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《**Original**》

A Modified Borresen's Coarse-Mesh Solution to the LRA-BWR Benchmark Problem

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LRA-BWR 비등수형로에 대한 수정 Borresen 모델 해

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Abstract

Computational accuracy of the modified Borresen's coarse-mesh diffusion theory scheme is investigated with the steady-state solutions of the two- and three-dimensional LRA-BWR benchmark problem. By comparing the numerical results available for the critical eigenvalue and power distribution of the LRA-BWR, it is shown that the modified scheme is capable of predicting the power distribution of the multi-dimensional BWR problem with an improved accuracy.

요 약

Borresen 소격확산이른에 대한 수정형 모델의 계산정확도를 다차원 LRA-BWR 문제를 중심으로 조사해 보았다. 동 LRA-BWR 원자로의 임계도와 출력분포에 대한 계산결과들을 비교함으로써 수정형 Borresen Model이 다차원 BWR의 출력계산의 계산정밀도를 향상시킬 수 있음을 보였다.

I. Introduction

The previous studies on the Borresen's 1.5 group diffusion theory scheme[1-3] indicate that the scheme offers a simple and inexpensive calculational method for the static LWR analysis problems but that the treatment of the thermal group flux need be modified in order to obtain the improved accuracy of the LWR computations without impairing the computational efficiency of the original scheme. The author proposed the modified Borresen's coarse-mesh[4-5] scheme in which the fine structure effects of the thermal flux distribution within the individual spatial node are taken into account, and the node-dependent thermal group weight factors are used, for computing the node-average thermal group fluxes. It has been demonstrated that the modified scheme can bring about a significant improvement in the computational accuracy of the two- and three-dimensional IAEA

benchmark problems.

The purpose of this article is to present a further investigation on the computational accuracy of the modified scheme with regard to the multi-dimensional LRA-BWR problem[6-7] which has become a useful benchmark problem for testing both the static and dynamic BWR analysis models. In the following, the brief outline for the essential modifications incorporated into the Borresen's scheme is described. Then the numerical results for the LRA-BWR problem are discussed in terms of the numerical accuracy and efficiency. Possible future applications of the modified Borresen's scheme are also indicated.

II. Computational Method

The equations to be solved in the modified Borresen's coarse-mesh scheme are the nodal coupling relations for the diffusion density of fast neutron group, $\Psi_i = \sqrt{D_i}\cdot\phi_i$,

$$-\sum_{4j}\Psi_j - R\sum_{2j}\Psi_j + \frac{p_i+(b+c\cdot r_i)q_i}{1-cq_i}\Psi_i = \frac{1}{\lambda}\frac{V_i}{\sqrt{D_i}\cdot(1-cq_i)}\left(\frac{\nu\Sigma_{fi}}{\sqrt{D_i}}\bar{\Psi}_i + \nu\Sigma_{fti}\bar{\phi}_{ti}\right) \quad (1)$$

where the node-average fast flux, $\bar{\phi}_i = \bar{\Psi}_i/\sqrt{D_i}$, is enumerated using an interpolation formula,

$$\bar{\phi}_i = b\phi_i + 2c\left(\sum_{4j}\phi_i^j + R\cdot\sum_{2j}\phi_i^j\right), \quad (2)$$

with the internodal surface flux ϕ_i^j given by the first-order finite difference approximation,

$$\phi_i^j = \frac{D_i\phi_i + D_j\phi_j}{D_i+D_j}, \quad (3)$$

The notations in the above have the same meaning as in references (1, 4, 5) except q_i, which is defined by

$$q_i = \frac{V_i\Sigma_{ti}}{D_i} \quad (4)$$

Solution of Eq. (1) requires a successive updating of the node-average thermal flux. As proposed in the original method, this is done by assuming the interpolation formula similar to Eq. (2) but with the node-dependent thermal group weight factors, b_{ti} and c_{ti},

$$\bar{\phi}_{ti} = b_{ti}\phi_{ti} + 2\cdot c_{ti}\left(\sum_{4j}\phi_{ti}^j + R\cdot\sum_{2j}\phi_{ti}^j\right) \quad (5)$$

The node-center thermal flux is determined by using the Borresen's assumption of the asymptotic thermal flux distribution, which is equivalent to the relation,

$$\phi_{ti} = (\Sigma_{ri}/\Sigma_{ati})\ \bar{\phi}_i. \quad (6)$$

As for the nodal interface thermal flux, ϕ_{ti}^j, the first-order approximation like Eq. (3) is no longer utilized. For in the coarse-mesh application the first-order approximation is not likely to provide an adequate description of the thermal spectral effect, particularly where strong thermal flux gradients can take place due to the presence of rodded fuel assemblies. In the modified Borresen's scheme, therefore, it is proposed to adopt the following analytical procedure in order to obtain an improved approximation for ϕ_{ti}^j.

