# 彈性体로 支持된 粘彈性体層에서의 剛性体의 運動解析

## A Study on the Sliding Rigid Indentor over the Viscoelastic Layer Supported by the Elastic Half-Space

南 俊祐* · 趙 宣彙** · 全 三杓***
J. W. Nam · S. W. Cho · S. P. Chun

### 要　約

강성체로된 牽引物體가 彈性無限平面으로 지지된 점탄성층 위를 미끄러져 갈때 접촉구간에서의 압력분포와 마찰 특성을 고찰하였다.

즉, 접촉구간에서의 강성체의 모양과 압력분포에 관한 적분 방정식을 구하고, 점탄성층의 두께가 접촉구간에 비하여 충분히 두꺼울 때 압력분포와 마찰계수의 근사해를 구하였다. 압력분포의 모양은 점탄성층의 물성을 표시하는 지수값, 즉 $\alpha<1/2$, $\alpha=1/2$, $\alpha>1/2$ 에 따라서 크게 다르다.

한편, 수치해석에 의하면 마찰 계수에 대한 근사해는 강성체의 미끄럼 속도, 점탄성 층의 두께, 탄성체의 영율($E_o$)과 점탄성층의 時效性彈性계수($E_v$)의 比, 즉 $E_o/E_v$ 에 따라 변화함을 알 수 있다.

즉, 탄성체가 점탄성층에 비하여 딱딱하면 할수록, 또 강성체 속도가 느리면 느릴수록 마찰계수는 작아진다.

그리고 物性의 지수($\alpha$)가 커지면 커질수록 근사해의 수렴 속도는 느려지게 되고 지수($\alpha$)가 1에 가까와지면 점탄성층의 탄성효과는 점성효과에 비하여 거의 무시할 수 있으며 근사해는 의미가 없어지게 된다.

### NOMENCLATURE

| | | |
|---|---|---|
| $A_1, A_2, B_1, B_2$ | : | Arbitrary functions of p. |
| B(p, q) | : | Beta function. |
| Eo | : | Young's Modulus of elastic half-space [kgf/cm²] |
| Ev | : | Quasi-elastic modulus of viscoelastic layer [$kgf/cm^2 sec^{\alpha}$] |
| Go | : | Shear modulus of elastic half-space [kgf/cm²] |
| G(t) | : | Shear relaxation modulus of viscoelastic layer. |
| E(t) | : | Relaxation modulus of viscoelastic layer in uniaxial tension. |
| Cf | : | Frictional coefficient [–] |
| K | : | Derived constant (=3-4n) [–] |

* : 國民大·工大 機械設計學科
** : 서울大 · 工大 機械設計學科
*** : 서울大 · 大學院

| | | |
|---|---|---|
| P | : | Dimensionless total load [−] |
| U | : | Sliding velocity of rigid indentor [!/sec] |
| C | : | S-coordinate of apex of cylindrical indentor [cm] |
| k | : | Coloumb's frictional coefficient [−] |
| h | : | Thickness of viscoelastic layer [cm] |
| Ch(t),Sh(t) | : | Hyperbolic functions. |
| u | : | Displacement x-component [cm] |
| v | : | Displacement y-component [cm] |
| sgn(p) | : | Sign of P [+. −] |
| S | : | Galilean variable, $s=x+U_t$ [cm] |
| f(s) | : | Shape function of rigid indentor |
| g(s) | : | Dimensionless pressure distribution in the contact interval $= \frac{3}{2} \frac{\sigma_y(s,o)}{U^{\alpha}E_v \{(1-d)\}}$. |
| $\bar{g}(p,y)$ | : | Fourier transform function $= (2\pi)^{-1/2} \int_{-\infty}^{\infty} g(s,y).e^{-ips}\, ds.$ |
| $\Lambda(t)$ | : | Bulk relaxation modulus of viscoelastic layer. |
| $\Gamma(x)$ | : | Gamma function. |
| $\theta$ | : | Dimensionless thickness of viscoelastic layer. = h/w [−] |
| v(t) | : | Poisson's ratio of viscoelastic layer [−] |
| $\sigma_x, \sigma_y, \tau_{xy}$ | : | Stress components [kgf/cm$^2$] |
| $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ | : | Strain components [−] |

## 1. Introduction

Sliding and rolling contact problems in linear viscoelastic subject have received considerable attention during the last decades.

