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《**Original**》

# Study on the Numerical Analysis of Nuclear Reactor Kinetics Equations

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# 원자로 동특성 방정식의 수치해석에 관한연구

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## Abstract

A two-step alternating direction explicit method is developed to solve the space-dependent reactor kinetics equations in two space dimensions. As a special case in the general class of alternating direction implicit methods, this method is analyzed for accuracy and stability.

To test the validity of this method it is compared with the implicit-difference method used in the TWIGL program. It is shown that the two methods are closely related.

The time dependent neutron fluxes of the pressurized water reactor (PWR), during control rod insertion, and, of the CANDU-PHW reactor, in case of postulated loss of coolant accident, are obtained from the numerical calculation results.

## 요 약

2차원 다군 확산 이론에 의한 원자로 동특성 방정식의 해를 구하기 위해서 two-step alternating direction explicit method를 도입하였다. Alternating direction implicit method의 특별한 경우로써 이 방법의 정확도 및 안전성을 해석하였다.

이 방법의 타당성을 시험하기 위해서 TWIGL 전산조직에 사용한 implicit difference method와 비교하여 두 방법의 결과가 일치함을 알았다.

이 방법을 이용하여 가압경수형 원자로(PWR)의 제어봉 삽입시의 중성자 신속의 시간변화와, CANDU-PHW 원자로의 가상된 냉각재상실 사고시의 중성자 신속의 시간변화를 계산하여 이들 원자로의 제어능력을 확인하였다.

## I. Introduction

In recent years, considerable effort has been devoted to the development and analysis of methods which will solve the time-dependent multigroup neuton diffusion equation in one or more spatrial dimensions.

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The power reactors are so large that they can not be analyzed by point kinetics equations.

As is well known. when one wishes to describe with great detail the transient behaviour of large nuclear power reactors, it is necessary to work with multidimensional calculations.

For safety purposes, transient analysis of the large liquid-metal-cooled fast breeder reactors must include detailed neutronics calculations.

The purpose of this work is to devise an adequate technique for calculating time-dependent reactor kinetics equations in two space dimensions.

The techniques, for the multigroup form of the time-dependent diffusion equations in one or more space dimensions, can be divided into two broad categories. One is the direct solution technique, the other is the indirect solution technique. The indirect solution techniques include: time-synthesis[1,2], space-time synthesis [3-8], quasistatic [9-13], and nodal methods.[14-19]

The time-synthesis approximation is derived by expanding the neutron flux in known functions of spatial dimension, with unknown expansion coefficients of time.

The space-time synthesis equations are derived by expanding the neutron ffux in known functions of two spatial dimensions with unknown combining coefficients which depend on the remaining spatial dimension and time.

In the quasi-static method, the neutron flux is factored into a shape function that is slowly varying with time and an amplitude function that varies more rapidly with time.

In the nodal method, when a reactor can readily be visualized as consisting of several cores, or regions, with a weak neutronic interaction, it may be possible to adequately represent kinetics behaviour by a system of coupled point reactor equations, one for each core.

These indirect methods are characterized by relatively small computational time requirements and by a lack of definitive error bounds on the final solutions.

The direct solution techniques, however, solve the equations by finite differencing in space and time. They are characterized by fairly definitive error bounds and are thus valuable as numerical standards against which the indirect techniques may be measured.

As computational capabilities increase, the direct methods also become practical tools for solving two dimensional kinetics equations.

In one dimension, the GAKIN[20] and WIGLE [21] codes represent successful applications of direct methods. In two dimensions, the WIGLE method has been extended into the code TWIGL [22]. This code is limited to two neutron groups.

A somewhat different separation of the neutron balance operator for two-dimensional problems has deen developed by Denning, etal[24].

The basis of this method, known as the alternating direction explicit method, is to separate the neutron balance operator so that the terms describing the coupling among fluxes at different mesh point in time $(t+\triangle t)$ are based on only a two-point difference.

Using the known value of the flux on the boundary, the algorithm becomes explicit when the mesh is transversed sequentially. The direction of the spatial coupling is alternated on successive time steps.

Reed and Hansen,[25] and Ferguson and Hansen[41] bave fond that the exponential transformed alternating direction explicit method significantly reduces the truncation error. But this method shows oscillations having no physical meaning. The alternating direction implicit method has also been examined for two-dimensional problems by Varga[26] and by Wight and Hansen[27].

If the two-dimensional spatial mesh is swept

to solve for new fluxes point by point, the method is termed explicit. If a whole row or column of points is solved simultaneously for new fluxes, it is termed implicit.

Hageman and Yasinsky[28] have developed a varient of this method, known as ADI-B², in which the coupling term in the other direction is replaced by a buckling evaluated from fluxes at time t.

The alternating direction implicit method is stable but has a large truncation error. Wight and Hansen[29,30] have found that the exponential transformation method reduced the truncation error, but the method was no longer unconditionally stable.

The equations are finite-differenced on a fixed spatial mesh before they are solved. From the finite differenced form of the equations, the two-step alternating direction explicit solution technique is presented and analyzed for accuracy and stability.

The time dependent neutron flux distributions of the pressurized water reactor(PWR) and the CANDU-PHW reactor are obtained from the results.

## II. The Nuclear Reactor Kinetics Equations.

The multigroup diffusion equation for the space- and time-dependent neutron flux is given by[31]

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\boldsymbol{r},t)=\nabla\cdot D_g(\boldsymbol{r},t)\cdot\nabla\phi_g(\boldsymbol{r},t)+\sum_{g'=1}^{G}\Sigma_{gg'}(\boldsymbol{r},t)\phi(\boldsymbol{r},t)+\sum_{i=1}^{I}f_{gi}\,C_i(\boldsymbol{r},t). \quad (1)$$

$(1\le g\le G)$

and the delayed-neutron precursor density is given by

$$\frac{\partial}{\partial t}C_i(\boldsymbol{r},t)=-\lambda_i C_i(\boldsymbol{r},t)+\sum_{g'=1}^{G}P_{ig'}(\boldsymbol{r},t)\phi_{g'}(\boldsymbol{r},t), \quad (2)$$

$(1\le i\le I)$

where the symbols are defined in Nomenclature.

