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Design of a New PN Sequence Waveform for Spread Spectrum Communication

(대역 확산 통신에 쓰이는 새로운 PN 시퀀스 파형의 설계)

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要 約

본 논문에서는 대역 확산 다중통신 시스템에 쓰이는 가상 잡음 시퀀스의 설계에 관하여 연구하였다. 구형파의 일반형인 새로운 파형을 제안하여 분석한 결과 파형폭이 줄어듦에 따라 다중통신간섭(multiple access interference)이 0으로 접근하였다. 이 파형은 구형파나 정현파에 비해 훨씬 좋은 특성을 나타낸다.

Abstract

This paper is concerned with the design of a pseudonoise (PN) sequence used in direct-sequence spread spectrum multiple-access (DS/SSMA) communication systems. Here we propose a new waveform, a generalized version of rectangular waveform, which can reduce the multiple access interference to zero as the pulse width becomes narrower. It gives far better performance than either rectangular or sine waveform.

I. Introduction

In recent years, there has been considerable interest in communication systems with spread spectrum and multiple access capabilities. The spread spectrum system can be used in many areas such as secure communication, ranging and so forth. For satellite applications, spread spectrum modulation yields a conceptually simple solution to the requirement of lowering radio frequency interferences among the users. In satellite communication a multiple access technique is normally used so that many users can share the bandwidth and transmission capability of a single wideband repeater.

In the spread spectrum multiple access (SSMA) system, one important subsystem is the pseudonoise (PN) sequence generator. Generation of the PN sequence has been studied by many researchers.[1],[2] In this paper we are concerned with the design of a PN sequence generator with emphasis on the code chip waveform. Until the late 1970's only rectangular waveform has been considered for direct sequence SSMA (DS/SSMA) system analysis. In 1980, however, the use of sine waveform was proposed for SSMA analysis.[3] It has been found that the use of sine waveform instead of rectangular waveform decreases the multiple access interference by about 12%.

In this paper, we propose a new code chip wave form expressed as $\psi(t) = \sqrt{T_C/\epsilon}\,\pi((t-\frac{T_C}{2})/\epsilon)$, where T_C is the code bit duration. If ϵ is

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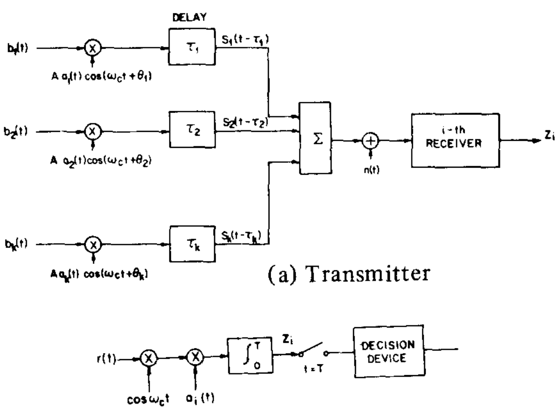
substituted by T_c, this reduces to the rectangular waveform. Thus, the proposed waveform can be regarded as the generalized version of rectangular waveform. In this paper we analyze the performance of the DS/SSMA system using the proposed waveform. By doing so, it will be shown that the multiple access interference can be reduced to zero by reducing ϵ to zero (i.e., impulse waveform), thus yielding far better performance than that with the conventional waveforms.

In what follows, we first describe the system model of DS/SSMA. Then, we propose a new code chip waveform and analyze the system performance with the proposed waveform. Finally, we make conclusions.

II. System Model

The DS/SSMA system model to be considered is shown in Fig. 1. In the figure, $b_i(t)$



(a) Transmitter

(b) Receiver.

Fig. 1. DS/SSMA model

is the binary data signal, $a_i(t)$ is the spectral spreading signal (i.e., PN sequence), and τ_i and θ_i are the relative time delay and phase angle, respectively. The data signal can be expressed as

$$b(t) = \sum_{\ell=-\infty}^{\infty} b_\ell \, \pi\left(\frac{t-\frac{T}{2}-\ell T}{T}\right)$$

where T is the data bit duration, b_ℓ is the binary data sequence (i.e., $b_\ell \in \{+1, -1\}$ for each ℓ) and $\pi((t-\frac{T}{2})/T)$ is the rectangular pulse of duration T which starts at t=0. Similarly, the spectral spreading signal can be expressed as

$$a(t) = \sum_{j=-\infty}^{\infty} a_j \, \psi(t - jT_c)$$

where T_c is the code bit duration, aj is a periodic sequence of elements of $\{+1, -1\}$, and $\psi(\cdot)$ is a time-limited (limited to $[0, T_c]$) signal, commonly referred to as the code chip waveform for which

