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Estimated Action Potentials During Repolarization Phase from the Body Surface Electrocardiogram

(心電圖의 再分極相에서의 活動電位의 推定)

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要 約

심전도는 심장근육세포와 흉분세포들의 활동전위의 전파에 따른 전계에 의해 발생한다. 활동전위의 재분극상은 임상적 요인에 매우 민감하다.

따라서, 역심전도에 관한 본 논문에서는, 디지탈 신호 추정방법으로 심전도의 재분극상에서의 일정활동전위(uniform action potential)를 추정하는 방법을 연구하였다.

추정된 정상인의 활동전위는 임상적 자료와 비슷한 재분극상을 보였다.

Abstract

The body surface ECG(electrocardiogram) is produced by the electric fields caused by the propagation of action potentials within the myocardial cells. The repolarization phase of the action potential is very sensitive to factors of clinical importance.

Therefore, in this paper of the inverse electrocardiography, we studied a method of estimating the uniform action potentials during repolarization phase from the body surface ECG using digital signal identification techniques.

The estimated action potential of a normal was similar to that of clinical data in the repolarization phase.

I. Introduction

The t-wave of the ECG is determined by the repolarization process in the excitable and conductive heart muscles. During this repolarization period, the intracelluar potential of the excitable cells returns slowly to its negative resting potential starting from the large and fast action potential condition of the depolarization phase.

There are two approaches about the analysis of the T-wave. One is the reconstruction of the T-wave with the dipolar source three-dimensional heart model and the associated formulas.[9] The other is the reconstruction of the action potential with variations of the Hodgkin-Huxley mode.[10]

We have studied a new method of finding

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the action potential of the repolarization phase with the body surface ECG and the digital system identification method.[8]

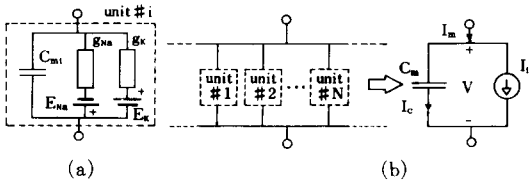

Fig. 1. (a) The equivalent circuit for the Hodgkin-Huxley units.
(b) The global circuit for the repolarization process.

The uniform action potential (V) is considered to arise in an idealized condition when an action potential is initiated in homogeneous segment groups of cardiac muscle such that the potential changes are uniform through the whole muscle without further propagation.[1]

This conidition is closely satisfied in the repolarization phase, as the shape of the repolarization is not critically dependent on whether the propagation occurs or not.[1] At this condition of no propagation, the membrane current (I_m) becomes zero in Fig. 1, and the capacitance current (c) becomes equal and opposite to the net ionic current (I_i). Thus, the uniform action potential and the net ionic current are related as follows;

$$-C_m \frac{dV}{dt} = I_i \quad [\mu A/cm^2] \tag{1}$$

where C_m [$\mu F/cm^2$] is the membrane capacitance.

For the relationship between the surface ECG as the output and the electric current in myocardial tissue as the input, we can make the following simplifications based upon the physiological considerations:

i) The body channel from the myocardial tissue's electric current flow to the surface ECG can be approximated as a linear time-invariant system.[4],[7]

ii) The magnitude of the net ionic current in the repolarization phase is extremely small as compared with the impulse-like depolarization current.[1] (Noble and Tsien, 1972)

The time-invariance in the first condition provides that the system transfer functions relating the myocardial tissue's electric current to the surface ECG are same both for the repolarization and the depolarization process. Also, the linearity means that the two independent sources in the input make an additive and homogeneous output field. Then using the above two simplifications, the system function of the body channel can be obtained from the impulse input of the depolarization phase, and, this system function, in turn, can be used to estimate the action potential of the repolarization phase from the T-wave of ECG.

In the present paper, we present a method of estimating the uniform action potential of the repolarization phase from the surface ECG.

II. Methods

1. Parameter Indentification

The net ionic current through the whole cardiac cycle can be represented as the sum of the fast and large impulse ($\delta(n)$) occurring during only the depolarization time and a slow and small ionic current, u(n), as follows;

$$i(n) = \delta(n) + u(n) \tag{2}$$

In the discrete-domain, the input-output relation between the body surface ECG and the net ionic current is described by the system function, H(z), as follows;

$$G(z) = H(z)\, I(z) \tag{3}$$

where G(z) is the z-transform o the sampled ECG, and I(z) is the z-transform of i(n) of eq.(2).

