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Error Performance of Binary FSK Fast Frequency Hopping(BFSK/FFH) Systems in the Presence of Partial-Band Noise Jamming

(部分 帶域 電波 防害下에서의 바이내리 FSK 周波數 急跳躍 通信 시스템의 誤差 性能에 관하여)

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要 約

本 論文에서는 부분 대역 전파 방해와 열잡음하에서 바이내리 FSK 변조와 noncoherent 수신을 이용한 周波数 急跳躍 스프레드 스펙트럼 通信 시스템을 썼을 때의 誤率 導出에 대한 解析을 보였다. 가장 효과적인 전파방해의 경우 誤率性能을 수치 해법으로 구하였으며 그 결과를 비트当 도약数(L)을 파라미터로 하여 신호 비트에너지 対 전파 방해 밀도의 比(E_b/N_J)의 함수로 図示하였다.

Abstract

This paper presents a complete analysis for the derivation of the probability of error for a fast (or multiple-hops per bit) frequency hopping spread spectrum system employing binary FSK modulation and noncoherent reception in the presence of partial-band noise jamming and thermal noise. The worst-case error rate performances were obtained numerically and presented as a function of E_b/N_J with L as a parameter, where E_b/N_J and L are the signal bit energy-to-jamiming density ratio and the number of hops per bit, respectively.

I. Introduction

Spread spectrum techniques are employed to protect the communication signals from detection, demodulation and intentional or unintentional interferences (or jamming) by spreading the RF bandwidth well beyond what is required to transmit the data. There are two fundamental techniques for achieving bandwidth spreading: "direct sequence (DS) pseudonoise modulation" and "frequency hopping (FH)". The direct sequence modulation system is known to be less vulnerable to detection than the FH system. However, this technique has the disadvantage of requiring coherent detection over a large spread bandwidth and the bandwidth spreading is limited by current technology of approximately 300 M chips/sec.

The frequency hopping technique, on the other hand, can achieve extremely wide bandwidths and the waveforms can be detected noncoherently. Thus, the FH technique is considered a practical means for achieving low-detectability or covertness of transmissions. To improve the degree of covertness, high rate frequency-hopping systems are employed, in which the hopping rate is greater than the data

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rate in general. These are referred to as "fast hop" or "multi-hops per bit" systems.

In this paper we will study the error rate performance of a binary frequency-shift keying (BFSK) fast frequency hopping spread spectrum system in the partial-band noise jamming environment.

Historically, the use of the multi-hops (L-hops) per bit transmission scheme in communications stems from the diversity concept originally introduced to combat fading channels.[1],[2],[3] Viterbi and Jacobs[4] are the first ones who analyzed the error rate performance of the fast hopping system with noncoherent (square-law combining) reception on a channel perturbed by partial-band noise jamming. They neglected thermal noise and employed the Chernoff bounding technique to obtain an upper bound to the error probability. Milstein et al.[5] considered both jamming and thermal noise for the case of single-hop per bit (L = 1). A study on multiple FSK signal reception considered in [6] employed multi-hops per bit strategy in both thermal noise and multiple access interference. This work is conceptually similar to a special case (wideband jamming) of our problem.

We will present in this paper a complete analysis for the performance of L-hops/bit spread spectrum system employing binary frequency-shift keying (BFSK) modulation in both wideband and optimum jamming strategies.

II. Waveform and Receiver Description

A binary FSK signal is generated by assigning a burst of sinusoids at one of two frequencies f_1 ("space") and f_2 ("mark"), according to the incoming binary data of rate $1/T_b$ bits/sec where T_b is the bit period. The selected frequency f_i, i=1,2, is then hopped to one of the frequency channels (commonly termed as frequency cells) within the total system bandwidth W and remains there for the duration of τ seconds (hop period). At the end of τ seconds, the frequency is hopped again to one of the frequency cells, and this process is repeated $L = T_b/\tau$ times where T_b is assumed to be an integral multiple of τ. Thus the hopping rate of this fast frequency hopping waveform is L hops/bit. Let B be the bandwidth occupied by a single hop (or a single frequency cell). Then the intrinsic value of B is approximately $1/\tau$, so that $B\tau = 1$. The total number of frequency cells to which carrier can be hopped is W/B and the modulation bandwidth for the binary FSK is 2B.

