate may be slower due to some neglected factors. They include pool swelling,⁽⁵⁾ the intake of saturated liquid from the annulus, nearly uniform level in each channel by the mixing of two-phase mixture among channels.

Here, the boiling-off rate is derived by consideration of only a hot channel. If we assume that the core level is uniform in channels, we can recalculate the critical time with this assumption which is very reasonable and gives more exact values of the critical time. As shown from Eq(32) in Appendix B, the revised critical time, t_c' is equal to t_c multiplied by $\frac{2}{\pi} \times \frac{q'_{\text{max}}}{\bar{q}'}$.

Sample Calculations for TMI-2 Accident

Assuming the nominal power shape, we obtain the following equation from Eq(2).

$$q^*(x) = \frac{\int_{H/2-x}^{H/2} q'(y) dy}{\int_{H/2-x}^{H/2} dy} \cong q'_{\text{max}} \frac{H}{\pi} \frac{1}{x} [1 - \cos(\frac{\pi}{H}x)] \quad \text{Eq(8)}$$

$$q^*(x) \cong q'_{\text{max}} \frac{H}{\pi} \frac{1}{x} [1 - \cos(\frac{\pi}{H}x)] \quad \text{Eq(9)}$$

i) Decay heat⁽⁶⁾: For $t_c=200$ sec, $\frac{P}{P_0} = 0.095t^{-0.26}$

For the TMI-2 accident, it is believed that the uncovering of core began from around 1.5 hours after shutdown.

At $t=1.5$ hr, $\frac{P}{P_0} \cong 0.01$

The initial average linear heat generation rate over the core

$$q'(t=0) = 6 \text{kw/ft}$$

$$q'(t=1.5\text{hr}) = 0.06 \text{kw/ft}$$

ii) By the assumption of the nominal power shape, the local power peaking factor, F_q^N is equal to 3.638.

From the definition of power peaking factor,

$$q'_{\max} = F_q^N \cdot \bar{q}' = 0.22 \text{kw/ft}$$

Now we can calculate the heat flux at the top of core.

$$q''_{\text{top}} = q''_{\max} \cos\left(\frac{\pi H}{2H_e}\right)$$

Here, the extrapolated length, H_e is roughly

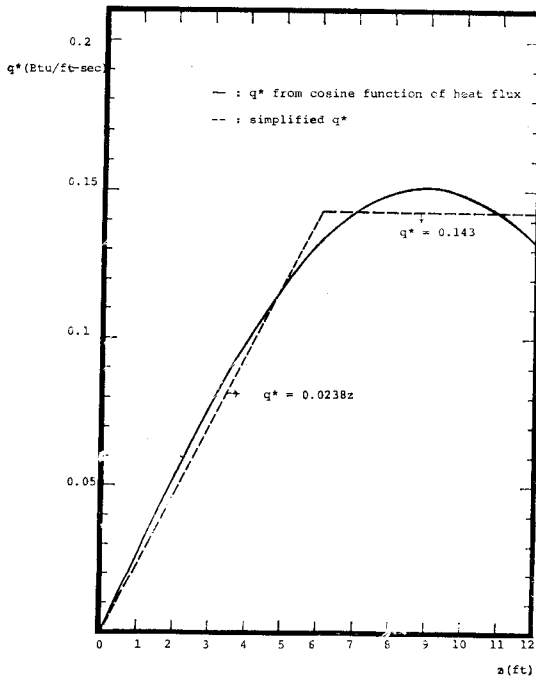


Fig. 2. Profile of q^* with Respect to z

equal to $(H+2L_{th})$.

L_{th} : thermal diffusion length, 0.0935ft,

$$H_e = 12.187 \text{ft},$$

$$q''_{\text{top}} = 0.0467 \text{kw/ft}^2$$

By Eq(8), Fig.2 can be drawn for the uncovered level. The graph of $q^*(x)$ versus x is the same if the variable, z is replaced by the variable, x . Based on the facts that the heat transfer coefficient in steady condition is conservative and applicable in transient conditions,⁽⁷⁾ and that the amount of entrained liquid in the core with low heat flux is very small,

$$Nu = 0.023 Re_b^{0.8} Pr_b^{0.4}, \quad \text{Eq(10)}$$

where b : property at the bulk temperature

Here, Pr_b is set as 0.9.