Consider a one-dimensional thermal group diffusion equation for a two-region slab representing two adjacent half nodes;

$$-D_{ti}\frac{d^2\phi_{ti}(x)}{dx^2} + \Sigma_{ati}\phi_{ti}(x) = S_i \quad \text{for } -h/2 \le x \le 0$$

$$-D_{tj}\frac{d^2\phi_{tj}(x)}{dx^2} + \Sigma_{atj}\phi_{tj}(x) = S_j \quad \text{for } 0 \le x \le \frac{h}{2} \quad (7a, b)$$

with the boundary conditions,

$$\phi_{ti}(-h/2) = \phi_{ti} = S_i/\Sigma_{ati}$$

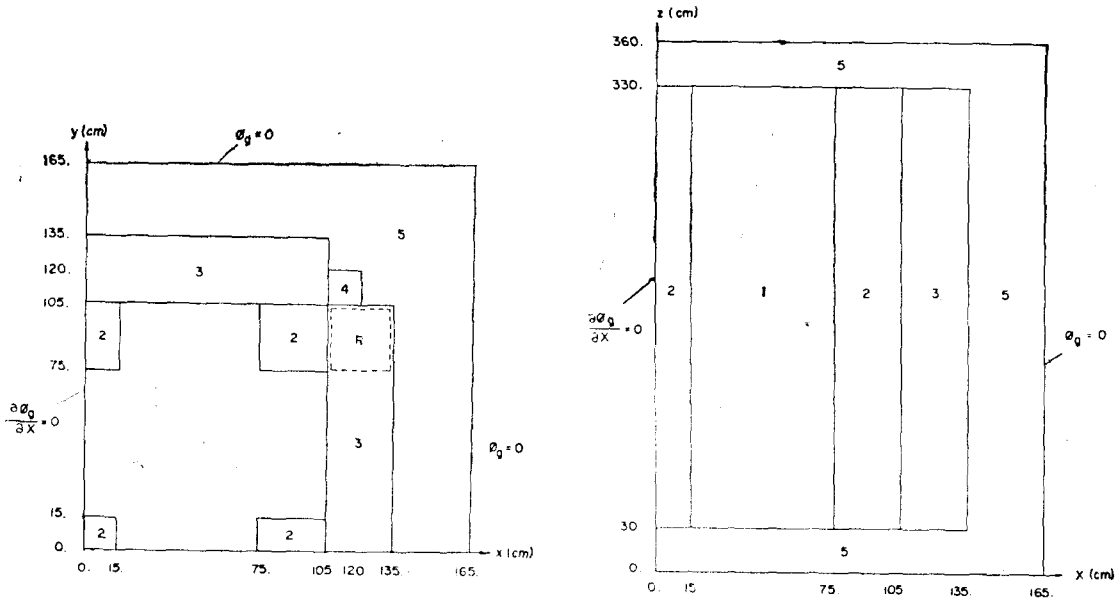
$$\phi_{tj}(h/2) = \phi_{tj} = S_j/\Sigma_{atj} \quad (8a, b)$$

and the interface continuity conditions,

$$\phi_{ti}(0) = \phi_{tj}(0) = \phi_{ti}^j$$

$$-D_{ti}\phi_{ti}'(0) = -D_{tj}\phi_{tj}'(0) \quad (9a, b)$$

Solving Eq. (9) on the assumption that the slowing down source, S, is determined by the node-average fast flux across each node, we find the analytical expression for the nodal



(a) Quadrant of Reactor Horizontal Cross Section (b) Vertical Cross Section, $y=0$

Fig. 1. The LRA-BWR Kinetics Benchmark Problem

interface thermal flux,

$$\phi_{tj}{}^{j}=\frac{W_i\phi_{ti}+W_j\phi_{tj}}{W_i+W_j} \tag{10}$$

where

$W_i=D_{ti}K_i/\tanh\ (K_ih/2)$

with $K_i=\sqrt{D_{ti}/\Sigma_{ati}}$

Eq. (10) is the desired analytical expression for the thermal group flux at the x-directed surfaces of the node i. By considering the one-dimensional thermal group problem in the y and z direction, similar expressions are obtained for the thermal group fluxes at the y- and z-directed surfaces of the node i.