Also, using the time-invariance and the linearity, the discrete system function H(z) between the sampled ECG and the net ionic current is represented by eq.(4).[4],[7]

$$H(z) = -\frac{a_0 + \sum_{j=1}^{M-1} a_j z^{-j}}{1 + \sum_{i=1}^{N-1} b_i z^{-i}} = \frac{A(z)}{B(z)} \qquad (4)$$

where $a_0 \neq 0$. Then, the transform of eq.(2) becomes,

$$I(z) = 1 + U(z). \qquad (5)$$

and $U(z) = u_0 + u_1 z^{-1} + \ldots + u_{k-1} z^{-K+1}$, where $u_0, u_1, \ldots, u_{k-1}$ is the discrete ionic current and K is the total date number.

Combining eq.(3), (4) and (5),

$$(a_0 + a_1 z^{-1} + \ldots + a_{M-1} z^{-M+1})(1 + U(z))$$

$$= (1 + b_1 z^{-1} + \ldots + b_{N-1} z^{-N+1})(g_0 + g_1 z^{-1} + \ldots + g_{K-1} z^{-K+1}) \qquad (6)$$

where $\{g_0, g_1, \ldots, g_{K-1}\}$ is the sampled ECG data.

The net current u_i is zero during the depolarization phase of QRS period, and the order M in eq.(4) is much smaller than the total sample number during QRS period. Also, themagnitudes of u_i are extremely small during the repolarization period as compared with the impulse-like depolarization current. Therefore, the left side coefficients of eq.(6) can be approximated as follows;

$$\begin{aligned}
&a_0(1 + u_0) \cong a_0 \\
&a_0 u_1 + a_1(1 + u_0) \cong a_1 \\
&\quad\vdots \qquad\qquad \vdots \\
&a_0 u_{M+1} + \ldots + a_{M-1}(1 + u_0) \cong a_{M-1} \\
&a_0 u_M + \ldots + a_{M-1} u_1 \triangleq e_M \\
&a_0 u_{M+1} + \ldots + a_{M-1} u_2 \triangleq e_{M+1} \\
&\quad\vdots \qquad\qquad \vdots \\
&a_0 u_{K-1} + \ldots + a_{M-1} u_{K-M} \triangleq e_{K-1}
\end{aligned} \qquad (7)$$

where e_i (i = M,M+1,⋯,K-1) represents the error terms[5],[6] caused by the non-negligible net ionic current during the long repolarization period. ($M \ll n \leq K-1$)

Let $\underline{a} = (a_0\ a_1\ \cdots\ a_{M-1})^T$, $\underline{b} = (1\ b_1\ \cdots\ b_{N-1})^T$, $\underline{e} = (e_M\ e_{M+1}\ \cdots\ e_{K-1})$ and

$$\underline{U} = \begin{bmatrix} u_M & u_{M+1} & \cdots & u_{K-1} \\ u_{M-1} & u_M & \cdots & u_{K-2} \\ \vdots & \vdots & & \vdots \\ u_1 & u_2 & \cdots & u_{K-M} \end{bmatrix}^T$$, then we can derive eq.(8),

$$\underline{e}^T \underline{e} = \underline{a}^T \underline{U}^T \underline{U}\, \underline{a}. \qquad (8)$$

And we can estimate the system parameters a_i's and b_i's by minimizing $\underline{u}^T \underline{u}$, the diagonal terms of $\underline{U}^T \underline{U}$, where $\underline{U}^T \underline{U}$ is a positive definite symmetric matrix. Eq.(6) is reduced to eq.(9) using the relation between the z-transform and the convolution.[5],[6]

$$\sum_{n=0}^{N-1} b_n g_{i-n} = \begin{cases} a_i & \text{for } i = 0,1,2,\cdots,M-1 \\ e_i & \text{for } i = M,M+1,\cdots,K-1 \end{cases} \qquad (9)$$