The boundary condition for Eqs. (1) and (2) are the homogeneous Neumann or Dirichlet type[32]. At internal interfaces, continuity of the flux and normal component of the neutron current is required. An initial flux distribution in energy and space must be specified.

Equations (1) and (2) can be compacted into the matrix form[33].

$$\frac{\partial}{\partial t}\boldsymbol{\Phi}(\boldsymbol{r},t)=\boldsymbol{M}(\boldsymbol{r},t)\boldsymbol{\Phi}(\boldsymbol{r},t) \quad (3)$$

where $\boldsymbol{\Phi}$ is column matrix,

$$\boldsymbol{\Phi}(\boldsymbol{r},t)=\begin{pmatrix}\phi_1(\boldsymbol{r},t)\\ \phi_2(\boldsymbol{r},t)\\ \vdots\\ \phi_G(\boldsymbol{r},t)\\ C_1(\boldsymbol{r},t)\\ \vdots\\ C_I(\boldsymbol{r},t)\end{pmatrix} \quad (4a)$$

and $\boldsymbol{M}(\boldsymbol{r},t)$ is a matrix,

$$\boldsymbol{M}(\boldsymbol{r},t)=\left(\begin{array}{ccc|cc} v_1(\nabla\cdot D_1\nabla+\Sigma_{11}) & v_1\Sigma_{12}\cdots & v_1\Sigma_{1G} & v_1f_{11} & v_1f_{1I}\\ v_2\Sigma_{21} & v_2(\nabla\cdot D_2\nabla+\Sigma_{22})\cdots & v_2\Sigma_{2G} & v_2f_{21} & v_2f_{2I}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ v_G\Sigma_{G1} & v_G\Sigma_{G2} & v_G(\nabla\cdot D_G\nabla+\Sigma_{GG}) & v_Gf_{G1} & v_Gf_{GI}\\ \hline p_{11} & p_{12}\cdots & p_{1G} & -\lambda_1 & 0\\ \cdots & \cdots & \cdots & 0 & \ddots\\ p_{I1} & p_{I2}\cdots & p_{IG} & 0 & -\lambda_I \end{array}\right) \quad (4b)$$

The equations (1) and (2) continuous in both spatial and temporal variabels.

Using the finite difference method,[34] the spatially discretized form of Eqs. (1) and (2) can be written as

$$\frac{d\Psi_g}{dt}=D_g\Psi_g+\sum_{g'=1}^{G}T_{gg'}\Psi_{g'}+\sum_{i=1}^{I}F_{gi}\,C_i, \quad (5)$$

$(i\le g\le G)$

and

$$\frac{dC_i}{dt}=-\Lambda_i C_i+\sum_{g'=1}^{G} P_{ig'}\Psi_{g'} \qquad (6)$$

$$(1\leq i\leq I)$$

where $\Psi_g$ is column matrix,

$$\Psi_g=\begin{pmatrix}\phi_{g,1}\\ \phi_{g,2}\\ \vdots\\ \phi_{g,N}\end{pmatrix} \qquad (7)$$

$D_g$ is the matrix corresponding to the difference operator, $T_{gg'}$ contains a term representing the intergroup transfer processes, and $F_{gi}$ represents the transfer of delayed neutrons into group $g$ due to decay in precursor group $i$, $\Lambda_i$ contains the precursor decay constants, while $P_{ig'}$ represents the production of delayed precursor $i$ due to fission in group $g'$.

The system of ordinary differential equations (5) and (6) can be written as the matrix-differential equation,

$$\frac{d\Psi}{dt}=A\Psi \qquad (8)$$

where $A$ is square and of the order $N_X(G+I)$ matrix, $N$ is the number of spatial mesh points, $\Psi$ and $A$ have been defined as:

$$\Psi=\begin{pmatrix}\Psi_1\\ \Psi_2\\ \vdots\\ \Psi_G\\ C_1\\ \vdots\\ C_I\end{pmatrix} \qquad (9)$$

and

$$A=\left(\begin{array}{cccc|cc} D_1+T_{11} & T_{12} & \cdots & T_{1G} & F_{11}\cdots & F_{1I}\\ T_{21} & D_2+T_{22} & \cdots & T_{2G} & F_{21}\cdots & F_{2I}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ T_{G1} & T_{g2} & \cdots & D_G+T_{GG} & F_{G1}\cdots & F_{GI}\\ \hline P_{11} & P_{12} & \cdots & P_{1G} & -\Lambda_1 & 0\\ P_{21} & P_{22} & \cdots & P_{2G} & & \ddots\\ \cdots & \cdots & \cdots & \cdots & & \\ P_{I1} & P_{I2} & \cdots & P_{IG} & 0 & -\Lambda_I \end{array}\right) \qquad (10)$$

For later reference, several matrices are defined here as follows:

$$D=\left(\begin{array}{cccc|cc} D_1 & 0 & \cdots & 0 & & \\ 0 & D_2 & \cdots & 0 & & \\ \cdots & \cdots & \cdots & \cdots & & 0\\ 0 & 0 & & D_G & & \\ \hline & & & & -\Lambda_1 & 0\\ & 0 & & & 0 & -\Lambda_I \end{array}\right) \qquad (11a)$$

$$U=\left(\begin{array}{ccccc|cc} 0 & T_{12} & \cdots & T_{1G} & & F_{11} & \cdots & F_{1I}\\ 0 & 0 & \cdots & T_{2G} & & F_{21} & \cdots & F_{2I}\\ \cdots & \cdots & \cdots & \cdots & & \cdots & & \\ 0 & 0 & & 0 & & F_{G1} & & F_{GI}\\ \hline & & & & & & & \\ & 0 & & & & & 0 & \end{array}\right) \qquad (11b)$$

$$L=\left(\begin{array}{ccccc|cc} 0 & 0 & \cdots & 0 & & & \\ T_{21} & 0 & \cdots & 0 & & & \\ \cdots & \cdots & \cdots & \cdots & & 0 & \\ T_{G1} & T_{G2} & \cdots & 0 & & & \\ \hline P_{11} & P_{12} & \cdots & P_{1G} & & & \\ \cdots & \cdots & \cdots & \cdots & & 0 & \\ P_{I1} & P_{I2} & \cdots & P_{IG} & & & \end{array}\right) \qquad (11c)$$

and

$$T=A-(D+L+U) \qquad (11d)$$

For any period of time, $\Delta t$, during which all terms in $A$ are constant, Eq (8) will be the form of solution

$$\Psi(\Delta t)=\exp(A\Delta t)\Psi(0) \qquad (12)$$

where $\Psi(0)$ is the initial condition.

