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《**Original**》

A Computer Code Development for Updating Reliability Data Using Bayes' Theorem and Its Application

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Bayes 정리를 이용한 신뢰도 자료 평가용 전산코드 개발 및 응용

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Abstract

A computer code, BERD (Bayesian Estimation of Reliability Data), has been developed and tested in order to update the data for the reliability analysis of safety related systems in a specific nuclear power plant.

The code has been used to derive the plant-specific data for reliability analysis of the auxiliary feedwater system of a pressurized water reactor. The prior information for components selected was taken from the U.S. Reactor Safety Study, WASH-1400, and the operating experiences from published licensee event reports. The results show that the updated data are well fitted to log-normal distribution curves and the error factors are reduced significantly.

요 약

특정 원자력발전소 안전성 계통의 신뢰도 분석을 위한 자료평가의 목적으로 전산코드를 개발하였으며 그 유용성을 입증하였다.

가압 경수로 보조급수 계통 신뢰도 분석을 위하여 개발된 전산코드를 이용하여 관련자료를 평가하였다. 이를 위하여 부품고장률의 선분포는 미국의 원자력안전성 연구보고서, 특정 발전소의 운전경험은 기발간된 인허가자 사상보고서에서 얻었다. 분석결과 후분포는 대수정규분포 곡선에 잘 접철되며 분포의 오차인자들은 현저히 감소하는 것으로 나타났다.

1. Introduction

The use of PRA(Probabilistic Risk Assessment) technique in evaluating nuclear power plant accident risks has been increased since WASH-1400, the report of the RSS (Reactor Safety Study), was published in October 1975.(1)

Although the result of the study has been discussed widely, actual application of the methodology in the evaluation of nuclear power reactor safety has not been recognized until the

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accident at the Three Mile Island occured. Subsequent to the event at TMI unit 2 in March of 1979, attitude concerning the usefulness of the PRA methodology has changed since the USNRC discovered that WASH-1400 had in fact analyzed a reactor accident similar to that occured at TMI.

Presently, several studies on individual power station have been completed or are in progress in the United States and the USNRC has prepared a comprehensive procedure guide.[2] There is no doubt that the PRA would play important roles in nuclear safety assessment in the near future.

The data analysis is an essential part in performing the PRA. However, considering the amount of work involved in PRA study, various activities should be seperated to perform. Therefore, the data analysis may be best accomplished by separating it from other probabilistic models. Apostolakis and Kaplan[3] showed that certain correlations among the failure rates must be accounted for in the probabilistic analyses so as to prevent uncertainties from underestimating. This fact indicates that proper treatment of data analysis as an integral part of the PRA is essential.

There are, in general, three types of data available; the general engineering knowledge of the design and the manufacture of the equipment in question, the historical performance of the component in other plants similar to the one in question, and the past experience in the specific plant.[4] First two types of data above constitute the "generic" data and third the "plant-specific" data, respectively. Those data are available, to varying degree of detail, for basic events such as hardware failure rates, initiating events, human error rates, test and maintenance unavailabilities, and abnormal environments, etc. Therefore, the main objective of data analysis is to integrate and to synthesize these three types of data.

The generic and the industry-wide data sources often provide a range of numbers, instead of single value, for each failure rate. For example, the RSS gives 5th and 95th percentiles of a log-normal distribution for most data which can be interpreted as "population variability curves", that is, as the curves representing the variations of the performance of individual components within the population considered.

Here we are mainly interested in the third type, that is, the plant-specific data, in order to analyse the reliability of safety related systems of a particular plant. However, only the information known at the beginning is a state of uncertainty on prior information and failure rate. Only after we have gained some operating experience with specific component, we can compute a posterior distribution which may be different from the population variability curve, and may in fact be thought of as identifying our posterior state of knowledge about the population range where our specific component falls.