The heat transfer coefficient at the top,

$$h = \frac{K_{\text{top}}}{D_h} Nu \quad \text{Eq(11)}$$

In order to obtain heat transfer coefficient, it is necessary to calculate the velocity of coolant at the top.

By Eqs (4) and (5),

$$V_g = \frac{V_g(p)}{h_{fg}(p) \cdot A_f} q^*(x) \cdot x \quad \text{Eq(12)}$$

Now we can calculate the coolant temperature at the top associated with the uncovered level by Eqs (2), (6), (10), (11), and (12).

Fig. 3 is drawn from the data of Table 1. It should be noted that the coolant temperature, heat transfer coefficient, and clad temperature at the top of core are nearly independent of pressure. But the velocity at the top is sensitively dependent on pressure due to the dependence of specific volume on pressure. Fig. 3 tells us that the coolant temperature at the top abruptly rises with the level drop. We must make sure that the maximum cladding temperature will not exceed 2,200°F at an uncovered level in order to determine the critical uncovered level.

Table 2 made by Eqs (25) and (26) in Appendix A indicates that at $z=6$ ft, the maximum cladding temperature approaches closely the critical cladding temperature, 2,200°F. It can be

Table 1. Cladding Temperature at the Top vs. Uncovered Level

P psi	h _g Btu/lb	ρ _g lb/ft ³	z=2ft				3			
			T ^{top} _{cool}	V ^{top}	h ^{top}	T ^{top} _{cl}	T ^{top} _{cool}	V ^{top}	h ^{top}	T ^{top} _{cl}
			°F	ft/sec	Btu/hrft ² F	°F				
400	1205.5	0.86	518	2	6.6	757	645	2.28	7.3	861
600	1204	1.3	554	1.4	6.9	783	650	1.59	7.4	863
800	1199	1.76	572	1.18	7.4	785	660	1.2	7.5	870
1000	1192	2.24	599	1.12	8.5	785	666	1	7.8	868

P psi	5				6				7	
	T ^{top} _{cool}	V ^{top}	h ^{top}	T ^{top} _{cl}	T ^{top} _{cool}	V ^{top}	h ^{top}	T ^{top} _{cl}	h ^{top} Btu/lb	T ^{top} _{cool}
400	1250	2.8	8.7	1431	1796	2.8	8.7	1977	2850	>2200
600	1220	1.8	8.5	1406	1742	1.9	8.9	1919	2792	>2200
800	1150	1.4	8.5	1337	1607	1.4	8.5	1793	2428	>2200
1000	1110	1.24	9.3	1280	1560	1.2	9	1735	2344	>2200

said that the critical uncovered level exists when the uncovered level, z is just above 6ft. Conservatively, we can set the critical uncovered level, z_c as 6ft.

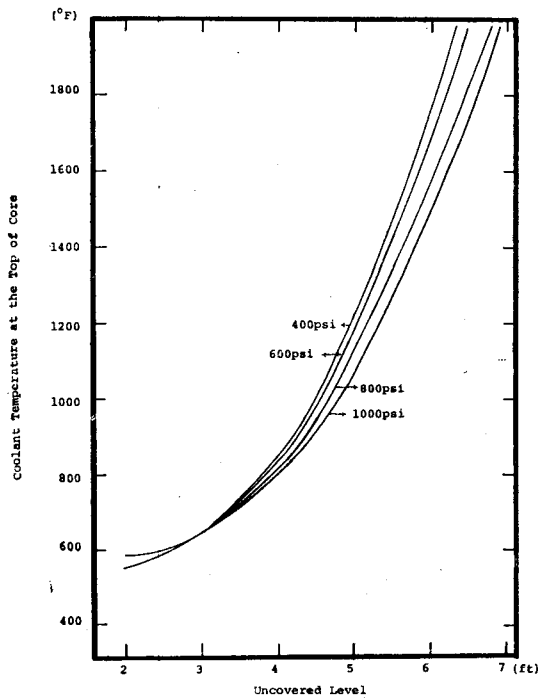


Fig. 3. Coolant Temperature at the Top of Core vs. System Pressure and Uncovered Level