Because this differential equation has the property of stiffness[35], any attempt to represent the time derivative in Eq (8) by a finite difference approximation will require that a relatively small time step be taken in order to follow the more rapidly varying components of the equation.

The $A$ matrix is a real, essential positive, and irreducible[26]. It states that exp $(A\,\Delta t)$ is positive where $t>0$.

In the two dimensional space the finite difference approximation to the $\nabla\cdot D\nabla$ operator contained in $A$ is consistent and accurate to order $\Delta x^2$ and $\Delta y^2$, i.e.

$$A\Phi=M\Phi+0(\Delta x^3)+0(\Delta y^3) \qquad (13)$$

## III. A Two-step Alternating Direction Explicit Method

To obtain a solution for Eq. (8), the time derivative is replaced by two successive forward differences over a time interval, $\Delta t$ $(=2h)$.

Let the time step start at $t=0$ so that the initial value is $\Psi(0)$. For the two halves of the time step, each of duration $h$, split $A$ as follows;

$$A=A_1+A_2 \tag{14a}$$

and

$$A=A_3+A_4 \tag{14b}$$

where

$$\begin{aligned} A_1&=U+D_Y \\ A_2&=T+L+D_X \\ A_3&=U+D_X \\ A_4&=T+L+D_Y \end{aligned} \tag{15}$$

The matrix $D_Y$ contains the two strips of $D$ which lie above the diagonal plus one-half of each term on the diagonal. The matrix $D_X$ contains the two strips below the diagonal plus the remainder of each diagonal term.

The differential equation is written by the two step finite difference equation,

$$\begin{aligned} \frac{\Psi(h)-\Psi(0)}{h} &= A_2\Psi(h)+A_1\Psi(0) \\ \frac{\Psi(2h)-\Psi(h)}{h} &= A_4\Psi(2h)+A_3\Psi(h) \end{aligned} \tag{16}$$

or, equivalently,

$$(I-hA_2)\Psi(h)=(I+hA_1)\Psi(0) \tag{17a}$$

$$(I-hA_4)\Psi(2h)=(I+hA_3)\Psi(h) \tag{17b}$$

where $\Psi(h)$ is an intermediate vector computed on the first half-step. The name alternating-direction explicit derives from the diffusion term where one direction is being handled explicitly in one half-step, and the other direction is handled in the next half-step.

By substituting Eq. (17a) into Eq. (17b) we obtain the formal expression for the advancement matrix $\boldsymbol{B}(h)$:

$$\begin{aligned} \Psi(2h)&=(I-hA_4)^{-1}\ (I+hA_3)\ (I-hA_2)^{-1} \\ &\quad (I+hA_1)\ \Psi(0) \\ &\equiv \boldsymbol{B}(h)\ \Psi(0) \end{aligned} \tag{18}$$

where

$$\begin{aligned} \boldsymbol{B}(h)&=(I-hA_4)^{-1}\ (I+hA_3)\ (I-hA_2)^{-1} \\ &\quad (I+hA_1) \end{aligned} \tag{19}$$

A general algorithm equation is obtained as follows,

$$\Psi^{N+1}=\boldsymbol{B}(h)\Psi^N \tag{20}$$

with the initial value, $\Psi^0=\Psi(0)$

Equations (19) and (20) represent an arbitrary alternating-direction explicit method.

Although it is termed a two-step method because two successive finite differences are taken to advance the solution over time $\Delta t$, it is essential to think of the two operators which advance the solution over each half-step $h$ as inseperable from each other.

The solution is thus said to be advanced over one step during time $\Delta t$, even though the entire space and energy mesh has been swept twice.

The first half-step is carried out by forward substitution, sweeping from one corner of the mesh to the diagonally-opposit corner and from the highest energy group to the lowest. The second half-step reverses the direction of the spatial sweep.

## IV. Accuracy and Stability of the Method

### 1. Steady State Property.

For the steady state reactor,

$$\frac{d\Psi^0}{dt}=A\Psi^0=0 \tag{21}$$

$$\Psi^1(2h)=\boldsymbol{B}(h)\Psi^0=\Psi^0 \tag{22}$$

where $\Psi^0=\Psi(0)$

which is the exact solution, independent of $h$.

Thus, operation on a $\Psi^0$ which represents a just-critical configuration by a $\boldsymbol{B}(h)$ formed from an $A$ containing the just-critical parameters will result in no change in $\Psi^0$.

This can be shown by writing Eq. (19)

$$\boldsymbol{B}(h)=(I-hA_4)^{-1}(I+hA_3)\,(I-hA_2)^{-1}$$

$(I+hA_1)$.

Using the splitting relations defined in Eq. (14), this becomes

$$\boldsymbol{B}(h)=(I-hA_4)^{-1}[I-h(A_4-A)](I-hA_2)^{-1}[I-h(A_2-A).$$

Since $A\Psi^0=0$,

$$\boldsymbol{B}(h)\Psi^0=(I-hA_4)^{-1}[I-h(A_4-A)](I-hA_2)^{-1}(I-hA_2)\Psi^0.$$
$$=(I-hA_4)^{-1}(I-hA_4)\Psi^0=\Psi^0$$

### 2. Temporal Truncation Error

This property is concerned with how well the advancement matrix $\boldsymbol{B}(h)$ approximates the exact discrete solution operator $e^{2hA}$. For sufficiently small values of $h$, the difference between the solution computed using $\boldsymbol{B}(h)$ and that computed using $e^{2hA}$ over a time step $\Delta t$ varies approximately as a single power of $h$. As shown below, $\boldsymbol{B}(h)$ agrees with the expansion of $e^{2hA}$ through terms of order $h^2$.