Through an experimental work on the friction between a rolling rigid sphere and balsam wood surfaces with varying lubrication, Atack and Tabor showed the resistance to be independent of the state of lubrication, and concluded that the rolling friction arose primarily from viscoelastic properties of balsam wood (1).

And similiar results were obtainied from the experiments with other materials, i.e. rubbery and glassy polymers.

The studies on this phenomenum were made by Hunter(2), Morland (3), (4), Golden (5), and Nachmann, Walton and Schapery (6), (7) by using the linear viscoelastic model.

Walton, Nachmann and Schapery treated sliding contact problems by direct Fourier transformation for the power-law (8) viscoelastic half-space, and derived a closed-form of analytical solution (6).

In this paper, it was attempted to suggest the approximate method to obtain the pressure distribution and frictional coefficient under a sliding rigid indentor over the viscoelastic layer supported by elastic half-space.

## 2. Formulation of Problem

As shown in Fig. 1., a rigid indentor is sliding with steady translational velocity U to the right over the viscoelastic layer, whose thickness is $\theta$ termed dimensionlessly, supported by elastic half-space.

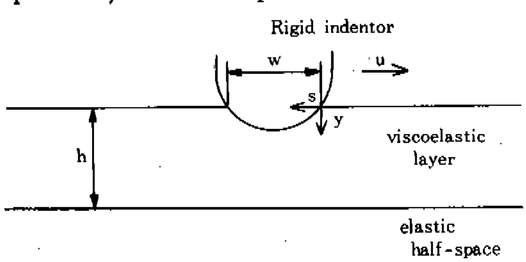


Fig. 1. Sliding rigid indentor over the viscoelastic layer supported by elastic half-space.

### 2-1. Basic assumptions

In order to simplify the problem, the following assumptions are submitted.

1) The particular problem interested in this paper is the two dimensional problem.
2) The sliding of a rigid indentor is the steady translational motion i.e., U is contant.

### 2-2. Formulation of an integral equation

The mathematical formulation on force balance is as follows:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0 \quad \cdots\cdots (2-1)$$

$$\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=0 \quad \cdots\cdots (2-2)$$

Relations for linear elastic stress-displacement are (8):

$$\sigma_x=\frac{2Go}{1-2\eta}\left[(1-\eta)\frac{\partial u}{\partial x}+\eta\frac{\partial v}{\partial y}\right] \quad \cdots\cdots (2-3)$$

$$\tau_{xy}=Go\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right] \quad \cdots\cdots (2-4)$$

$$\sigma_y=\frac{2Go}{1-2\eta}\left[\eta\frac{\partial u}{\partial x}+(1-\eta)\frac{\partial v}{\partial x}\right] \quad \cdots\cdots (2-5)$$

Relations for the linear viscoelastic stress-strain relation are:

$$\sigma_x=\int_{-\infty}^{t}\Lambda(t-\tau)\frac{\partial\Delta}{\partial\tau}d\tau+2\int_{-\infty}^{t}G(t-\tau)\frac{\partial\varepsilon_x}{\partial\tau}d\tau \quad \cdots\cdots (2-7)$$

$$\sigma_y=\int_{-\infty}^{t}\Lambda(t-\tau)\frac{\partial\Delta}{\partial\tau}d\tau+2\int_{-\infty}^{t}G(t-\tau)\frac{\partial\varepsilon_y}{\partial\tau}d\tau \quad \cdots\cdots (2-8)$$

$$\tau_{xy}=\int_{-\infty}^{t}G(t-\tau)\frac{\partial\gamma_{xy}}{\partial\tau}d\tau \quad \cdots\cdots (2-9)$$

where, $\varepsilon_x=\frac{\partial u}{\partial x}$ $\quad \varepsilon_y=\frac{\partial v}{\partial y}$

$$\gamma_{xy}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}$$

$$\Delta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$$

Owing to the governing equations (2-1)~(2-9), stresses and displacements at the boundary could be obtained (9).

For algebraic convenience, the particular problems in which the viscoelastic layer is supposed to be imcompressible i.e., $\nu = 1/2$ and no Coloumb's friction exists i.e., k = 0, are discussed in this article.