III. Numerical Results and Discussions

Numerical effectiveness of the modifications incorporated into the Borresen's scheme is

Table 1. Material Properties of LRA-BWR Problem

Region	Material	Group, g	D (cm)	Σ_a (cm^{-1})	$\nu\Sigma_f$ (cm^{-1})	Σ_r (cm^{-1})
1	Fuel 1 with Rod	1	1.255	0.008252	0.004602	0.02533
		2	0.211	0.1003	0.1092	
2	Fuel 1 without Rod	1	1.268	0.007181	0.004609	0.02767
		2	0.1902	0.07047	0.08675	
3	Fuel 2 with rod	1	1.259	0.008002	0.004663	0.02617
		2	0.2091	0.08344	0.1021	
4	Fuel 2 without rod	1	1.259	0.008002	0.004663	0.02617
		2	0.2091	0.073324	0.1021	
5	reflector	1	1.257	0.0006034	0.0	0.04754
		2	0.1592	0.01911	0.0	

*Axial buckling of 10^{-4} cm^{-2} for all compositions in 2-D problem

investigated with the computations of critical eigenvalue and power distribution of the two- and three-dimensional LRA-BWR benchmark problem[7]. Fig. 1 shows the horizontal and vertical cross sections of the BWR. The reactor contains a total of 312 fuel assemblies, each fuel assembly having a width of 15cm. Table 1 lists the two group diffusion theory parameters that represent the distinct material composition of the fuel assemblies including the reflector.

The static computations of the LRA-BWR problem are conducted for two different core configurations; one with the control rods inserted and the other with the control rods withdrawn. Shown in Figs. 2~4 are the normalized assembly power densities obtained from two- and three-dimensional computation. For the purpose of computations available for the assembly power densities of the LRA-BWR are also presented.

Fig. 2 shows a comparison of two-dimensional, octant-core computation of the modified Borresen's scheme and solutions of the fine-mesh finite difference KIDD code and the nodal expansion IQSBOX code[9] for the normalized assembly power densities and k_{eff} of the LRA-BWR with the control rods inserted. Table 2 presents a summary of comparison of the two-dimensional computations with the reference solution of the KIDD code,[10] which is obtained with the width of 0.75cm. It is noted that the modified scheme with one node per fuel assembly predicts the assembly power densities with the