A Taylor series expansion of the exact operator yields

$$e^{2hA}=I+2hA+2h^2A^2+\cdots\cdots \tag{23}$$

The Eq. (19) is

$$\begin{aligned}\boldsymbol{B}(h)&=(I-hA_3)^{-1}(I-hA_3+hA)(I-hA_1)^{-1}(I-hA_1+hA)\\&=\{I+(I-hA_3)^{-1}\cdot hA\}\{I+(I-hA_1)^{-1}\cdot hA\}\\&=I+h(I-hA_3)^{-1}A+h(I-hA_1)^{-1}A+h^2(I-hA_3)^{-1}(I-hA_1)^{-1}A^2\\&=I+h\{2I+hA_1+hA_3\}A+h^2\{I+hA_3)(I+hA_1)\}A^2\end{aligned} \tag{24}$$

For symmetric splitting, this equation is

$$\boldsymbol{B}(h)=I+2hA+2h^2A^2+0(h^3) \tag{25}$$

For any other splitting, the terms of order $h^2$ remain as in Eq.(24).

### 3. Stability

To ensure stability, it is necessary that the solution remain bounded for finite times and finite time steps. The definition for which the advancement matrix, $\boldsymbol{B}(h)$, is stable is that[25,27]

$$\|\boldsymbol{B}(h)\|\leq b \tag{26}$$

for

$$0\leq h\leq\tau, \qquad 0\leq t\leq T$$

where $b$ is a constant.

This condition can be satisfied if the solution grows by no more than a factor $(1+K\Delta t)$ with each time step[36], where $K$ is a constant. $\boldsymbol{B}(h)$ contains quantities of the form $hv_g D_g/\Delta x^2$, arising from the approximation of the diffusion operator, which become very large as $\Delta x^2$ becomes very small. If we fix $\Delta x$ and merely shrink $\Delta t$, then the method will be stable. The real problem exists when both $\Delta t$ and $\Delta x$ approach zero together. To examine this case we shall require that $\Delta t$ and $\Delta x$ are related so they approach zero together with the ratio $r=hv_gD_g/\Delta x^2$ held fixed.

If any value of the constant $r$ may be used, the method is said to be unconditionally stable.

## V. Numerical Results

A computer code which is named KIND has been developed to calculate the method of chapter III in two dimensional geometry. Several trial problems have been run with this code. Problem 1 was for simplified pressurized water reactor cylindrical geometry and problem 2 was for the Wolsung-1 Nuclear Generating Ssation (CANDU-PHW).

**CASE 2.** PWR Reactor

The test case 1 is for a simplified PWR (Pressurized Water Reactor) cylindrical reactor as depicted in Fig. 1.

The inverse group speeds used were $1/v_1=1\times10^{-7}$ and $1/v_2=5\times10^{-6}$(sec/cm). One group of delayed neutrons was considered with $\beta=0.0064$ and $\lambda=0.08$.

This problem began with the rods out of the seed 3 region.

The initial (critical) flux distribution for the exact solution was a steady state solution for

**Table 1. Lattice Parameters of Test Case 1.**

| | Seed 1 | Seed 2 | Seed 3 (R.I.) | Seed 3 (R.O.) | In Blkt. | Out Blkt. | Refl. | Water |
|---|---|---|---|---|---|---|---|---|
| $D_1$ | 1.367 | 1.404 | 1.377 | 1.377 | 1.291 | 1.297 | 1.412 | 1.625 |
| $\Sigma_{a1}$ | 0.01099 | 0.00661 | 0.01061 | 0.006551 | 0.008494 | 0.008400 | 0.001483 | 0.000365 |
| $\Sigma_{r1}$ | 0.01309 | 0.01582 | 0.01309 | 0.01586 | 0.01116 | 0.01114 | 0.005923 | 0.03797 |
| $\nu\Sigma_{f1}$ | 0.007142 | 0.006602 | 0.006046 | 0.006046 | 0.002677 | 0.002681 | 0 | 0 |
| $D_2$ | 0.3825 | 0.3778 | 0.3384 | 0.3384 | 0.4565 | 0.4572 | 0.7162 | 0.2750 |
| $\Sigma_{a2}$ | 0.1511 | 0.09493 | 0.1339 | 0.09039 | 0.04975 | 0.04727 | 0.006549 | 0.01083 |
| $\nu\Sigma_{f2}$ | 0.194451 | 0.147798 | 0.171219 | 0.171219 | 0.058203 | | | |

the material parameters shown in Table 1.

The transient problem was stimulated by a step-wise scram, at t=0, of the group of control rods in the cross-hatched seed 3 region. This created an asymtotic shutdown reactivity of$\sim$1.6%.

The time step used for this problem was $5\times 10^{-5}$ sec. Figures 2 through 4 display the calculation results which have been compared with the results of the TWIGL code which has been widely applied in the United States over a number of years.

The calculation results were in close agreement between the two methods for this problem.