Table 2. Maximum Cladding Temperature at the Uncovered Level 6ft

P _{psia}	T ^{top} _{cl} °F	y _m ft	T ^{max} _{cl}
400	1977	4	2173
600	1919	4	2110
800	1793	3.78	2015
1,000	1735	3.74	1864

In order to obtain the critical time with the above value of the critical uncovered level, Eq (7) is integrated by the use of the simplified q*(x) profile on Fig. 2.

If x_c ≥ 6,

$$t_c = \rho_f A_f h_{fg} \int_{x_c}^x \frac{dx}{0.143x} = 7\rho_f A_f h_{fg} \ln\left(\frac{x}{x_c}\right) \quad \text{Eq(13-a)}$$

If x_c < 6,

$$t_c = \rho_f A_f h_{fg} \left[\int_6^x \frac{dx}{0.143x} + \int_{x_c}^6 \frac{dx}{0.0238x^2} \right] = \rho_f A_f h_{fg} \left[7\ln\left(\frac{x}{6}\right) + 42\left(\frac{1}{x_c} - \frac{1}{6}\right) \right] \quad \text{Eq(13-b)}$$

Here since x_c is equal to 6ft, Eq(13-a) is taken to calculate the critical time.

From Eq(32) in Appendix B

$$t_c' = \frac{2}{\pi} \frac{q'_{max}}{q'} \cdot t_c$$

$$= 15.5 \rho_f A_f h_{fg} \ln\left(\frac{x}{x_c}\right) \quad \text{Eq(14)}$$

Fig. 4 can be drawn by using Eq(14). For simplicity, the critical time is calculated with constant pressure which results in constant properties, ρ_f and h_{fg} . Actually pressure continues to decrease during boil-off. But Fig. 4 is useful and directly applicable. If the level drops from the top of the core to $z=1$ ft and pressure decreases from 1,000 psi to 800 psi, we can know the new critical time is 380 sec at $z=1$ ft and 800 psi. As pressure decreases, the critical time increases at the same uncovered level. Therefore, the critical time with constant pressure is the minimum value of the actual critical time with decreasing pressure.

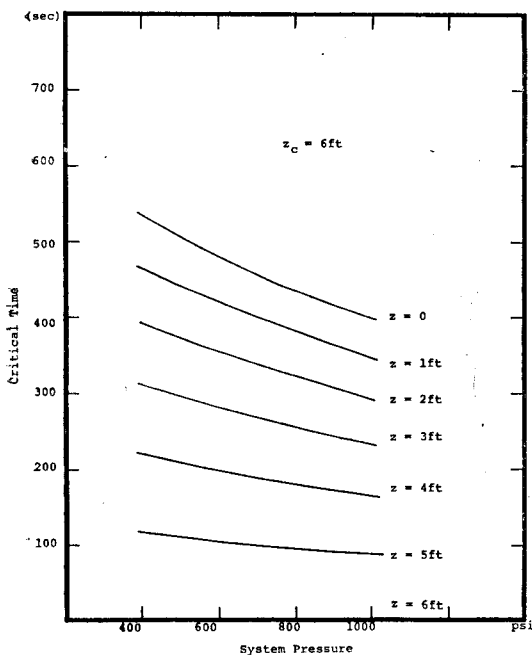


Fig. 4. Critical Time vs. System Pressure

Now with the measurable parameters of the coolant temperature and system pressure, we can draw the map to show the uncovered level of the core and the critical time. In order to obtain the uncovered level with the variable of the

critical time, we can use Eq (14).