From the continuity of stresses and displacements i.e., u, v at the interface between the viscoelastic layer and elastic half-space, and the boundary conditions at the surface of viscoelastic layer on which a rigid indentor is sliding, the integral equation is obtained as following (6):

$$f'(s)=\frac{1}{(2\pi)}\int_{-\infty}^{\infty}dp\,\Phi_o(h)\frac{(ip)^{1-\alpha}}{|p|}e^{ips}\int_0^1 g(\xi)e^{-ip\xi}d\xi \quad \cdots\cdots (2-10)$$

where, $g(\xi)=\frac{3}{2}\frac{\sigma_y(\xi,0)}{U^{\alpha}E_{\nu}\Gamma(1-\alpha)}$

Here the function $\phi_O(h)$ is to be put as

$$\phi_0(h)=\frac{\phi_3(h)+i\,\phi_4(h)}{\phi_1(h)+i\,\phi_2(h)} \quad \cdots\cdots (2-11)$$

where,

$$\Phi_1(h)=4K(ch^2(|h|)+sh^2(|h|))+2[(K+1)ch(|h|)sh(|h|)-(K-1)h^2]\cdot t\cos\left(\frac{\alpha\pi}{2}\right)\left(\frac{|h|}{\theta}\right)^{\alpha}+(sh^2(|h|)-h^2)t^2\cos(\alpha\pi)\left(\frac{|h|}{\theta}\right)^{2\alpha}$$

$$\Phi_2(h)=2[(K+1)ch(|h|)sh(|h|)-(K-1)h^2]t\sin\left(\frac{\alpha\pi}{2}\right)\left(\frac{|h|}{\theta}\right)^{\alpha}+(sh^2(|h|)-h^2)t^2\sin(\alpha\pi)\left(\frac{|h|}{\theta}\right)^{2\alpha}$$

$$\Phi_3(h)=4K(ch(|h|)sh(|h|)-(|h|)+[2(K-1)|h|$$

$$+(K+1)\left(ch^2(|h|)+sh^2(|h|)\right)\big]\cdot t\cos\left(\frac{\alpha\pi}{2}\right)\left(\frac{|h|}{\theta}\right)^{\alpha}+\left(|h|+ch(|h|)sh(|h|)t^2\cos(\alpha\pi)\left(\frac{|h|}{\theta}\right)^{2\alpha}\right.$$

$$\Phi_4(h)=\big[2(K-1)|h|+(K+1)\left(ch^2(|h|)+sh^2(|h|)\right)\big]t\sin\left(\frac{\alpha\pi}{2}\right)\left(\frac{|h|}{\theta}\right)^{\alpha}+\left(|h|+ch(|h|)sh(|h|)\right)t^2\sin(\alpha\pi)\left(\frac{|h|}{\theta}\right)^{2\alpha}$$

where, $t=\dfrac{KU^{\alpha}E_v\Gamma(1-\alpha)}{3G_0}$

## 3. Thick layer problem.

After mathematical treatments the following integral equation is obtained.

$$f'(s)=\frac{\Gamma(1-\alpha)}{\pi}\left[\int^{\prime}\frac{g(\xi)d\xi}{(s-\xi)^{1-\alpha}}-\cos(\alpha\pi)\int_0\frac{g(\xi)d\xi}{(s-\xi)^{1-\alpha}}\right]-\frac{1}{\pi}\cdot\theta^{\alpha-1}\int_0^1 d\xi g(\xi)\int_0^1 dh\,h^{-\alpha}\left[\left(\frac{\phi_1\phi_3+\phi_2\phi_4}{\phi_1^2\ \ \phi_2^2}+1\right)\sin\beta+\frac{\phi_1\phi_4-\phi_2\phi_3}{\phi_1^2+\phi_2^2}\cos\beta\right]\cdots\cdots(3\text{-}1)$$

where, $\beta=\dfrac{h}{\theta}(s-\xi)-\dfrac{\alpha\pi}{2}$

Since,

$$\frac{\phi_1\phi_3+\phi_2\phi_4}{\phi_1^2+\phi_2^2}-1,\quad \frac{\phi_1\phi_4-\phi_2\phi_3}{\phi_1^2+\phi_2^2}$$

are exponentially small for large h, the integral equation may be written as follows,