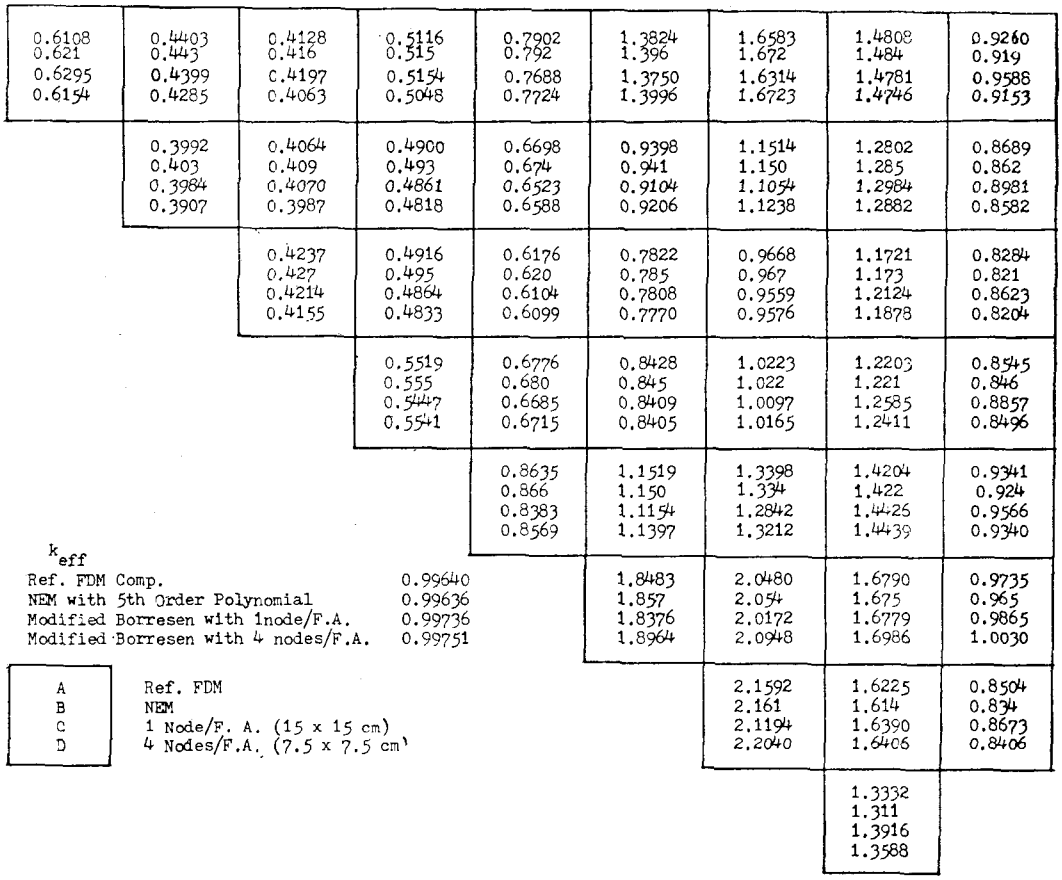


Fig. 2. Normalized Assmbly Power Densities for the Two-Dimensional Static LRA-BWR Benchmark Problem with Control Rods Inserted.

Table 2. Summary of Results for the Octant-Core, Two-Dimensional LRA-BWR Problem

Method	Original	Borresen	Modified	Borresen
Mesh width, cm	15	7.5	15	7.5
Number of nodes per fuel assembly	1	4	1	4
Eigenvalue, k_{eff}	0.99843	0.99856	0.99736	0.99751
Maximum/mean[a)] relative errors in power densities, %	9.98/3.49	4.60/1.87	4.38/1.87	2.60/1.33
Relative error[a)] in k_{eff}, %	0.20	0.22	0.10	0.11
Execution time, sec (Computer System)	2.4	13.4	2.7	13.4

(a) The reference power densities and k_{eff} ane taken from the computation of the KIDD code with 0.75cm mesh width.

maximum and mean relative errors of 4.79% and 1.93%, respectively. It is also noted that computations with four nodes per fuel assembly brings down these errors to 2.65% and 1.33%. As for the effective multiplication factor, the two computations predict almost identical results with the relative errors of roughly 0.10%.

Figures 3 and 4 present the three-dimensional results of the modified Borresen's scheme and the two-dimensional results of the fine-mesh KIDD code for the normalized power densities, which are computed in the octant-core symmetry and the quarter-core symmetry, respectively. Examination of these results indicates that the three-dimensional computations of the modified scheme predict the power distribution of the LRA-BWR with the numerical accuracy similar to that observed already in the two-dimensional computations.