$$T_c^{-1} \int_0^{T_c} \psi^2(t)dt = 1. \qquad (1)$$

The received signal may be written as

$$r(t) = n(t) + \sum_{k=1}^{K} S_k(t-\tau_k)$$

$$= n(t) + \sum_{k=1}^{K} A a_k(t-\tau_k)\, b_k(t-\tau_k) \cdot \cos(\omega_c t+\phi_k) \qquad (2)$$

where

$$\phi_k = \theta_k - \omega_c \tau_k$$

and n(t) is an additive white Gaussian noise process. The i-th receiver is assumed to be a correlation receiver (or matched filter) whose impulse response is matched to the i-th signal. Thus the output of the i-th receiver is

$$Z_i = \int_0^T r(t)a_i(t) \cos\omega_c t dt \qquad (3)$$

$$= \eta_i + \frac{1}{2}AT\{b_0^{(i)} + \sum_{\substack{k=1\\k\neq i}}^{K} I_{k,i}(\underline{b}_k, \tau_k, \phi_k)\} \qquad (4)$$

where η_i is a random variable given by

$$\eta_i = \int_0^T n(t)a_i(t) \cos\omega_c t dt,$$

and $I_{k,i}(\cdot)$ is the multiple-access interference at the output of the i-th receiver due to the k-th signal.

As in [4], we define the signal-to-noise ratio (SNR) for the i-th receiver by

$$SNR_i = E\{Z_i | b_o^{(i)} = +1\} \cdot [\mathrm{var}\{Z_i | b_o^{(i)} = +1\}]^{-1/2} \quad (5)$$

It has been shown in [3], [7] that

$$SNR_i = \{N_o/A^2T + \sum_{\substack{k=1\\k\neq i}}^{K} \sigma_{k,i}^2\}^{-1/2}$$

$$= \{N_o/2E_b + \sum_{\substack{k=1\\k\neq i}}^{K} \sigma_{k,i}^2\}^{-1/2} \quad (6)$$

where $E_b = \frac{1}{2}A^2T$,

$$\sigma_{k,i}^2 = \mathrm{var}\{I_{k,i}(\underline{b}_k, \tau_k, \phi_k)\}$$

$$= T^{-3}\{\mu_{k,i}(0)m_\psi + \mu_{k,i}(1)m'_\psi\}, \quad (7)$$

$$\mu_{k,i}(n) = \sum_{\ell=1-L}^{L-1} C_k(\ell)C_i(\ell+n),$$

L is one code period, $C_k(\cdot)$ is the aperiodic autocorrelation function for the k-th signature sequence given by [4], [5], [6],

$$C_u(\ell) = \begin{cases} \sum_{j=0}^{L-1-\ell} U_jU_{j+\ell}, & 0 \leq \ell \leq L-1 \\ \sum_{j=0}^{L-1+\ell} U_{j-\ell}U_j, & 1-L \leq \ell < 0 \\ 0, & |\ell| \geq L, \end{cases}$$

$$m_\psi = \int_0^{T_c} R_\psi^2(s)ds, \quad (8)$$

$$m'_\psi = \int_0^{T_c} R_\psi(s)\hat{R}_\psi(s)ds, \quad (9)$$

$$\hat{R}_\psi(s) = \int_0^{T_c} \psi(t)\psi(t-s)dt, \quad (10)$$

$$R_\psi(s) = \int_0^{s} \psi(t)\psi(t+T_c-s)dt, \quad 0 \leq s \leq T_c \quad (11)$$

and N_o is Gaussian noise spectral density. Here, one can see that $\sigma_{k,i}^2$ is dependent on the correlational property of a code and the code chip waveform.

For the rectangular chip waveform, we have

$$\psi(t) = \pi((t - \frac{T_c}{2})/T_c),$$

$$m_\psi = T_c^3/3, \quad (12)$$

$$m'_\psi = T_c^3/6, \quad (13)$$

and thus we can reduce (7) to

$$\sigma_{k,i}^2 = (\frac{T_c}{T})^3 \{2\mu_{k,i}(0) + \mu_{k,i}(1)\}/6. \quad (14)$$

Also, for the sine chip waveform, we have

$$\psi(t) = \sqrt{2}\sin(\pi t/T_c)\,\pi((t - T_c/2)/T_c), \quad (15)$$

$$m_\psi = T_c^3(15+2\pi^2)/12\pi^2, \quad (16)$$

and $m'_\psi = T_c^3(25-\pi^2)/12\pi^2$, (17)

and thus we can reduce (7) to

$$\sigma_{k,i}^2 = 0.88(\frac{T_c}{T})^3 \{2\mu_{k,i}(0) + 0.2954\mu_{k,i}(1)\}/6. \quad (18)$$

Thus, one can see from (18) that by using sine pulses instead of rectangular pulses it is possible to decrease the multiple access interference by about 12%.