$$f'(s)=\frac{\Gamma(1-\alpha)}{\pi}\left[\int_s^1\frac{g(\xi)d\xi}{(\xi-s)^{1-\alpha}}-\cos(\alpha\pi)\int_0^s\frac{g(\xi)d\xi}{(\xi-s)^{1-\alpha}}\right]-\sum_{n=0}^{\infty}\frac{a_n}{\theta^{n+1-\alpha}n!}\int_0^1 g(\xi)(s-\xi)^n d\xi\cdots(3\text{-}2)$$

where,

$$a_n=\frac{1}{\pi}\int_0^{\infty}dh\,h^{n-\alpha}\left[\left(\frac{\phi_1\phi_3+\phi_2\phi_4}{\phi_1^2+\phi_2^2}-1\right)\sin\left[\left(n-\alpha\right)\frac{\pi}{2}\right]+\frac{\phi_1\phi_4-\phi_2\phi_3}{\phi_1^2+\phi_2^2}\cdot\cos\left[(n-\alpha)\frac{\pi}{2}\right]\right]\cdots\cdots(3\text{-}3)$$

If it is assumed that viscoelastic layer is thick i.e., $\theta\gg 1$, f(s) and g(s) can be expressed as asymptotic expansions in terms of gage functions $\varepsilon^{p_1}$, $\varepsilon^{p_2}$, etc.

$$f'(s)=f_0'(s)+\varepsilon^{p_1}f_1'(s)+\varepsilon^{p_2}f_2'(s)+\cdots\cdots(3\text{-}4)$$

$$g(s)=g_0(s)+\varepsilon^{p_1}g_1(s)+\varepsilon^{p_2}g_2(s)+\cdots\cdots(3\text{-}5)$$

where, $\varepsilon=1/\theta$

$p_n=n-\alpha$

Thus, the expansion terms are found in the uncoupled integral equations as,

$$f_0'(s)=\frac{\Gamma(1-\alpha)}{\pi}\left[\int_s^1\frac{g_0(\xi)d\xi}{(\xi-s)^{1-\alpha}}-\cos(\alpha\pi)\int_0^s\frac{g_0(\xi)d\xi}{(s-\xi)^{1-\alpha}}\right]\cdots\cdots(3\text{-}6a)$$

$$f_1'(s)+a_0\int_0^1 g_0(\xi)d\xi)=\frac{\Gamma(1-\alpha)}{\pi}\left[\int_s^1\frac{g_1(\xi)d\xi}{(\xi-s)^{1-\alpha}}-\cos(\alpha\pi)\int_0^s\frac{g_1(\xi)d\xi}{(s-\xi)^{1-\alpha}}\right]\cdots\cdots(3\text{-}6b)$$

$$f_2'(s)+a_1\int_0^1 g_0(\xi)(s-\xi)d\xi+a_0\int_0^1 g(\xi)d\xi=\frac{\Gamma(1-\alpha)}{\pi}\left[\int_s^1\frac{g_2(\xi)d\xi}{(\xi-s)^{1-\alpha}}-\cos(\alpha\pi)\int_0^s\frac{g_2(\xi)d\xi}{(s-\xi)^{1-\alpha}}\right]\cdots\cdots(3\text{-}6c)$$

In each case of integral equations (3-6), the right-hand side is always the same generalized Abel-type integral operator.

Therefore, if the coefficients $a_n$ are computed by numerical integration and the shape of indentor i.e., f'(s) is given, approximation to normal traction and frictional coefficient can be found.

## 4. Example and discussions.

The problem for a cylindrical indentor of radius R is considered.

$$f(s) = -\frac{(s-c)^2}{2R} + d \quad \text{for} \quad 0 \leq s < 1 \quad \cdots\cdots (4\text{-}1)$$

Here, only three terms in the asymptotic expanisions of f'(s) and g(s) are sought, and the rest terms are neglected (10).