$$x = x_c \exp(A),$$

where $A = t'_c / (15.5 \rho_f A_f h_{fg})$

By converting x into z ,

$$z = 6[2 - \exp(A)] \quad \text{Eq(15)}$$

Combining Eqs (3), (4), and (5), we can obtain the enthalpy at the top with the variable z ,

$$h_{top} = h_g(p) + \frac{q^*(z) \cdot z}{q^*(x) \cdot x} h_{fg}(p)$$

From the fact that $q^*(z)$ is equal to $0.0238 z$ from the top to 6ft,

$$h_{top} = h_g(p) + \frac{0.0238 z^2}{1.6 - 0.0238 z^2} h_{fg}(p) \quad \text{Eq(16)}$$

Putting Eq(15) into Eq(16), we can calculate the enthalpy at the top.

$$h_{top} = h_g(p) + \frac{0.858[2 - \exp(A)]^2}{1.6 - 0.858[2 - \exp(A)]^2} h_{fg}(p)$$

Eq(17)

The map, Fig. 5 can be drawn using Eqs (15) and (17). This map provides the uncovered level and the critical time for the operator in terms of measurable parameters, the coolant temperature and system pressure. In order to use this map the operator should know the maximum time required to accomplish a system correction. The maximum time includes diagnosis time, decision making time, system manipulation time, and time from system response to actual correction of system which the operator intends to accomplish. If the maximum time is 100 sec, the critical time, which is called in abstract as operator time margin, would exceed 100 sec. Therefore, the operator must take a corrective action before the coolant temperature reaches the line of $t_c=100$ sec. on the map for the system pressure. Also with this map the operator can determine what corrective actions he can take in order to prevent core damage. For example, more depressurization is desirable if the uncovered level is expected to exceed the critical uncovered level when the makeup rate exceeds the fluid loss rate through break due to

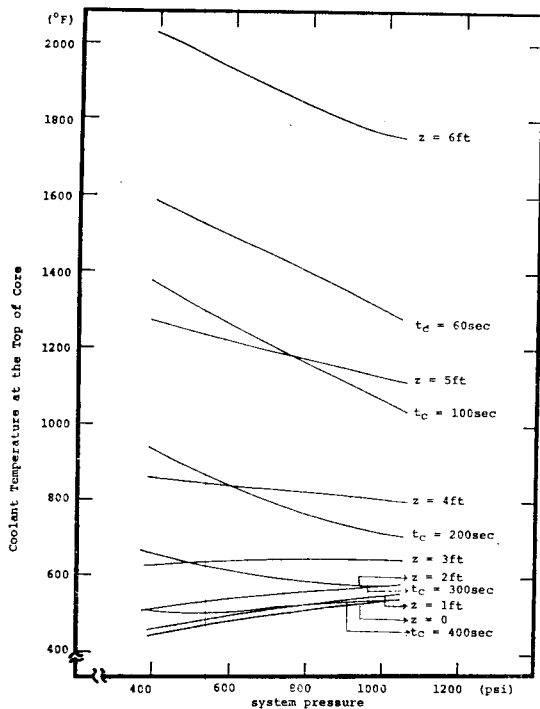


Fig. 5. Map of Operator Time Margin and Uncovered Level Determined by Measurable Parameters

more depressurization. If more depressurization is not possible, the isolation of broken loop and/or the intermittent operation of primary pumps can be considered in order to prevent core damage.

3. Conclusion and Recommendation

We can predict the velocity of superheated steam, uncovered length and critical time with instruments which can measure the coolant temperature up to 2,200°F. For TMI-2 the predicted critical uncovered level is 6ft. If the operators had had the map drawn here and the instruments to indicate the coolant temperature up to 2,200°F, the operators would have known the uncovered level of the core. Thus, the operators would have restarted primary pumps earlier after pump trip and operated intermittently until the makeup rate exceeded the

fluid loss rate through the relief valve by more depressurization.