III. Performance Analysis for the Proposed Waveform

From (7) through (11) one can see that the multiple access interference, $\sigma_{k,i}^2$, depends on how much the chip waveform overlaps with its time-shifted version. Here we propose a new chip waveform as

$$\psi(t) = \sqrt{T_c/\epsilon}\,\pi((t - \frac{T_c}{2})/\epsilon)$$

which satisfies (1).

We evaluate m_ψ and m'_ψ for this waveform and find $\sigma_{k,i}^2$ as follows. From (11), we obtain

$$R_{\psi}(s) = \int_{0}^{s} \psi(t)\psi(t+T_c - s)dt$$

$$= (\frac{T_c}{\epsilon}) \int_{0}^{s} \pi(\frac{t-\frac{T_c}{2}-s}{\epsilon})\, \pi(\frac{t+\frac{T_c}{2}-s}{\epsilon})\, dt$$

$$= \begin{cases} (\frac{T_c}{\epsilon}) \cdot \{S - (T_c - \epsilon)\}, & T_c - \epsilon \leq S \leq T_c \\ 0 , & 0 \leq S < T_c - \epsilon . \end{cases} \quad (19)$$

Also, from (10) we obtain

$$\hat{R}_{\psi}(s) = \int_{s}^{T_c} \psi(t)\psi(t-s)dt$$

$$= (\frac{T_c}{\epsilon}) \int_{s}^{T_c} \pi(\frac{t-\frac{T_c}{2}}{\epsilon})\, \pi(\frac{t-\frac{T_c}{2}-s}{\epsilon})dt$$

$$= \begin{cases} -s + \epsilon , & 0 \leq s \leq \epsilon \\ 0 , & \epsilon < s < T_c . \end{cases} \quad (20)$$

The functions $R_{\psi}(s)$ and $\hat{R}_{\psi}(s)$ are shown in Fig. 2.

Using (8), (9), (19) and (20), we obtain

$$m_{\psi} = \int_{0}^{T_c} R^2_{\psi}(s)ds$$

$$= \int_{T_c-\epsilon}^{T_c} (\frac{T_c}{\epsilon})^2 \{s - (T_c - \epsilon)\}^2 \, ds \quad (21)$$

$$= \frac{T_c^2 \epsilon}{3}$$

and $m'_{\psi} = \int_{0}^{T_c} R_{\psi}(s)\hat{R}_{\psi}(s)ds.$

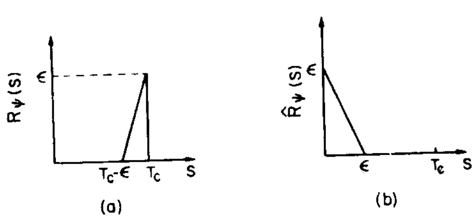


Fig. 2. (a) $R_{\psi}(s)$ and (b) $\hat{R}_{\psi}(s)$ functions.

If $\epsilon \leq \frac{T_c}{2}$, it is shown in Fig. 2 that $R_{\psi}(s)$ and $\hat{R}_{\psi}(s)$ do no overlap. Therefore, we have

$$m'_{\psi} = 0.$$

But, if $\epsilon > \frac{T_c}{2}$, we have

$$m'_{\psi} = (\frac{T_c}{\epsilon})^2 \int_{T_c-\epsilon}^{\epsilon} \{s - (T_c - \epsilon)\} \cdot \{-s + \epsilon\} ds$$

$$= (\frac{T_c}{\epsilon})^2 \left[-\frac{T_c^3}{6} + \frac{4}{3}\epsilon^3 + T_c^2 \epsilon - 2T_c \epsilon^2 \right] .$$

Thus, we obtain

$$m'_{\psi} = \begin{cases} 0 , & 0 \leq \epsilon \leq \frac{T_c}{2} \\ (\frac{T_c}{\epsilon})^2 \left[-\frac{T_c^3}{6} + \frac{4}{3}\epsilon^3 + T_c^2\epsilon - 2T_c\epsilon^2 \right] , & \frac{T_c}{2} < \epsilon \leq T_c . \end{cases} \quad (22)$$

From (7), (21) and (22) we obtain

$$\sigma^2_{k,i} = \begin{cases} \frac{1}{3}(\frac{T_c}{T})^2 \mu_{k,i}(0)(\frac{\epsilon}{T_c}), & 0 \leq \epsilon \leq \frac{T_c}{2} \\ (\frac{T_c}{T})^3 \left[\frac{1}{3}(\frac{\epsilon}{T_c}) \mu_{k,i}(0) + \{ -\frac{1}{6}(\frac{T_c}{\epsilon})^2 + \frac{4}{3}(\frac{\epsilon}{T} \ldots + \frac{\epsilon}{T_c} - 2 \} \mu_{k,i}(1) \right], & \frac{T_c}{2} < \epsilon \leq T_c . \end{cases} \quad (23)$$

Here, if we substitute ϵ by T_c in (23) (this is the rectangular wave-form case), we have

$$\sigma^2_{k,i} = (\frac{T_c}{T})^3 \left[\frac{1}{3}\mu_{k,i}(0) + \frac{1}{6}\mu_{k,i}(1) \right]$$

which is identical to (14).