The solutions of first terms equation (3-6a), differently behave for $\alpha < 1/2$, $\alpha = 1/2$, and $\alpha > 1/2$.

when $\alpha < 1/2$, $\alpha > 1/2$.

$$g_0(s) = \frac{R^{\alpha-1}}{\Gamma(1-\alpha)} \int_s^1 \xi^{\frac{1}{2}-\alpha}(1-\xi)^{-\frac{1}{2}}\, d\xi$$

$$\frac{R^{\alpha-1}\Gamma\left(\frac{1}{2}\right)\left(\frac{1}{2}-\alpha\right)}{\Gamma(1-\alpha)\,\Gamma(2-\alpha)\,\Gamma\left(\alpha+\frac{1}{2}\right)}$$

$$\int_s^1 \frac{\xi^{-\frac{1}{2}}(1-\xi)^{\alpha-\frac{1}{2}}}{(\xi-s)^{\alpha}}\, d\xi \cdots\cdots\cdots\cdots (4\text{-}2)$$

where,

$$\int_0^1 (\xi - c_0)\,\xi^{-1/2}(1-\xi)^{-\alpha-1/2} d\xi = 0 \cdots\cdots (4\text{-}3)$$

$$\text{i.e.,}\ c_0 = \frac{1}{2\,(1-d)} \cdots\cdots\cdots\cdots (4\text{-}4)$$

when $\alpha = 1/2$,

$$g_0(s) = 2[(1-s)/\pi R^2]^{1/2} \cdots\cdots\cdots\cdots (4\text{-}5)$$

$$\text{with}\ C_0 = 1 \cdots\cdots\cdots\cdots (4\text{-}6)$$

The preceding work exhibits the first approximations to P and Cf as following

$$p_0 = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}-\alpha\right)R^{\alpha-2}}{(1-\alpha)\,\Gamma(3-\alpha)} \cdots\cdots\cdots\cdots (4\text{-}7)$$

$$C_{f0} = \alpha(2-\alpha)/R\,(1-\alpha)\,(3-\alpha) \cdots\cdots\cdots\cdots (4\text{-}8)$$

From the second terms equation (3-6b), no bounded solution exists unless $C_1 = a_0 P_0 R$ and then $g_1(s) = 0$, (10). Therefore, the second approximation to p and $C_f$.

$$P_1 = 0 \cdots\cdots\cdots\cdots (4\text{-}9)$$

$$C_{f1} = -a_0 P_0 \cdots\cdots\cdots\cdots (4\text{-}10)$$

The solution of third terms equation (3-6c) may be found from eqs. (4-2), (4-3) and (4-5) by the simple expedient of replacing R by 1 and $g_0(s)$ by $-g_2(s)/a_1 P_0$.

$$c_2 = \frac{Ra_1}{2(1-\alpha)}\left(RC_{f0} + (c_0-1)P_0\right) \cdots\cdots (4\text{-}11)$$

From this last results, third approximation to P and Cf are obtained as follows,

$$P_2 = -1/a_1 \cdots\cdots\cdots\cdots (4\text{-}12)$$

$$C_{f2} = \frac{a_1}{2(1-\alpha)}\left[RC_{f0}\frac{0}{f} + (c_0-1)P_0\right] \cdots\cdots\cdots (4\text{-}13)$$

Finally, the approximations to g(s), c,p, Cf, are obtained as,

$$g(s) = \left(1 - \frac{\varepsilon^{P_1}}{R^{\alpha-1}\alpha_1 p_0}\right) g_0(s) \cdots\cdots\cdots (4\text{-}14)$$

$$c = c_0 - \varepsilon^{P_1} a_0 P_0 R + \frac{\varepsilon^{P_1} Ra_1}{2(1-\alpha)}\,(RCf_0 + (c_0-1)P_0) \cdots\cdots\cdots (4\text{-}15)$$

$$P = P_0 \frac{\varepsilon^{P_1}}{a_1} \cdots\cdots\cdots\cdots (4\text{-}16)$$

$$Cf = Cf_0 - \varepsilon^{P_1} a_0 P_0 + \frac{\varepsilon^{P_1} a_1}{2(1-\alpha)}\left[RC_{f0} + (c_0-1)P_0\right] \cdots\cdots\cdots (4\text{-}17)$$

According to the sign of $a_0$ in (4-15), (4-17) the shift of the apex of indentor and the variation of the frictional coefficient due to finiteness of layer are expected.

Fig. 2. represents the pressure distribution

expressed demensionlessly in the contact interval of indentor sliding over layer.

The curves go off very differently with the values of α.