There are only a few limited studies for updating the plant-specific data.[5,6,7] However, these results are neither available to us nor appropriate to be adapted here. Therefore, it is necessary to develop a methodology to obtain the plant-specific failure rate data that can be used in the related study along with a computer code programming. The basic procedure developed is first to establish a "prior" probability distribution for each failure rate of components using existing generic information, and then to specialize this distribution with specific operating experiences by adapting Bayes' theorem.

We begin in Section 2 with a brief description on Bayes' theorem, prior distribution, and likelihood functions. In Section 3 we describe a computer code, BERD.

Section 4 presents the plant-specific evaluation result on reliability data that will be used for related studies on the auxiliary feedwater system of a pressurized water reactor along with brief descriptions on generic data sources. The conclusion is presented in Section 5.

2. Theory

(1) Bayes' Theorem

The basic approach for updating the generic distributions in order to obtain plant-specific parameter estimates is to apply Bayes' theorem.[8] If a failure rate of a component, λ, which is defined as the number of failures per unit time, is the parameter of interest, we can update the datum using Bayes' theorem, which states that:

$$f(\lambda|E)=\frac{f(\lambda)L(E|\lambda)}{\int_0^\infty f(\lambda)L(E|\lambda)d\lambda} \quad (1)$$

where, $f(\lambda|E)$ = posterior distribution, the probability density function of λ, which is conditional on the evidence E,

$f(\lambda)$ = the prior distribution without having the evidence E, and

$L(E|\lambda)$ = likelihood function, the probability distribution of the evidence E for a given value of λ.

If the parameter is the probability of failure on demand, p, rather than a failure rate per unit time, λ, then λ is simply replaced by p in Eqn. 1. Only problem left in applying Eqn. 1 is to choose proper likelihood function as well as prior distribution. However, the right side of Eqn. 1 cannot be, in general, integrated analytically to be expressed in a closed form for posterior distribution. Therefore, numerical integration or a discrete approximation to the continuous distribution is required to adapt the theorem.

The discrete form of Bayes' theorem is given by

$$f(\lambda_i|E)=\frac{f(\lambda_i)L(E|\lambda_i)}{\sum_{j=1}^{n} f(\lambda_j)L(E|\lambda_j)} \quad (2)$$

where, $f(\lambda_i|E)$, $f(\lambda_i)$, and $L(E|\lambda_i)$ are discretized posterior distribution, discretized prior distribution and discretized likelihood function, respectively for given discrete set of λ_i and n is the number of discretized intervals in evaluating Eqn.2.

(2) Prior Distribution

We have considered only log-normal prior distribution at present due to two reasons.[4] First, log-normal distribution is frequently used as a prior distribution for failure rates, especially when the failure rates typically encountered are too low to make a logarithmic transformation attractive. This happens quite often for nuclear grade components. Next, the final objective of present study is to establish a PRA methodology based on WASH-1400 where the log-normal distribution is used for most components. Now we will discuss the prior distribution function briefly.

A random variable λ, such as failure rate, is identified as a log-normal distribution, if $x=\ln\lambda$ is normally distributed. By means of a simple logarithmic transformation of variable, it can be easily shown that a log-normal pdf (probability distribution function) becomes

$$f(\lambda;\xi,\sigma^2)=\frac{1}{\sigma\lambda\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}(\ln\lambda-\xi)^2\right], \quad (3)$$

where $\xi=E(\ln\lambda)$ and $\sigma^2=Var(\ln\lambda)$.

Suppose that two symmetrically located percentiles are specified for the log-normal distribution, i.e., $\lambda_{1-\gamma}$ and λ_γ where $0<\gamma<0.05$. Then, $p(\lambda<\lambda_\gamma)=p(\lambda>\lambda_{1-\gamma})=\gamma$. The geometric mean of the percentiles and the error factor are defined as $M=(\lambda_\gamma\lambda_{1-\gamma})^{1/2}$ and $EF=(\lambda_{1-\gamma}/\lambda_\gamma)^{1/2}$, respectively. With these notations, $\xi=\ln M$ and σ^2