**CASE 2.** Wolsung-1 Nuclear Generating St-a-tion

Case 2 was foɪ a reactvitiy accident analysis

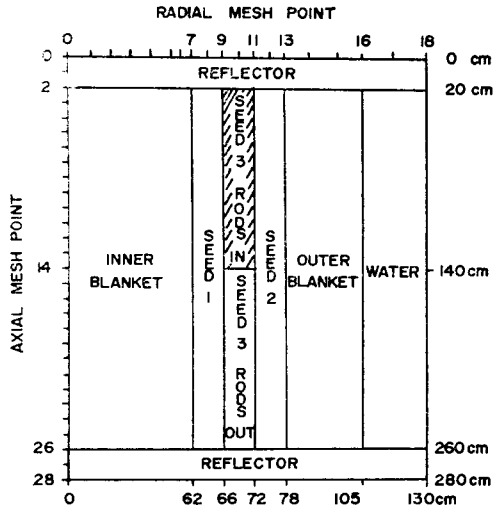


**Fig. 1. PWR Cylindrical Geometry.**

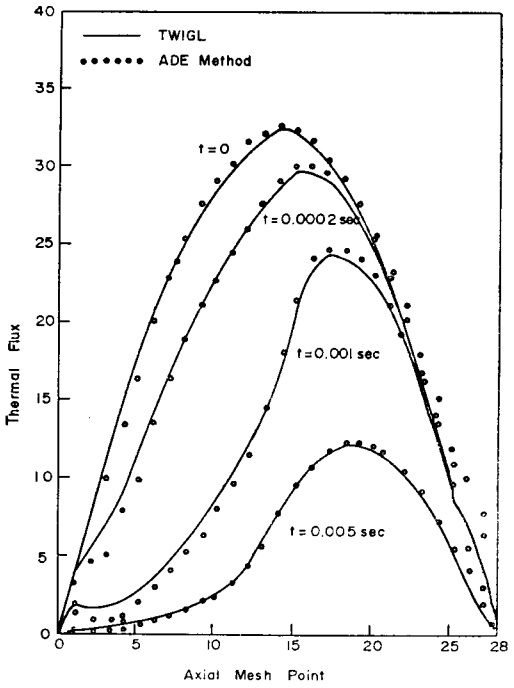


**Fig. 2. Thermal Flux at Radial Mesh Point 5, PWR Problem.**

of Wolsung-1 (CANDU-PHW) Nuclear Generating Station.

The reactor core is nearly cylindrical, 594.4 cm in length and 314.3cm in equivalent diameter.

It is made up of 380 calandria tubes or fuel channels. The fuel channels are arranged on a regular square lattice with center to center spacing of 28.575cm. The pressure tube holds 12 fuel bundles each of which has 37 fuel elements. $D_2O$ coolant is pumped through the fuel

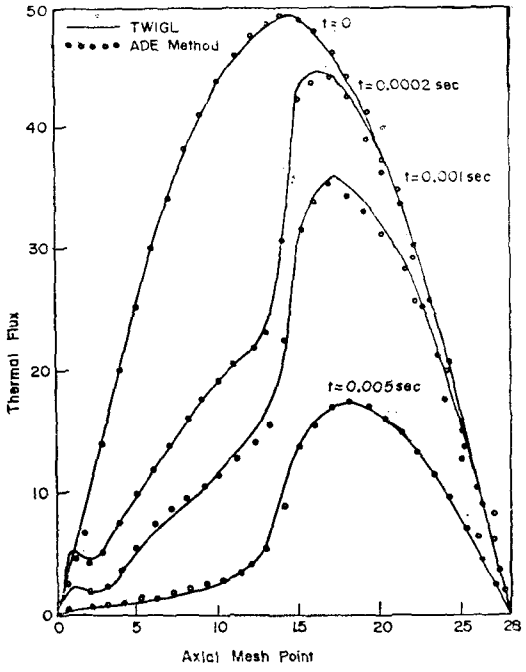


**Fig. 3. Thermal Flux at Radial Mesh Point 10, PWR Problem.**

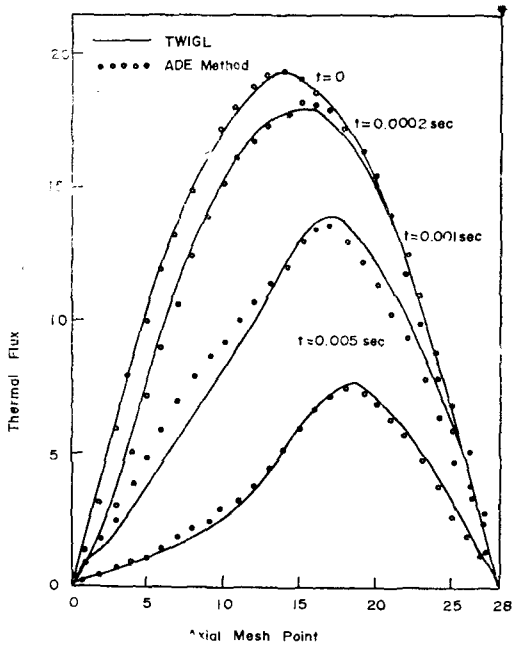


**Fig. 4. Thermal Flux at Radial Mesh Point 15, PWR Problem.**

elements.

The air gap between the calandria tube and the pressure tube serves to insulate the hot heavy water coolant in the pressure tube from the cold heavy water moderator surrounding the calandria tube. Fig. 5 shows the initial core configuration[37] which consists of two regions; depleted inner fuel region of 124 channels and undepleted outer fuel region of 256 channels. The reactor core is surrounded by the heavy water reflector. The power reactor must be installed with two independent shutdown systems according to regulations set by the United State Nuclear Regulatory Commission.[38] Two shutdown systems have also been installed in the Wolsung-1 Nuclear Generating Station.

One system is the shutoff rods (28 rods) whose reactivity worth ranges between -74 and -80 milli-k for the configurations with which the reactor is expected to operate for significant lengths of time. The other is a poison injection system which is designed to give a negative reactivity insertion rate which would be as effective as the shutoff rod system. The negative reactivity inserted on completion of the injection is greater than -300 milli-k.