The existence of roots (a_i's and b_i's) is guaranteed for a condition of $K > N + M - 1$, i.e. the data size is larger than the sum of orders N, M and eq.(9) can be represented in matrix form as follows;

$$\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{M-1} \\ \hline e_M \\ e_{M+1} \\ \vdots \\ e_{K-1} \end{bmatrix} = \begin{bmatrix} g_0 & & & 0 \\ g_1 & g_0 & & \\ \vdots & \vdots & \ddots & \\ g_{M-1} & g_{M-2} & & g_0 \\ \hline g_M & g_{M-1} & \cdots & g_1 \\ g_{M+1} & g_M & \cdots & g_2 \\ \vdots & \vdots & & \vdots \\ g_{K-1} & g_{K-2} & & g_{K-N} \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ \vdots \\ b_{N-1} \end{bmatrix} \qquad (10\text{-a})$$

or

$$\begin{bmatrix} \underline{a} \\ \hline \underline{e} \end{bmatrix} \qquad \begin{bmatrix} \underline{H}_1 \\ \hline \underline{H}_2 \end{bmatrix} \begin{bmatrix} \underline{b} \end{bmatrix} \qquad (10\text{-b})$$

where $\underline{a}, \underline{b}, \underline{e}$ H , and $\underline{H}_2$ are the associated vectors and matrices.

As the minimization of $\underline{u}^T\underline{u}$ means that the diagonal terms of matrix $\underline{U}^T\underline{U}$ has the minimization condition for arbitrary values of a_i's, this condition is same as minimizing $\underline{e}^T\underline{e}$ of eq.(8). In this case, we can obtain the solutions of eq.(10-b) using the Shanks' method.[5],[6];

$$\underline{b}^* = -(\underline{H}_3{}^T\underline{H}_3)^{-1}\ \underline{H}_3{}^T\underline{h}^1 \tag{11}$$

$$\underline{b} = \begin{bmatrix} 1 \\ \hline \underline{b}^* \end{bmatrix} \tag{12}$$

$$\underline{a} = \underline{H}_1\ \underline{b} \tag{13}$$

where $\underline{h}^1$, $\underline{H}_3$ are the first column vector and the residual matrix of $\underline{H}_2$, respectively.

2. Depolarization Process

The above estimation of H*(z) from $\underline{a}$ and $\underline{b}$ of eq.(4) is used to obtain the impulse response, d*(n), of the depolarization process, which is related to the QRS waveform of the body ECG. The optimal gain K_p is introduced to compensate the effect of decreasing magnitude of the impulse response due to variations of the measured magnitudes in ECG's R-waveform.

The QRS portion related to the ventricular depolarization process, d*(n), is given in eq.(14),

$$d^*(n) = DFT^{-1}(K_p \cdot H^*(z)) = \sum_{r=0}^{K-1} h(n-r)\ K_p\delta(n). \tag{14}$$

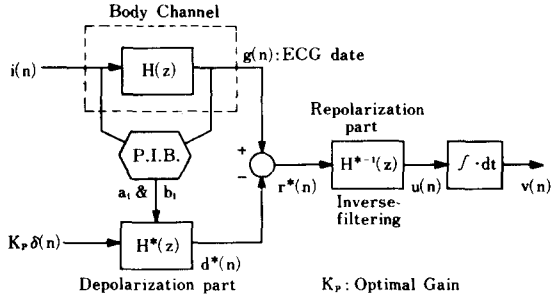

Fig. 2. The schematic diagram of the whole system (P.I.B. = Parameter Identification Block).

where h(n) is the inverse transform of H*(z).

This computation procedure of depolarization process d*(n) is shown as the parameter identification block (P.I.B.) and the depolarization part in the schematic diagram of Fig. 2.

3. Inverse Filtering and Repolarization Process

The repolarization process is computed using the above computed H*(z) and d*(n) as shown in Fig. 2. Since the ECG can be represented as the sum of the depolarization process, d(*n), and the repolarization process, r*(n), the measured ECG can be represented as follows;

$$g(n) = d^*(n) + r^*(n) \tag{15}$$

Also, from eq.(2),

$$g(n) = \sum_{s=0}^{K-1} h(n-s)\ i(s)$$

$$= \sum_{s=0}^{K-1} h(n-s)\ (K_p\delta(s) + u(s)). \tag{16}$$

Then, from eq.(14) and (16), the T-wave of ECG is related to the computed repolarization process, r*(n), as the repolarization input current, u(n), produces the T-wave.

$$r^*(n) = g(n) - d^*(n) = \sum_{s=0}^{K-1} h(n-s)\ u(s) \tag{17}$$

In the transform domain, eq.(17) is represented as,

$$R^*(z) = H^*(z)\ U(z) = \frac{A(z)}{B(z)}\ U(z) \tag{18}$$

In the transform domain, eq.(17) is represented as,

Then, we can compute the net ionic current of the repolarization process of u(n) using the following Gauss-elimination method.