$=\ln(EF/z_{1-\gamma})$, where $z_{1-\gamma}$ is the $100(1-\gamma)$ th percentile of a standard normal distribution. Therefore, characteristic parameters of a log-normal distribution can be obtained using following relations:[2]

$$\text{Mean} = \exp(\xi+\sigma^2/2) \quad (5)$$

$$\text{Mode} = \exp(\xi+\sigma^2) \quad (6)$$

$$\text{Median} = \exp(\xi) \quad (7)$$

$$\text{Variance} = \exp[(2\xi+\sigma^2)][\exp(\sigma^2)-1]. \quad (8)$$

It is further observed that M is the median of a log-normal distribution and that the two percentiles are $\lambda_{1-\gamma}=EF\cdot M$ and $\lambda_\gamma=M/EF$.

(3) Likelihood Function

We have considered three types of likelihood functions, i.e., binomial, Poisson, and Pascal distribution functions. These functions are described below.

Binomial Distribution Function: A binomial distribution is the distribution of the number of failures, r, out of n independent demands or trials, on each component which has a constant failure-on-demand probability, p. Given this statistical framework, the likelihood function in Eqn. 1 is the binomial distribution given by

$$L(E|p)=\frac{n!}{r!(n-r)!}p^r(1-p)^{n-r} \quad (9)$$

for $r=0, 1, 2, \ldots\ldots, n$ and the parameter p has a value between 0 and 1. If the parameter p is small enough, then Eqn. 9 will usually be most conveniently approximated by the Poisson distribution,

$$L(E|p)=\frac{1}{r!}(np)^r\exp(-np) \quad (10)$$

where r can be any nonnegative integer value because the number of demands is so large in comparison with the number of failures.

Poisson Distribution Function: A common assumption in reliability models is that independent of a common exponential (constant failure rate) behavior. It follows that the distribution of the number of failures, r, in fixed total operating time, T, has Poisson distribution. In this case the likelihood function used in Eqn. 1 is the Poisson density function given by

$$L(E|\lambda)=\frac{1}{r!}(\lambda T)^r\exp(-\lambda T) \quad (11)$$

where λ denotes the constant failure rate.

Pascal Distribution Function: In binomial distribution the sample size is held fixed at n and r is a random variable. However, if the conditional probability, p, is given so that n trials will be required to produce s failures, a Pascal distribution should be selected, which is given by

$$L(n|p)=\frac{(n-1)!}{(s-1)!(n-s)!}p^{(n-s)}(1-p)^s \quad (12)$$

where, $n=2,\ 2+1, \ldots$ and $s=1, 2, \ldots$ However, since this kind of sampling is rarely taken in nuclear power plants, the use of this distribution is limited.

3. Computer Code BERD

A general purpose computer code, BERD, has been developed in order to update the generic or the industry-wide reliability data using plant-specific operating experience with component of interest. Since prior distribution depends on the specific data source and the failure modes are somewhat different depending on the data collection method employed, the code has been programmed to add various prior distributions, posterior distributions, and likelihood functions with user supplied subroutines even though present version is limited to log-normal distribution as prior information.

As mentioned previously, since Eqn. 1 cannot be, in general, applied analytically, BERD employs discretized parameters in order to use Eqn. 2. The level of discretization can be expanded as much as available computer memories. We have tested up to several hundreds of discretizations. However, we have prefixed it to 10 levels considering computer time and we achieved sufficient accuracy. The discretizing interval is computed so as to have a symmetric form in histogram typed plot. In the case that the posterior distribution locates near the tail of prior distribution, the discretizing range may be revised according to the intent of analysists. BERD is programmed for the CDC machine at KAERI, and is run on interactive mode in order to give analysist full flexibilities.

The brief computation procedures are as follows:

Step 1: Give selected prior distribution parameters to compute other prior parameters. Since the present version of BERD accept log-normal distribution only, a set of 5th and 95th percentile values or median and standard deviation can be given in order to compute the parameters given in Eqns. 5 to 8.