In case 2, the reactivity transient represents a nonuniform loss-of-coolant (LOCA) of half

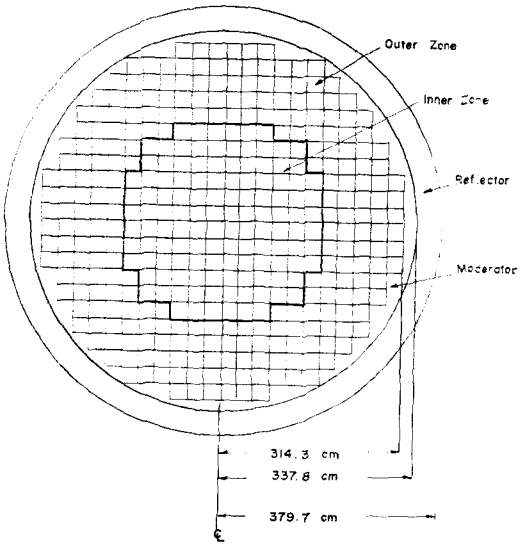


**Fig. 5. Side View of the Wolsung-1 Core.**

**Table 2. Lattice Parameters of Test Case 2.**

| Region | Group g | Dg (cm) | $\Sigma_g$ ($cm^{-1}$) | $\nu\Sigma_{fg}$ ($cm^{-1}$) | $\Sigma_{1\to2}$ ($cm^{-1}$) |
|---|---|---|---|---|---|
| 1-12 | 1 | 1.321 | $1.009\times10^{-2}$ | 0.0 | $1.009\times10^{-2}$ |
| | 2 | 0.8837 | $7.788\times10^{-5}$ | 0.0 | |
| 13, 14, 15, 18, 19, 20 | 1 | 1.274 | $8.1556\times10^{-3}$ | 0.0 | $7.3928\times10^{-3}$ |
| | 2 | 0.94133 | $4.083\times10^{-3}$ | $4.63\times10^{-3}$ | |
| 16, 17 | 1 | 1.274 | $8.1556\times10^{-3}$ | 0.0 | $7.3934\times10^{-3}$ |
| | 2 | 0.94126 | $4.1014\times10^{-3}$ | $4.578\times10^{-3}$ | |

$v_1=7.14\times10^6$cm/sec, $v_2=2,725\times10^5$cm/sec.

**Table 3. Delayed Neutron Data of Test Case 2.**

| Group $i$ | $\beta_i$ | $\lambda_i$ ($sec^{-1}$) |
|---|---|---|
| 1 | $2.95\times10^{-3}$ | 0.100612 |
| 2 | $1.165\times10^{-3}$ | 0.1355 |
| 3 | $1.033\times10^{-3}$ | 0.1218 |
| 4 | $2.350\times10^{-3}$ | 0.3175 |
| 5 | $7.80\times10^{-4}$ | 1.389 |
| 6 | $1.97\times10^{-4}$ | 3.784 |

core and a subsequent asymmetric insertion of the shutoff rods. The transient is approximated by a linear increase of 0.7% in the nentron yield per fission for 0.4 sec, followed by an increase in the the thermal absorption cross section of 74 milli-k in one-half of the core during the following 1.8 sec.

Fig. 6 shows the mesh space of the reactor core and the two group cell parameters. Additional data on the problem is given in Tables 2 and 3[37].

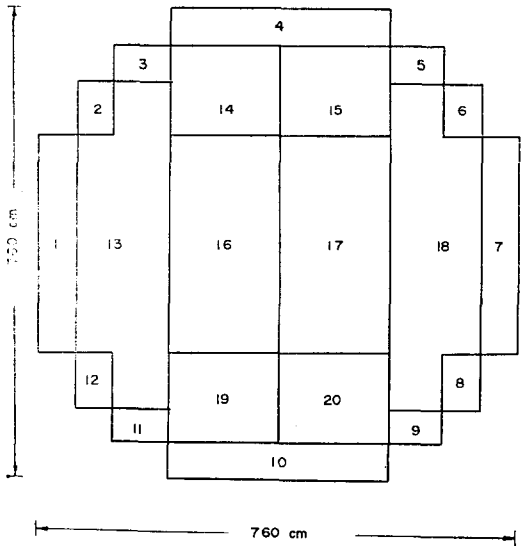


**Fig. 6. Mesh Space of Reactor Core.**

**Table 4. Relative Average Thermal Flux of Test Case 2.** (The values of $\bar{\phi}$ are in relative units)

| Time (sec) | $\bar{\phi}_2(t)^a$ ($\Delta t=0.01$ sec) | $\bar{\phi}_2(t)^b$ ($\Delta t=0.005$ sec) |
|---|---|---|
| 0.0 | 1.000 | 1.000 |
| 0.2 | 1.914 | 1.991 |
| 0.4 | 2.573 | 2.577 |
| 0.6 | 2.588 | 2.498 |
| 0.8 | 2.158 | 2.094 |
| 1.0 | 1.802 | 1.806 |
| 1.2 | 1.265 | 1.319 |
| 1.4 | 0.865 | 0.922 |
| 1.6 | 0.677 | 0.724 |
| 1.8 | 0.604 | 0.635 |
| 2.0 | 0.559 | 0.578 |
| 2.2 | 0.524 | 0.535 |
| CYBER 73 CPU (sec) | 186 | 249 |

a. $\nu\Sigma_f$ has been increased during response time.
b. $\Sigma_{a2}$ has been decreased during response time.

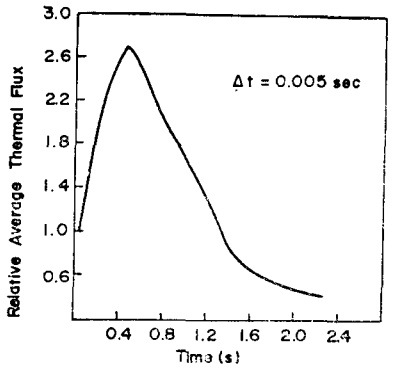


**Fig. 7. Time-Dependent Thermal Flux, Wolsung 1. NGS.**

The mesh spacing of X-and Y-directions is 28.575cm for the inner and outer core regions and 32.7cm for the reflector region.

The solution was calculated using various time steps with the two-step alternating direction explicit method.

The transient results for problem 2 are presented in Table 4 and Figures 7 through 11.

The results were indirectly compared with the results of another method[39,40]. The relative average thermal flux in Fig. 8 has she same pattern in the results of the improved quasi-static method.