Thus, it can be said that the proposed wave-form is a generalized version of the previously mentioned rectangular waveform.[7] Since $\mu_{k,i}(0)$ is normally much greater than $\mu_{k,i}(1)$, we see from (23) that the multiple access interference, $\sigma^2_{k,i}$, decays almost linearly as ϵ varies from T_c to zero, thus reducing error probability or increasing multiple access capability. Particularly, if ϵ is zero (i.e., impulse waveform), the multiple access interference becomes zero regardless of the number of users. Numerical result showing SNR of the i-th receiver as a function of ϵ/T_c is given in Fig. 3. The correlation parameters $\mu_{k,i}(0)$

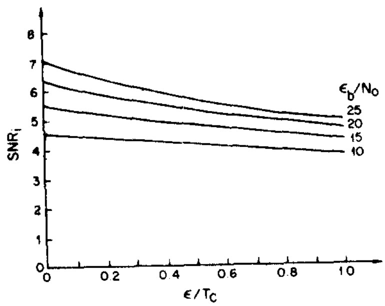


Fig. 3. SRN_i variation vs. ϵ/T_c.

and $\mu_{k,i}(1)$ have been obtained for AO/LSE m-sequences of period 31 and are given in Table 1.[8]

Table 1. Correlation parameters AO/LSE m-sequences of period 31.

k,i	$\mu_{k,i}(0)$	$\mu_{k,i}(1)$
1,2	967	-88
1,3	1015	-40
2,3	1015	-120

It has been shown in recent papers[9] that for most systems of interest a very accurate estimate of the average probability of errors can be obtained using SNR. This approximation, which was suggested in [4], is

$$P_{e,i} \simeq Q(SNR_i)$$

where $P_{e,i}$ is the average probability of errors for the i-th signal and Q is the Q function which is given by[11]

$$Q(k) \triangleq \frac{1}{\sqrt{2\pi}} \int_k^{\infty} e^{-\lambda^2/2}\, d\lambda \, .$$

Quantitative results on the accuracy of this

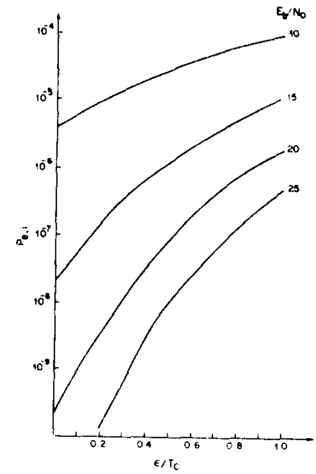


Fig. 4. Error probability of i-th receiver vs. ϵ/T_c.

approximation have been obtained by Yao [9]. Numerical result showing error probability of the i-th receiver as a function of ϵ/T_c is shown in Fig. 4. In this figure it is seen that there is a great difference in error probability (10^2–10^4) between the rectangular (ϵ=T_c) and the impulse (ϵ=0) wave-forms. The multiple access capability (i.e., the number of users who can communicate with the same channel at the same time) variation as a function of the waveform pulse width, ϵ, is shown in Fig. 5.

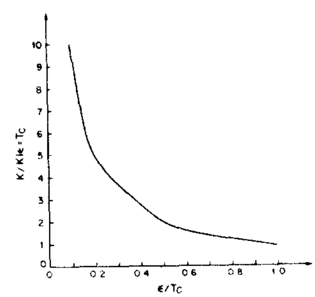


Fig. 5. Multiple access capability variation vs. waveform pulse width (Note: K denotes the number of users.).

IV. Conclusion

In this paper, the design of a PN code used in SSMA communication systems has been studied. We have proposed a new chip waveform that may be regarded as a generalized version of rectangular waveform. Using the proposed waveform, we have done analysis on a DS/SSMA system. We have found that the multiple access interference, $\sigma^2_{k,i}$, decays almost linearly as the pulse width, ϵ varies from T_c to zero, thus reducing error probability or increasing multiple access capability. Particularly, if ϵ is zero (i.e., impulse waveform), the multiple access interference becomes zero regardless of the number of users.

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