Step 2: Discretize prior distribution, and evaluate likelihood function and posterior distribution. Since BERD utilizes system library function for a standard normal distribution of the CDC machine, which gives the integration limit value from negative infinite with given probability, a search scheme to get the probability value for given integration limit is programmed by Fibonacci method.[9] After computing the likelihood function, the distributions are normalized since functional values are often too small for the computer to handle due to large negative exponent in Eqns. 9 to 12.

The posterior distribution obtained is fitted to log-normal distribution. The fitting procedure is as follows. Since a log-normal distribution as given in Eqn. 3 can be transformed to a standard normal distribution with variable transformation, we can get following cumulative normal distribution after some manipulation:

$$F(x_i)=\int_{-\infty}^{\alpha+\beta x_i}\frac{1}{\sqrt{2\pi}}\exp(-w^2/2)\,dw \qquad (13)$$

where $\alpha=-\mu/\sigma, \beta=1/\sigma,\ x_i=\ln\lambda_i$, and $\mu=\ln\xi$. Therefore, following relation holds.

$$E[f(p_i)]=\alpha+\beta x_i \qquad (14)$$

The fitting of posterior distribution data is simply to obtain parameters, α and β in Eqn. 14, and the parameters can be calculated by a standard least-square fitting method with proper weightings which are calculated as the inverse values of functional heights corresponding to the data points. The goodness-of-fit test employing χ^2-test procedure is performed after fitting the data.[10]

Step 3: Plot the data (prior and posterior distribution curves), and choose proper discretizing limit and go to step 2 if required, Otherwise, go to next case.

In order to verify the computation, we select two cases given by Apostolakis, et al.[5]

These are as follows:

Case 1. Diesel Generator-Does Not Start: The frequency of failure to start per demand, Q, is given by the RSS as a log-normal distribution with $Q_{05}=10^{-2}$ and $Q_{95}=10^{-1}$, 5th and 95th percentiles, respectively. The operating experiences for this particular case is 5 failures in 227 trials or demands.

Case 2. Diesel Generator- Does Not Continue to Run: In the RSS, the frequency of failure per hour is given as a log-normal distribution with $Q_{05}=3\times10^{-4}$ and $Q_{95}=3\times10^{-2}$. The operating experiences are 9 failures in 398.03 hours.

We have updated the reliability data using BERD for above two cases. The plotted results along with all the parameters are shown in Figs.1 and 2.

In Table 1, the summarized results for the above cases are shown, and compared with the previous study.[5] It is also shown that the posteriors are well fitted with log-normal distribution since χ^2 -tests are well within its limitation. Compared with our result(Table 1), the minor differences in these values are judged negligible. χ^2-test results are 9.60×10^{-3} and 1.23×10^{-2}, respectively for each case. Therefore, we judged that the data are well fitted on