From eq.(15), $A(z)\ U(z) = B(z)\ R^*(z) \triangleq T(z)$ (19)

where A(z) and B(z) are the numerator and the denominator polynomial of H*(z), respectively.

Then, by rearranging eq.(19), it becomes

$$T(z) = t_0 + t_1 z^{-1} + \cdots + t_{K-1} z^{-K+1} \tag{20}$$

where $t_0 = b_0 r_0$, $t_1 = b_0 r_1 + b_1 r_0$, ..., $t_{K-1} = b_0 r_{K-1} + b_1 r_{K-2} + \ldots + b_{N-1} r_{K-N}$.

Since K is much larger than N, the sequences related to $n \geq K$ period are excluded in both sides of the equation. The associated matrix is given in eq.(21).

$$\begin{bmatrix} a_0 & & & & 0 \\ a_1 & a_0 & & & \\ a_2 & a_1 & a_0 & & \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \\ a_{M-1} & a_{M-2} & & & \\ & a_{M-1} & \cdot & & a_0 \\ 0 & & \cdot & & \\ & a_{M-1} & \cdot\cdot\cdot\cdot & a_1 & a_0 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_{K-1} \end{bmatrix} = \begin{bmatrix} t_0 \\ t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ t_{K-1} \end{bmatrix} \tag{21}$$

From the above equation, u(n) can be computed using the numerical method of simultaneous equations, where the solution in eq.(21) is satisfied by a condition $a_0 \neq 0$. This condition is same as that the determinant of the matrix in the left side of eq.(21) can not be zero which is shown in eq.(4) previously.

As the sequence u(n) represents the net ionic current in repolarization phase in eq.(1), the uniform action potential V can be computed by the integration of u(n) with a constant value of C_m. The analysis procedures are summarized in the flow-chart of Fig. 3.

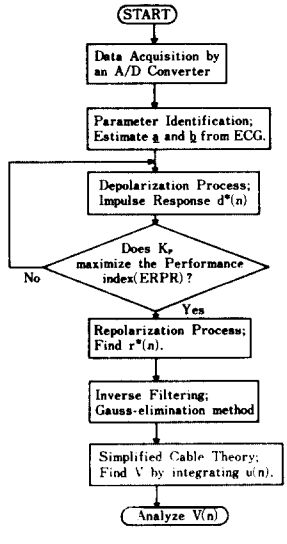

Fig. 3. The analysis procedures.

III. Results and Discussions

The uniform action potential, V(n), was computed from the measured ECG waveform, as shown in Fig. 4.

The ECG data of a normal (80 b.p.m.) were sampled every 5 msec through the 8-bit A/D converter installed in the APPLE II PLUS microcomputer system, and were normalized such that 1V has a magnitude of one. The total 128 sampled data for the whole cardiac phase were used for computation of the parameters a_i's and b_i's in eq.(4) for the fourth-order equal-pole-zero transfer function.

The optimal gain of compensator, K_p, was chosen when the following performance index ERPR (ECG signal to Repolarization signal Power Ratio) which is defined has the maximum value so that the fluctuation of r*(n) in the depolarization phase can be diminished:

$$\text{ERPR} \triangleq \sqrt{\frac{\sum_{n=0}^{K-1} g(n)^2}{\sum_{n=0}^{K-1} r^*(n)^2}} \tag{22}$$

As shown in Fig. 4, the computed results show that V(n) has a similar waveform as the reported uniform action potentials in Fig. 5.[1] The fluctuating u(n) in the vicinity of the R-wave is caused by the difference between the ECG and the impulse response in the depolarization process.

The initial value, V(0), for integration in

eq.(23) was adjusted such that V(n) around the flat region in the depolarization has the zero level.

$$V(n) = V(n-1) - \frac{\Delta t}{C_m} u(n) = V(n-1) - \frac{1}{C_m^*} u(n) \tag{23}$$

where $C_m^* = C_m/\Delta t$, and C_m^* is related to the membrane capacitance in eq.(1) which we selected as the proportional constant between the voltage-current relationship.