It is recommended to examine the possibility of the installation of level detectors in the pressure vessel. The state of the primary system can be known more effectively with the added level detectors in the pressure vessel. We cannot know the level of core before the uncovering of core with instruments for the measurement of the coolant temperature, because the coolant temperature is equal to the saturation temperature before the uncovering of core. But, the level detectors in the pressure vessel can indicate the level of core before and after the uncovering of core as well as the prediction of the coolant temperature and of the critical time by Eqs(15) and (16) after the uncovering of core. Also level detectors enable the operator to diagnose a small break LOCA within a few minutes.

4. References

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Appendix A

Calculation of the maximum clad temperature with the variable of the coolant temperature at the top with Fig. 1.

By Newton's cooling law and energy conservation, the following equations are obtained.

$$q''(y) = h[T_{cl}(y) - T_{cool}(y)] \quad \text{Eq(18)}$$

$$\begin{aligned} W_{rod} C_p dT &= q'(y) dy \\ &= q'_{max} \cos\left(\frac{\pi y}{H_e}\right) dy \end{aligned} \quad \text{Eq(19)}$$

By integrating Eq (19) from the surface of two-phase mixture to a position, y above the surface with the assumption of constant specific heat in the superheated region, we have the equation

$$\begin{aligned} T_{cool}(y) &= T_{cool}\left(\frac{H}{2} - z\right) \\ &+ \frac{H_e}{\pi C_p W_{rod}} q'_{max} \left[\sin\left(\frac{\pi y}{H_e}\right) \right. \\ &\left. - \sin\left(\frac{\pi\left(\frac{H}{2} - z\right)}{H_e}\right) \right] \end{aligned} \quad \text{Eq(20)}$$

We can obtain the clad temperature at a position, y by combining Eq (18) and (19).

$$\begin{aligned} T_{cl}(y) &= T_{cool}\left(\frac{H}{2} - z\right) + \frac{q'_{max}}{h\pi D_h} \cos\left(\frac{\pi y}{H_e}\right) \\ &+ \frac{H_e}{\pi C_p W_{rod}} q'_{max} \left[\sin\left(\frac{\pi y}{H_e}\right) \right. \\ &\left. - \sin\left(\frac{\pi\left(\frac{H}{2} - z\right)}{H_e}\right) \right] \end{aligned} \quad \text{Eq(21)}$$

The position of the maximum clad temperature, y_m can be determined differentiating Eq(21),

$$y_m = \frac{H_e}{\pi} \tan^{-1}\left(\frac{hD_h H_e}{C_p W_{rod}}\right) \quad \text{Eq(22)}$$

By subtracting T_{cl} at the top from T_{cl} at the position, y_m

$$\begin{aligned} T_{cl}(y_m) - T_{cl}^{top} &= \frac{q'_{max}}{\pi D_h} \left[\cos\left(\frac{\pi y_m}{H_e}\right) \right. \\ &\left. - \cos\left(\frac{\pi H}{2H_e}\right) \right] \\ &+ \frac{H_e}{\pi C_p W_{rod}} q'_{max} \left[\sin\left(\frac{\pi y_m}{H_e}\right) \right. \\ &\left. - \sin\left(\frac{\pi H}{2H_e}\right) \right] \end{aligned} \quad \text{Eq(23)}$$

From Eq(18)

$$T_{cl}^{top} = T_{cool}^{top} + q''_{top}/h_{top} \quad \text{Eq(24)}$$

Putting Eq(24) into Eq(23), we have

$$\begin{aligned} T_{cl}(y_m) &= T_{cool}^{top} + \frac{q''_{top}}{h_{top}} \\ &+ \frac{q'_{max}}{\pi D_h} \left[\cos\left(\frac{\pi y_m}{H_e}\right) - \cos\left(\frac{\pi H}{2H_e}\right) \right] \\ &+ \frac{H_e}{\pi C_p W_{rod}} q'_{max} \left[\sin\left(\frac{\pi y_m}{H_e}\right) \right. \\ &\left. - \sin\left(\frac{\pi H}{2H_e}\right) \right] \end{aligned} \quad \text{Eq(25)}$$