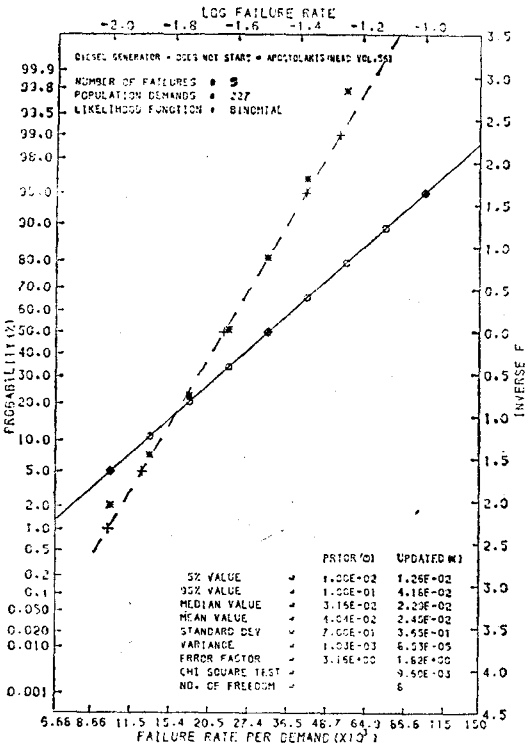

Fig. 1. Plotted Result of BERD Run for Case 1

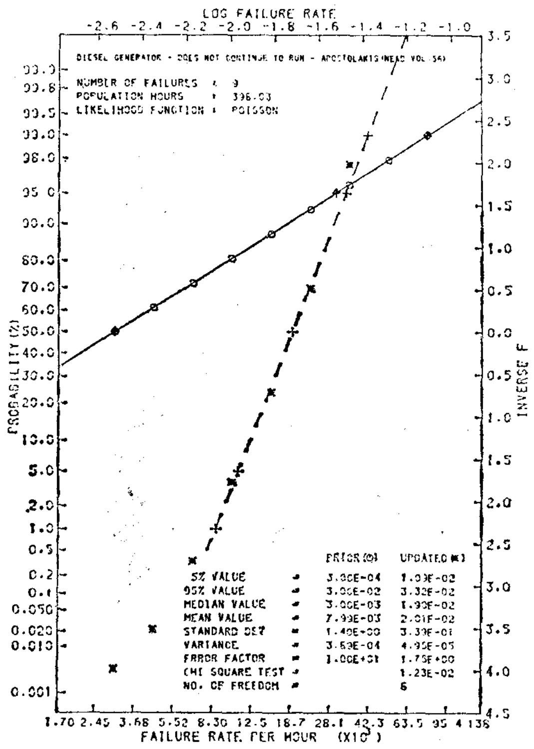

Fig. 2. Plotted Result of BERD Run for Case 2

Table 1. Summary of BERD Calculational Results

Distribution Parameter	Does Not Start		Does Not Continue to Run	
	Ref. 5	Present Study	Ref. 5	Present Study
5%	1.29×10^{-2}/d	1.26×10^{-2}/d	1.08×10^{-2}/hr	1.08×10^{-2}/hr
95%	4.15×10^{-2}/d	4.18×10^{-2}/d	2.25×10^{-2}/hr	3.32×10^{-2}/hr
Median	2.32×10^{-2}/d	2.29×10^{-2}/d	1.93×10^{-2}/hr	1.90×10^{-2}/hr
Mean	2.50×10^{-2}/d	2.45×10^{-2}/d	2.00×10^{-2}/hr	2.01×10^{-2}/hr
Variance	$8.20\times10^{-5}/d^2$	$8.53\times10^{-5}/d^2$	$4.70\times10^{-5}/hr^2$	$4.95\times10^{-5}/hr^2$
χ^2-Test	—	9.60×10^{-3}	—	1.23×10^{-2}

log-normal probability curves since the values are far below χ^2-test limit of $\chi^2_{0.95,9}=3.325$.[10]

4. Data Specialization

In order to update the plant-specific reliability data with operating experiences of the component in that plant, one must take proper generic data that establish prior distribution. However, for many components, there are no existing data sources with a content and format that allow the selection of a prior distribution. For example, it is not always specified what failure modes are represented, for what environments the data are applicable, etc. Therefore, it is sometimes evident that certain engineering judgement must be exercised in order to select proper prior distribution.

Two dominant sources of failure rates for nuclear industry, i.e., the RSS and IEEE Standard-500, are briefly discussed below.

WASH-1400 Appendix III (Failure Data):