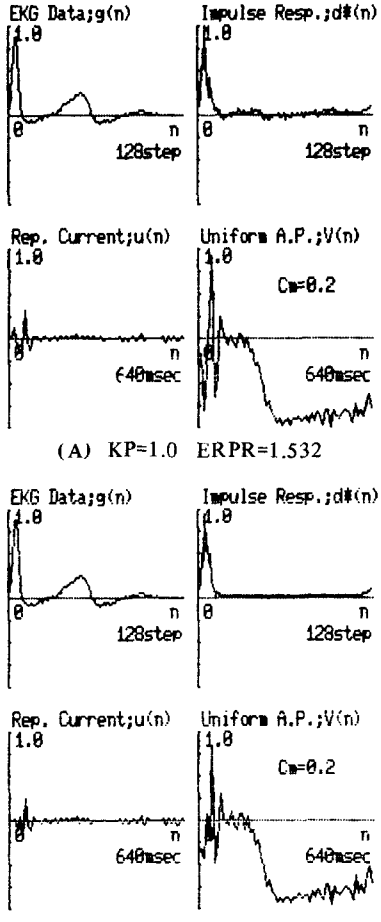

Fig. 4. A: The normal ECG data g(n), B: The impulse response d*(n), C: The repolarization input current (net ionic current) u(n), D: The uniform action potential V(n), the scale of C and D is a half of that of A and B.

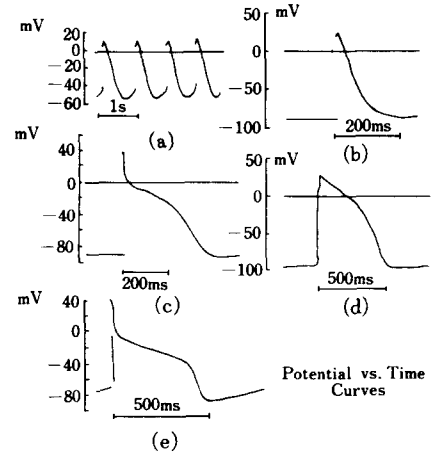

Fig. 5. Action potentials and pacemaker activity recorded in different parts of the heart.
(a) Recorded from frog sinus venosus (Hutter and Trautwein 1956). (b) Recorded from dog atrium by Hoffman and Suckling (from Weidmann 1956). (c) Recorded from dog Purkinje fibre by Draper and Weidnmann (1951) (photograph from Fokow and Neil (1971)). (d) Recorded from frog ventricle by Hoffman (from Weidmann 1956). (e) Recorded from sheep Purkinje fibre (Weidmann 1956).

Table 1. The estimated parameter data of the fourth-order system function H*(z).

Coefficients of the numerator a and b	
$a_0 = 2.740\times10^{-2}$	$b_0 = 1.000$
$a_1 = 2.329\times10^{-1}$	$b_1 = -1.500$
$a_2 = 1.791\times10^{-1}$	$b_2 = 5.469\times10^{-1}$
$a_3 = 1.325\times10^{-1}$	$b_3 = 3.253\times10^{-1}$
$a_4 = 4.743\times10^{-2}$	$b_4 = -2.394\times10^{-1}$
Estimated error of $H^*(z) = 2.668\times10^{-2}$	

Table 2. The optimal gain K_p and its ERPR.

Optimal gain K_p	ERPR
$K_p = 1.3$ (optimal)	ERPR = 1.592 (max.)
$K_p = 1.0$	ERPR = 1.532

It is shown in Fig. 4 that the resting potential of V(n) has negative value for proper V(0)'s. And the initial value, V(0), and C_m* are relative factors which make V(n) lie in the template region of the action potential. The analysis procedures were constructed with hte UCSD PASCAL (V2.1) language system. The computed parameters are summarized in Table 1, and 2, and as a result of characteristic equation in Table 1, H*(z) is stable as the poles are in the unit circle of the z-plane.

It is concluded that the estimated uniform action potential of normal ECG in Fig. 4 has a similar waveform and a rate as that in Fig. 5, but only the repolarization region is significant in V(n) of Fig. 4 because eq.(1) and eq.(23) mean much to the repolarization process.

Therefore, the suggested method will be helpful to analyze certain arrhythmias occuring in the repolarization area (T-wave) such as myocardial infarctions and ischemias, etc.

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