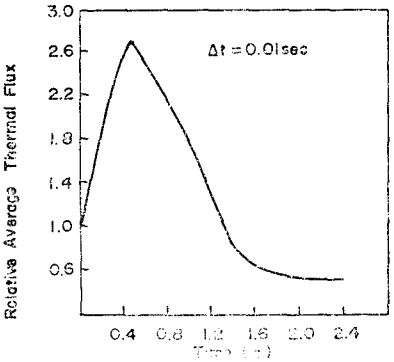


**Fig. 8. Time-Dependent Thermal Flux. Half-Core LOCA of Wolsung 1.**

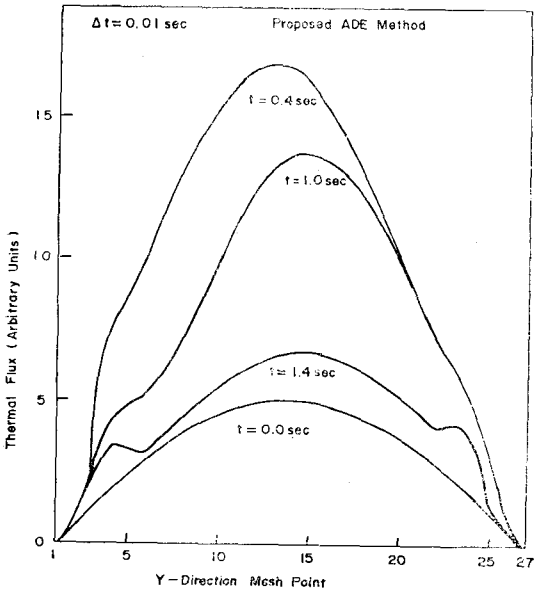


**Fig. 9. Time-Dependent Thermal Flux at X-Mesh Point 7, Wolsung-1. Problem.**

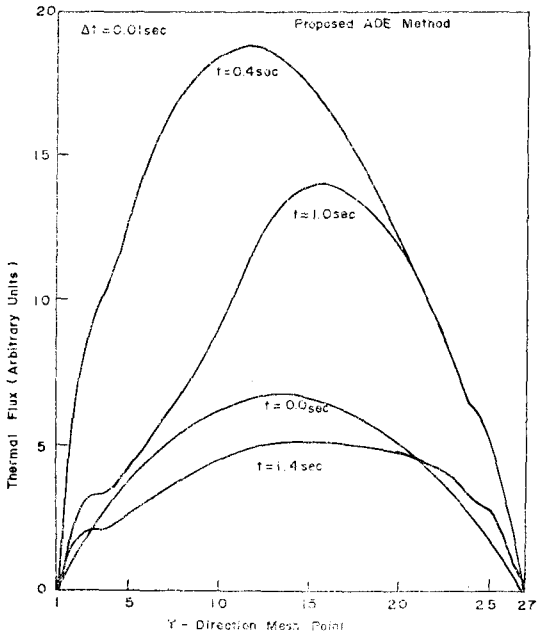


**Fig. 10. Time-Dependent Thermal Flux at X-Mesh Point 11, Wolsung-1 Problem.**

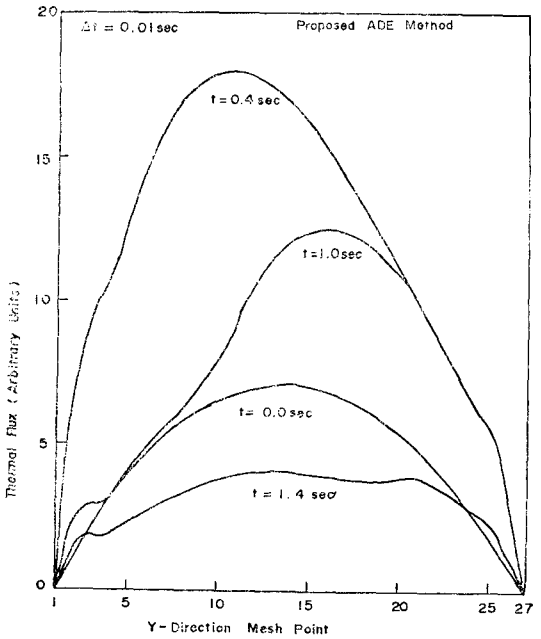


**Fig. 11. Time-Dependent Thermal Flux at X-Mesh Point 14, Wolsung-1 Problem.**

The relative average neutron power was increased during the response time (0.4 sec.

for Wolsung-1 problem) to 2.573 times of steady state and decreased in the subsequent insertions of the shutdown system.

The average neutron power at the time 1.4 sec. after the transient initiation was decreased to 0.865 times of steady state.

## VI. Discussions and Conclusions

Several important characteristics are easily observed in these numerical experiments.

There are two kinds of truncation errors in the nuclear reactor kinetics problem. One error came from the finite difference method of space and the other came from the time difference method.

The spatial and temporal truncation errors for the alternating direction explicit method have been shown to be within order $\Delta X^2$ and $h^2$, respectively. In the two test cases, the alternating direction explicit method has given acceptably accurate solutions, within a reasonable amount of time, with tolerable step sizes. It was shown to be consistent and stable with the reactor kinetics equations. Because transient results are extremely sensitive to input cross section values and input initial values, a very accurate initial flux distribution and eigenvalue must be used in the calculations. The calculation results of a simplified PWR cylindrical reactor between TWIGL and this method were comparable.

In the case of the CANDU-PHW reactor, these results were also comparable between this method and the improved quasi-static method.

As a conclusion it was found that this method represents reliable solutions to these two problems.