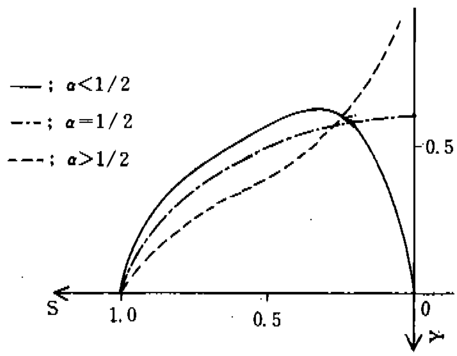


Fig. 2. Characteristics of normal component, g(s) to indentor surface along the contact interval.

The values of coefficients, $a_n$ are computed by numerical integration of the equation (3-3).

From the equations (3-3), (2-11) the values of coefficients, $a_n$ depend upon $t/\theta^{\alpha}$, if α is prescribed.

In this paper, the particular problem in which Poisson's ratio of elastic half-space, $\eta$ is 0.5 and the velocity of rigid indentor, U is 1000/sec, is considered.

Then, since $t/\theta^{\alpha} = \Gamma(1-\alpha)(U/\theta)^{\alpha}/(Eo/E_V)$, the values of coefficients, $a_n$ depend upon 1/(Eo/Ev).

From Table 1, Table 2, as the value of $E_O/E_V$ increases, the value of dominant coefficient, $a_o$ decreases.

And, as the value of $E_O/E_V$ approaches to infinity, the value of dominant coefficient, $a_o$ approaches to the value when rigid support exists.

From the equations (4-15), (4-16), (4-17) and Table 1, Table 2, the values of P,C, and $C_f$ are computed.

In order that the three terms approximation converges rapidly, the values of α and θ are chosen as 0.25 and 5x10, 0.5 and $5x10^3$, 0.75 and $5x10^7$.

The Fig. 3, shows that the apex of cylindrical indentor shifts to the right as $E_O/E_V$ increases.

The Fig. 4 shows that the frictional coefficient decreases, as $E_O/E_V$ increases.

When α is 0.75, the values of C and $C_f$ are nearly constant.

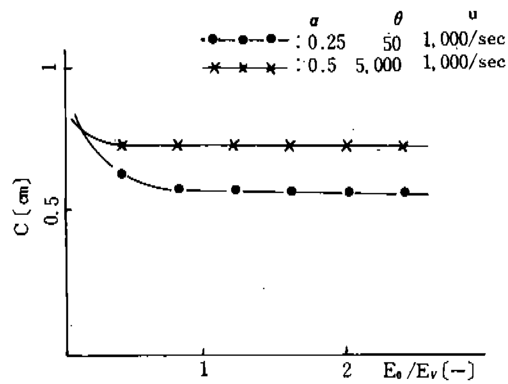


Fig. 3. Apex of cylindrical identor vs. ratio of moduli.

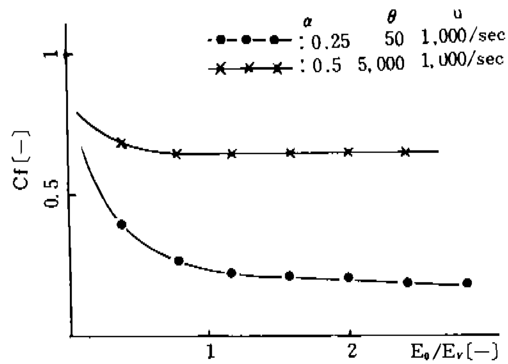


Fig. 4. Frictional coefficient vs. ratio of moduli.

Physically speaking, the harder elastic half-space material becomes, the lesser the frictional coefficient becomes, and ultimately the value of $C_f$ approaches to the value when rigid support exists.

The increase of $(\theta/U)^{\alpha}$ is equivalent to the increase of $E_O/E_V$.

Therefore, it can be shown that the value of $E_O/E_V$ has less influence on the friction coefficient, as the viscoelastic layer beocmes thicker.

Also, the faster the rigid indentor is sliding, the larger the frictional coefficient becomes.