Replacing y with $\frac{H}{2}$, we can calculate the coolant temperature at the top.

$$\begin{aligned} T_{cool}^{top} &= T_{cool}\left(\frac{H}{2} - z\right) + \frac{H_e}{\pi C_p W_{rod}} q'_{max} \cdot \\ &\left[\sin\left(\frac{\pi H}{2H_e}\right) - \sin\left(\frac{\pi\left(\frac{H}{2} - z\right)}{H_e}\right) \right] \end{aligned} \quad \text{Eq(26)}$$

Now we can calculate the maximum clad temperature with the coolant temperature at the top by Eqs(23) and (24) when the uncovered level is z . When the maximum clad temperature, $T_{cl}(y_m)$ reaches 2,200°F, the uncovered level, z and the coolant temperature at the top, respectively, are denoted by z_c and $T_{cool}^{c,top}$.

From Eq(25), the critical coolant temperature,

$$\begin{aligned} T_{cool}^{c,top} &= 2,200^\circ\text{F} - \frac{q''_{top}}{h_{top}} \\ &- \frac{q'_{max}}{\pi D_h} \left[\cos\left(\frac{\pi y_m}{H_e}\right) - \cos\left(\frac{\pi H}{2H_e}\right) \right] \\ &- \frac{H_e}{\pi C_p W_{rod}} q'_{max} \cdot \left[\sin\left(\frac{\pi y_m}{H_e}\right) \right. \\ &\left. - \sin\left(\frac{\pi H}{2H_e}\right) \right] \end{aligned} \quad \text{Eq(27)}$$

Appendix B

By energy and mass consideration over the whole core, we obtain

$$\begin{aligned} \frac{Q^*(x) \cdot x}{h_{fg}} &= -\frac{dM_t^{core}}{dt} = \rho_f \left(-\frac{dV_f}{dt}\right) \\ &= -\rho_f A_f N \cdot \frac{dx}{dt} \end{aligned} \quad \text{Eq(28)}$$

Q^* : average linear heat generation rate of all over the core;

N : the total number of fuel rods in the core;

A_f : flow area of a channel.

The total decay heat in the core with the average linear heat generation rate over the core, \bar{q}'

$$P = \bar{q}' \cdot N \cdot H$$

$$= Q'_{\max} \int_{-H/2}^{H/2} \cos\left(\frac{\pi y}{H_c}\right) dy$$

$$\cong Q'_{\max} \frac{2H}{\pi}$$

$$Q'_{\max} = \frac{\pi}{2} \cdot N \cdot \bar{q}'$$

$$Q^*(x) = \int_{-H/2}^{x-H/2} Q'_{\max} \cos\left(\frac{\pi y}{H_c}\right) dy / \int_{-H/2}^{x-H/2} dy$$

$$\cong Q'_{\max} \frac{H}{\pi} \cdot \frac{1}{x} \left[1 - \cos\left(\frac{\pi x}{H}\right) \right]$$

Eq(29)

Using Eqs.(9) and (29), we can get the following relationship

$$Q^*(x) = q^*(x) \cdot \frac{Q'_{\max}}{q'_{\max}}$$

$$= q^*(x) \cdot \frac{\bar{q}'}{q'_{\max}} \cdot \frac{\pi}{2} N \quad \text{Eq(30)}$$

From Eq(28),

$$dt = -\rho_f A_f h_{fg} \frac{dx}{q^*(x) \cdot x} \cdot \frac{2}{\pi} \cdot \frac{q'_{\max}}{\bar{q}'}$$

Eq(31)

Integrating Eq(31), we obtain the equation

$$\int_0^{t_c'} dt = \left(\frac{2}{\pi} \cdot \frac{q'_{\max}}{\bar{q}'} \right) \rho_f A_f h_{fg} \int_x^x \frac{dx}{q^*(x) \cdot x}$$

$$t_c' = \frac{2}{\pi} \cdot \frac{q'_{\max}}{\bar{q}'} \cdot t_c \quad \text{Eq(32)}$$