The computational accuracy of the modified

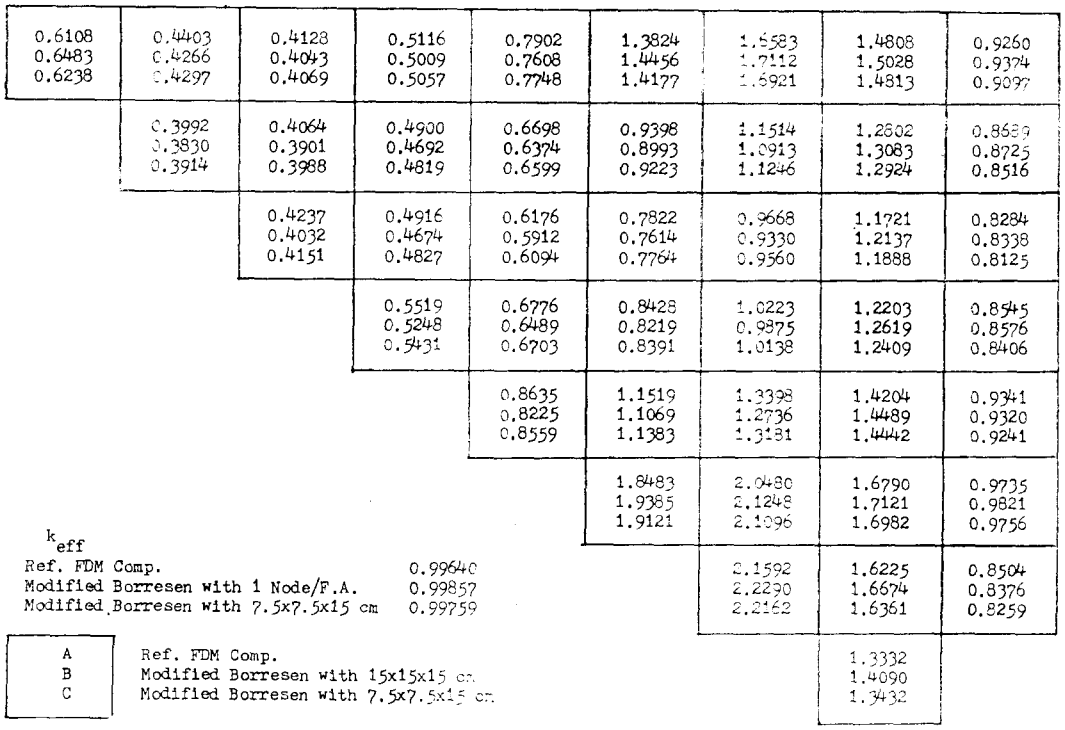


Fig. 3. Normalized Assembly Power Densities for the Three-Dimensional Static LRA-BWR Benchmark Problem with Control Rods Inserted.

0.1922 0.1857 0.1876	0.1515 0.1353 0.1421	0.1671 0.1534 0.1595	0.2385 0.2227 0.2300	0.4002 0.3718 0.3842	0.7286 0.7401 0.7334	0.9042 0.9165 0.9085	0.8367 0.8468 0.8266	0.5369 0.5447 0.5288
0.1456 0.1294 0.1362	0.1463 0.1302 0.1381	0.1763 0.1593 0.1680	0.2485 0.2272 0.2389	0.3754 0.3458 0.3633	0.5580 0.5218 0.5400	0.7215 0.6775 0.6979	0.8437 0.8686 0.8462	0.5906 0.5996 0.5742
0.1475 0.1335 0.1398	0.1628 0.1459 0.1544	0.2056 0.1854 0.1964	0.2903 0.2649 0.2800	0.4245 0.3946 0.4137	0.6008 0.5729 0.5914	0.8097 0.7764 0.7980	1.0387 1.0885 1.0550	0.7581 0.7759 0.7439
0.1920 0.1752 0.1829	0.2090 0.1883 0.1993	0.2641 0.2396 0.2537	0.3818 0.3503 0.3705	0.5799 0.5403 0.5688	0.8514 0.8142 0.8442	1.1771 1.1287 1.1689	1.5295 1.6025 1.5672	1.1210 1.1457 1.1114
0.2994 0.2698 0.2830	0.2907 0.2621 0.2779	0.3510 0.3229 0.3395	0.5224 0.4857 0.5107	0.8611 0.8013 0.8476	1.4069 1.3259 1.3849	1.9803 1.8554 1.9484	2.4902 2.5647 2.5507	1.7908 1.8083 1.7904
0.5210 0.5098 0.5151	0.4050 0.3672 0.3856	0.4519 0.4234 0.4394	0.6789 0.6474 0.6683	1.2292 1.1648 1.2057	2.4327 2.5199 2.5055	3.4216 3.5211 3.5218	4.1076 4.4734 4.2816	2.8070 3.0598 2.9300
0.6279 0.6079 0.6179	0.4984 0.4478 0.4722	0.5568 0.5149 0.5383	0.8152 0.7636 0.7965	1.4153 1.3155 1.3766	2.6788 2.7269 2.7363	3.6000 3.6651 3.6744	4.0418 4.4232 4.1823	2.6137 2.8452 2.7082
0.5682 0.5442 0.5485	0.5626 0.5463 0.5506	0.6695 0.6603 0.6625	0.9418 0.9277 0.9387	1.4244 1.3885 1.4231	2.0804 2.0372 2.0706	2.4845 2.4560 2.4684	2.5888 2.6818 2.5606	
0.3595 0.3430 0.3400	0.3858 0.3663 0.3655	0.4709 0.4477 0.4491	0.6435 0.6080 0.6182	0.8982 0.8435 0.8695	1.1375 1.0799 1.1139	1.1833 1.0779 1.0960		