Table 1. Related values of coefficient with $\alpha=0.25$, $\theta=50$

| $E_O/E_V$ \ $a_n$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|
| 0.10 | −0.355 | 2.451 | 0.671 | 3.885 |
| 0.25 | 0.049 | 1.447 | 0.365 | −3.096 |
| 0.50 | 0.083 | 0.864 | 0.138 | −2.334 |
| 0.75 | 0.134 | 0.581 | 0.013 | −1.815 |
| 1.00 | 0.161 | 0.404 | −0.067 | −1.420 |
| 1.25 | 0.178 | 0.280 | −0.124 | −1.105 |
| 1.50 | 0.190 | 0.187 | −0.166 | −0.844 |
| 1.75 | 0.198 | 0.114 | −0.198 | −0.625 |
| 2.00 | 0.204 | 0.055 | −0.224 | −0.436 |
| 2.25 | 0.209 | 0.006 | −0.244 | −0.273 |
| 2.50 | 0.230 | −0.357 | −0.261 | −0.129 |
| ∞ | 0.236 | −0.547 | −0.398 | 2.162 |

Table 2. Related values of coefficient with $\alpha-0.5$, $\theta=5,000$

| $E_O/E_V$ \ $a_n$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|
| 0.10 | 0.386 | 0.502 | 0.013 | −1.592 |
| 0.25 | 0.409 | −0.332 | −0.745 | −0.745 |
| 0.50 | 0.519 | −0.095 | −0.489 | −0.075 |
| 0.75 | 0.528 | −0.180 | −0.540 | 0.270 |
| 1.00 | 0.534 | −0.227 | −0.564 | 0.479 |
| 1.25 | 0.535 | −0.257 | −0.577 | 0.618 |
| 1.50 | 0.535 | −0.278 | −0.585 | 0.718 |
| 1.75 | 0.536 | −0.293 | −0.590 | 0.793 |
| 2.00 | 0.536 | −0.304 | −0.593 | 0.851 |
| 2.25 | 0.537 | −0.314 | −0.595 | 0.898 |
| 2.50 | 0.537 | −0.321 | −0.597 | 0.936 |
| ∞ | 0.538 | −0.391 | −0.605 | 1.311 |

## 5. Conclusion

It is studied, in this paper, how the pressure distribution and frictional properties appear differently in connection with the sliding velocity, $\alpha$ values of viscoelastic material specified with power law and its thickness of layer supported by elastic half-space when a rigid indentor is dragging over the layer.

In order to determine the appearance of pressure distribution and frictional coefficient along the contact interval, an adequate integral equation is formulated. Through the computation with the formulated mathematical equation, the followings are derived.

The pressure distribution deviates widely in dependance on the values of exponent $\alpha$, which represents the physical properties of the viscoelastic layer (Fig. 2).

And frictional coefficient varies with the velocity of the rigid indentor, thickness of viscoelastic layer and the ratio of Young's modulus of elastic half-space ($E_O$) to quasi-elastic modulus of viscoelastic layer ($E_V$), expressed in form of $E_O/E_V$ (Fig. 3,4).

## References


(1) D. Atack, D. Tabor, "The friction of wood," Proceedings of the Royal Society of London, London, England, Series A, Vol. 246, 1958, pp.539-555.

(2) S.C. Hunter, "The rolling contact of a rigid cylinder with a viscoelastic half-space," J. Appl. Mech., 1961.

(3) L.W. Morland, "A plane problem of rolling contact in linear viscoelasticity theory," J. Appl. Mech., 1962.

(4) L.W. Morland, "Exact solutions for rolling contact between viscoelastic

cylinders," Q.J. Mech. and Appl. Math., 1967.

(5) J.M. Golden, "Hysteric friction of a plane punch on a half plane with arbitrary viscoelastic behavior," Q.J. Mech. & Appl. Math., 1977.

(6) A. Nachman, J.R. Walton, R.A. Schapery, "The sliding of a rigid indentor over a power-law viscoelastic half-space," Q.J. Appl. Math., 1978.

(7) A. Nachman, J.R. Walton, "The sliding of a rigid indentor over a power-law viscoelastic layer," J. Appl. Mech., 1978.

(8) A.C. Pipkin, "Lectures on viscoelasticity theory," Springer Verlag, 1972.

(9) I.N. Sneddon, "Application of integral transforms in the theory of elasticity," Springer-Verlag, 1975.

(10) F.D. Gakhov, "Boundary-value problems," Pergamon press, 1966.