## Refernces.


1. S. Kaplan et al, Nucl. Sci. Eng. 18, 163 (1964).
2. M. Back et al, "On the Formulation of Time-Dependent Multichannel Synthesis Equatious", Dynamics of Nuclear Systems, D. L. Hetrick (ed), Univ. of Arizona Press, Arizona (1972).
3. J.B. Yasinsky, Nucl. Sci. Eng. 29, 381 (1967).
4. W.M. Stacey, Jr., "Spectral Synthesis Methods in Fast Reactor Dynamics", Dynamics of Nuclear Systems, D.L. Hetrick (ed), University of Arizona Press, Tucson, Arizona (1972).
5. W.M. Stacey, Jr., Nucl. Sci. Eng. 47, 449 (1972).
6. W.M. Stacey, Jr., Nucl. Sci. Eng. 34, 45 (1968).
7. Enrco Brega, Annals of Nuclear Energy, 5, 117 (1978).
8. W.M. Stacey, Jr., Nucl. Sci. Eng. 28, 438 (1967).
9. K.O. Ctt and D.A. Menelay, Nucl. Sci. Eng., 36, 402 (1969).
10. K.O. Ott, Nucl. Sci. Eng. 26, 563 (1966).
11. H.L. Dodds, Jr., Nucl. Sci. Eng. 59, 271 (1976).
12. D.A. Meneleyet al., "Fast Reactor Kinetics-The QX 1 Code", ANL-7769 (1971).
13. H.L. Dodds et al., Trans. Am. Nucl. Soc., 15, 786 (1972).
14. R. Avery, "Theory of Coupled Reactors", in Proceedings of the Second United Nations Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, Vol. 12, p. 182 (1958).
15. M. Komata, Nucl. Sci. Eng. 38, 193 (1969).
16. R.A. Shober et al, Nucl. Sci. Eng., 64, 582 (1977).
17. D.R. Harris and R.G. Fluharty, "Coupled-Region Kinetics Formulations", Dynamics of Nuclear Systems, S.L. Hetriok (ed.), Univ. of Arizona Press, Arizona (1972).
18. J.M. Sicilian, Nucl. Sci. Eng., 56, 291 (1975).
19. M. Shinkawa et al., Nucl. Sci. Eng., 67, 19 (1968).
20. K.F. Hansen and S.R. Johnson, "GAKIN, A Program for the Solution of the One-Dimensional, Multigroup, Space-Time Dependent Diffusion Equations, GA-7543, General Atomic (1967).
21. W.R. Cadwell et al., "WIGLE-A Program for Solution of the Two-Group, Space-Time Diffusion Equations in Slab Geometry", WAPD-TM-416.

Bettis Atomic Power Laboratory (1964).
22. J.B. Yasinsky et al, "TWIGL-A Program to Solve the Two-Dimensional, Two-Group, Space-Time Neutron Diffusion Equations with Temperature Feedback", WAPD-TM-743, Bettis Atomic Power Laboratory (1968).
23. B.K. Larkin, Math. Comp. **18**, 196 (1964).
24. R.S. Denning et al., Trans. Am. Nucl. Soc., **12**, 148 (1969).
25. W.H. Reed and K.F. Hansen, Nucl. Sci. Eng., **41**, 431 (1970).
26. R.S. Varga, "Matrix Iterative Analysis", Prentice-Hall, Englewood Cliffs, N. J. (1962).
27. A.L. Wight et al., Nucl. Sci. Eng., **44**, 239 (1971).
28. L.A. Hageman and J.B. Yasinsky, Nucl. Sci. Eng., **38**, 8 (1969).
29. A.L. Wight and K.F. Hansen, Trans, Am, Nucl. Soc., **12**, 620 (1969).
30. K.F. Hansen, "Finite-Difference Solutions for Space-Dependent Kinetics Equations", Dynamics of Nuclear System, D.L. Hetrick (ed.), Univ. of Arizona Press, Arizona (1972).
31. W.M. Stacy, Jr. "Space-Time Nuclear Reactor Kinetics", Academic Press, New York (1969).
32. M. Clark, Jr. and K.F. Hansen, "Numerical Methods of Reactor Analysis", Academic Press, New York (1964).
33. J.J. Duderstadt, and L.J. Hamilton, "Nuclear Reactor Analysis", John Wiley & Sons, Inc. New York (1976).
34. H. Greemspan et al., "Computing Methods in Reactor Physics", Gordon and Breach Sciene Publishers, New York (1968).
35. JEFFERT LEWINS, "Nuclear Reactor Kinetics and Control", Pergamon Press, Oxford. (1978).
36. S. Nakamura, "Computational Methods in Engineering and Science", John Wiley & Sons, New York (1977).
37. 600 MWe CANDU-PHW Wolsung-1 Nuclear Generating Station for KECO. Safety Report, June (1977).
38. 10 CFR Part 50, Appendix A, General Design Criteria.
39. F.N. McDonnel et al, Nucl. Sci. Eng., **64**, 95 (1977).
40. J.C. Luxat and G.M. Frescura, Nucl. Tech. **46**, 507 (1979).
41. D.R. Ferguson and K.F. Hansen, Nucl. Sci. Eng. **51**, 189 (1973).

## NOMENCLATURE

$g$ =index number of the energy group

$i$ =index number of the delayed neutron precursor group

$\phi_g$ =scalar neutron flux in energy group g(neutrons/cm$^2$sec)

$C_i$ =density of the $i^{th}$ precursor (cm$^{-3}$)

$v_g$ =speed of the neutrons in the $g^{th}$ group (cm/sec)

$D_g$ =diffusion coefficient for neutrons in group g. (cm)

$\chi_g$ =the fission spectrum yield in group g

$\nu_g$ =average number of neutrons per fission in group g

$\Sigma_{fg}$ =macroscopic fission cross section in group g

$\Sigma_{ag}$ =macroscopic absorption cross section in group g

$\Sigma_{sgg'}$=macroscopic scattering removal cross section from $g'$ to $g$

$\lambda_i$ =decay constant of the $i^{th}$ precursor (sec$^{-1}$)

$\chi_{gi}$ =energy spectrum of neutrons from the $i^{th}$ precursor

$\beta$ =delayed group yield fraction

$$\Sigma_{gg}=\chi_g\nu_g(1-\beta)\Sigma_{fg}-\Sigma_{ag}-\sum_{g'\neq g}\Sigma_{sg'g}$$

$$\Sigma_{gg'}=\chi_g\nu_g\Sigma_{fg'}(1-\beta)+\Sigma_{sgg'},\quad g'\neq g$$

$f_{gi}$ $=\lambda_i\chi_{gi}$=probanility (sec$^{-1}$) that $i^{th}$ precursor will yield a neutron in group $g$

$P_{ig'}$ $=\beta_i\nu_{g'}\Sigma_{fg'}$=production factor (cm$^{-1}$), for the $i^{th}$ precursor.