		K_{eff}
Ref. FDM Comp.	A	1.01553
Modified Borresen with 15x15x15 cm	B	1.01972
Modified Borresen with 7.5x7.5x15 cm	C	1.01763

Fig. 4. Normalized Assembly Power Densities for the Three-Dimensional LRA-BWR Benchmark Problem with Control Rods Withdrawn.

Borresen's scheme is much higher than that of the original scheme. As summarized in Table 2 already, the core power prediction of the modified scheme has much lower maximum and mean relative errors than that of the original scheme. In quantitative terms, Table 2 shows that the numerical accuracy of the modified scheme with one node per fuel assembly is comparable to the computational accuracy of the original scheme with four nodes per fuel assembly. From the standpoint of the computing time, the modified scheme does not impair the computational efficiency of the original scheme. The last row of Table 2 shows that two schemes require about equal execution time in computing the core power and the multiplication factor. Table 2 does not show the computing time required for the reference KIDD computations, yet it must be noted that about 4,000 seconds are spent for obtaining the reference KIDD solution with the mesh width of 0.75cm.

IV. Conclusion

The chief advantages of the Borresen's coarse-mesh scheme stem from the fact that it involves the relatively fewer number of the unknowns. What's more, the iteration matrix given by Eq. (1) has a very simple structure in that the off-diagonal elements are either unity or a constant. These facts considerably reduce the computer storage requirements and the computing time as well. With the improved computational accuracy achieved by the modifications

in the treatment of the thermal group flux, the scheme offers a useful calculational method of the LWR physics problems, particularly for those in which the detailed 3-D computations are essential.

References

1. S. Borresen, A Simplified Coarse-Mesh, Three Dimensional Diffusion Scheme for Calculating the Gross Power Distribution in a Boiling Water Reactor, Nucl. Sci. Eng., **44**, 37 (1971)
2. M.R. Wagner, Current Trends in Multidimensional Static Reactor Calculations, *Proc. Conf. Computing Methods in Nuclear Engineering*, Charleston, S.C., CONF-750413, vol.I, p. 1 (1975)
3. S. Langenbuch, W. Mauser and W. Werner, Coarse-Mesh Flux-Expansion Method for the Analysis of Space-Time Effects in Large Light Water Reactor Cores, Nucl. Sci. Eng. **63**, 437 (1977)
4. C.H. Kim and S.H. Levine, A Modified Borresen's Coarse-Mesh Scheme for the Computation of the Gross Power Distribution in the PWR. Trans. Am. Nucl. Soc., **39**, (1981)
5. C.H. Kim and S.H. Levine, A Modified Borresen's Coarse-Mesh Computation for a Three Dimensional PWR Benchmark Problem, Nuclear Technology, **61**, 49 (1983)
6. W. Werner, et al., Two and Three Dimensional BWR Kinetics Benchmark Problems, Trans. Am. Nucl. Soc., **23**, 215 (1976)
7. Argonne Code Center, Benchmark Problem Book ANL-7416, Supplement 2, ANL (1977)
8. M. Tsuiki and K. Aoki, Modified One-Group Diffusion Scheme for Calculating Gross Power Distribution in a BWR Core, Nucl. Sci. Tech. **11**, 369 (1974)
9. H. Finnemann, A Consistent Nodal Method for the Analysis of Space-Time Effects in Large LWR's, *Proc. of Joint NEACRP/CSNI Specialists' Meeting on New Developments in 3-D Neutron Kinetics and Review of Kinetics Benchmark Calculations*, Munich, Jan. 22-24 (1975), MRR 145.
10. S.K. Lee, et al., "KIDD- KAERI Improved Diffusion-Depletion Program for Nuclear Reactor Analysis," to be published.