Table 2. Prior and Updated Parameters for Diesel Generator Valve, and Pump

Component	Failure Mode	WASH-1400 Data 5%	95%	Median	EF	Operating Experience No. of Failures	Population Hours (Demands)
Diesel Generator (Complete Set)	Does Not Start	1×10^{-2}/d	1×10^{-1}/d	3.16×10^{-2}/d	3.19	1	(312)
						1	(72)
	Does Not Continue to Run	3×10^{-4}/hr	3×10^{-2}/hr	3.00×10^{-3}/hr	10.00	0	312.00
						0	72.00
Valve(Motor-Operated)	Fail to Operate	3×10^{-4}/d	3×10^{-3}/d	9.49×10^{-4}/d	3.16	2	(660)
	Fail to Remain Open	1×10^{-7}/hr	1×10^{-6}/hr	3.16×10^{-7}/hr	3.16	0	1445400.0
Valve[2] (Air-Operated)	Fail to Operate	1×10^{-4}/d	1×10^{-3}/d	3.16×10^{-4}/d	3.16	2	(1960)
	Fail to Remain Open	1×10^{-7}/hr	1×10^{-6}/hr	3.16×10^{-7}/hr	3.16	1	4304016.0
Valve(Manual)	Fail to Remain Open	3×10^{-5}/d	3×10^{-4}/d	9.49×10^{-5}/d	3.16	0	(672)
	Leak Externally	1×10^{-9}/hr	1×10^{-7}/hr	1.00×10^{-8}/hr	10.0	0	1471680.0
Valve(Check)	Fail to Open	3×10^{-5}/d	3×10^{-4}/d	9.49×10^{-5}/d	3.16	0	(576)
	Leak Internally	1×10^{-7}/hr	1×10^{-6}/hr	3.16×10^{-7}/hr	3.16	2	1261440.0
Standby Pump (Motor-Driven)	Does Not Start	3×10^{-4}/d	3×10^{-3}/d	9.49×10^{-4}/d	3.16	0	(630)
	Does Not Continue to Run	3×10^{-6}/hr	3×10^{-4}/hr	3.00×10^{-5}/hr	10.0	1	287856.0
Standby Pump (Turbine-Driven)	Does Not Start	3×10^{-4}/d	3×10^{-3}/d	9.49×10^{-4}/d	3.16	0	(210)
	Does Not Continue to Run	3×10^{-6}/hr	3×10^{-4}/hr	3.00×10^{-5}/hr	10.0	0	95952.0

Component	Failure Mode	Updated Data 5%	95%	Median	EF	χ^2-Test(1)	Remark
Diesel Generator (Complete Set)	Does Not Start	4.89×10^{-3}/d	1.98×10^{-2}/d	9.85×10^{-3}/d	2.01	2.0×10^{-3}	weekly testing
		9.27×10^{-3}/d	4.90×10^{-2}/d	2.13×10^{-2}/d	2.30	3.3×10^{-3}	monthly testing
	Does Not Continue to Run	2.14×10^{-4}/hr	5.40×10^{-3}/hr	1.07×10^{-3}/hr	5.02	5.0×10^{-3}	weekly testing
		2.74×10^{-4}/hr	1.25×10^{-2}/hr	1.85×10^{-3}/hr	6.77	3.5×10^{-3}	monthly testing
Valve(Motor-Operated)	Fail to Operate	5.64×10^{-4}/d	3.92×10^{-3}/d	1.49×10^{-3}/d	2.64	2.0×10^{-3}	
	Fail to Remain Open	8.96×10^{-8}/hr	7.28×10^{-7}/hr	2.55×10^{-7}/hr	2.85	1.9×10^{-4}	plugged
Valve[2] (Air-Operated)	Fail to Operate	1.88×10^{-4}/d	1.31×10^{-3}/d	4.97×10^{-4}/d	2.64	1.9×10^{-3}	
	Fail to Remain Open	1.08×10^{-7}/hr	6.61×10^{-7}/hr	2.67×10^{-7}/hr	2.48	2.9×10^{-3}	plugged
Valve(Manual)	Fail to Remain Open	2.95×10^{-5}/d	2.85×10^{-4}/d	9.16×10^{-5}/d	3.11	3.5×10^{-6}	plugged
	Leak Externally	9.92×10^{-10}/hr	9.09×10^{-8}/hr	9.50×10^{-9}/hr	9.58	3.8×10^{-5}	rupture
Valve(Check)	Fail to Open	2.95×10^{-5}/d	2.87×10^{-4}/d	9.21×10^{-5}/d	3.12	2.5×10^{-6}	
	Leak Internally	1.70×10^{-7}/hr	1.49×10^{-6}/hr	5.03×10^{-7}/hr	2.95	3.7×10^{-3}	
Standby Pump (Motor-Driven)	Does Not Start	2.62×10^{-4}/d	2.02×10^{-3}/d	7.28×10^{-3}/d	2.78	3.1×10^{-4}	including driver
	Does Not Continue to Run	1.68×10^{-6}/hr	1.63×10^{-5}/hr	5.23×10^{-6}/hr	3.11	1.9×10^{-2}	〃
Standby Pump (Turbine-Driven)	Does Not Start	2.84×10^{-4}/d	2.58×10^{-3}/d	8.56×10^{-4}/d	3.01	3.7×10^{-5}	including driver
	Does Not Continue to Run	1.29×10^{-6}/hr	3.44×10^{-5}/hr	6.66×10^{-6}/hr	5.17	1.7×10^{-3}	〃

(1) Number of freedoms for all cases are 9 ($\chi^2_{0.95,9}=3.325$)

(2) Failure records for Air-Operated Valves are for Westinghouse designed PWR.

This document contains the failure data used in the RSS, including raw data from the U.S. reactor experiences in the year of 1972, notes on test time, notes on maintenance time and frequency, the results of human reliability analyses, estimates of the frequency of initiating events, and some informations on common cause failures, etc.

From the assembled information, this appendix defines the assessed range as given for the diesel generators in Section 3.

IEEE Standard-500 Data Manual[11]: This document contains data for electronic, electrical, and sensing components. The reported values are obtained from the opinions of about 200 experts.

Each expert has reported a low, a recommended, and a high values for the failure rates under normal condition and a maximum value that would be applicable under all conditions including abnormal conditions. Each value was estimated by using geometric averaging method. Even though the standard does not specify any distribution at all, the averaging method itself indicates that it would be consistent to assume a log-normal distribution. In order to determine the distribution parameters, with the given information, Apostolakis, et al[5] suggested to use the recommended value as the median value, and to calculate error with the maximum and the low values assuming that distribution is log-normal.

We have selected some of components, i.e., diesel generator, valve, and pump, in updating reliabiliy data based on the following two requirements. First, in order to develop a PRA methodology we started to analyse the reliability of the auxiliary feedwater system given in WASH-1400(Surry unit 1). Next, we simplified the analyses by restricting only to mechanical components. Therefore, the failure modes for those components are also consistent with WASH-1400. The information on operating experiences with components in Surry unit 1 is obtained from the data summaries of licensee event reports for diesel generators,[12] pump,[13] and valves[14] at U.S. nuclear power plants published by the USNRC.

With the operating experiences and the prior information, we have run BERD to get the updated data. The results are given in Table 2 in condensed form. As shown in Table 2, the updated data are well fitted to log-normal distributions for all cases.

In Table 2, it can be easily identified that the overall data have shifted to reliable side except few cases, and the error factors have in general reduced. In addition, the χ^2-test results indicate that the posteriors are well fitted to log-normal distributions.

The reliability analysis of AFWS with two data sets, i.e., WASM-1400 data and the updated data, has been performed. The result shows that the over-all reliability of the AFWS is about same for both sets, but the uncertainty is reduced as expected. The result will be published later.[15]

5. Conclusion

We have developed a method for updating plant-specific reliability data based on Bayes' theorem, and demonstrated its usefulness.

All updated data are well fitted to log-normal distribution curves. Therefore, it is believed that the log-normal distribution assumed in WASH-1400 can be applied to the reliability data based on present operating experience. The error factors of resultant distributions have been reduced. Even though the USNRC reports[12,13,14] presented 95th percentile value of all the accessed data as the median for the cases with no failure record, we judge that the updated data should be used for consistency for other cases. Most of the data show that the reliability data are shifted to reliable sides.

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