Figure 1 shows a receiver block diagram for the BFSK/FH signal employing L-hops/bit strategy. The received carrier is down-converted (dehopped) by means of a frequency

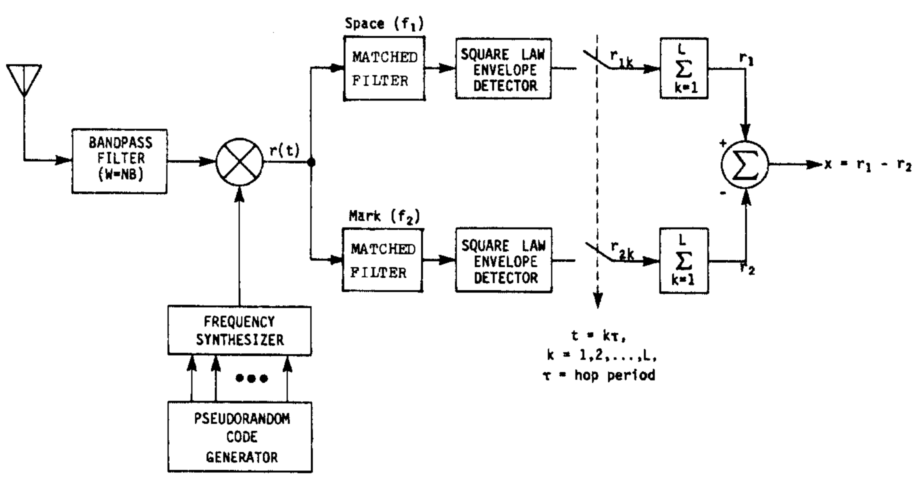

Fig. 1. Noncoherent receiver configuration for BFSK/FH signals with L hops/bit strategy.

synthesizer controlled by a pseudorandom code generator. The pseudorandom code is assumed to be synchronized and identical to that in the transmitter. The (dehopped) noisy baseband signal r(t) is then processed by the standard square-law combining binary FSK receiver as shown. Note that the matched filter-square law envelope detector in Fig. 1 is equivalent to the quadrature receiver which consists of a sum of the in-phase and quadrature correlator outputs. It is the purpose of this paper to assess the error rate performance of this receiver under both partial-band noise jamming and thermal noise assumptions.

III. Probability of Error Analysis

Preliminaries

The binary FSK signal s(t) is assumed to be, over a kth hop interval $(k\tau, (k+1)\tau)$,

$$s(t) = \begin{cases} \sqrt{2S}\cos(2\pi f_1 t + \theta_{1k}), & \text{(1a)} \\ \sqrt{2S}\cos(2\pi f_2 t + \theta_{2k}), & \text{(1b)} \end{cases}$$

where S is the received (average) signal power, f_1 and f_2 are the "space" and "mark" frequencies, respectively, and θ_{1k} and θ_{2k} are independent phases uniformly distributed on $(0, 2\pi)$. As usual, we assume the transmission is corrupted by thermal noise (additive white Gaussian) with spectral density $N_0/2$ (two-sided).

The partial-band noise jammer (also Gaussian) is assumed to have a total power J, which is uniformly distributed across a fraction γ of the total spread spectrum bandwidth W Hz. Thus, a specific hop is received jamming free with probability $1 - \gamma$ and perturbed by jamming noise of density $J/(\gamma W) = N_J/\gamma$, $N_J = J/W$, with probability γ. We assume that the two adjacent frequency cells of BFSK (modulation band) are jammed simultaneously with probability γ on each hop.

Since we assume that both the jammer noise and the thermal noise are Gaussian (and independent of each other), the resulting total noise (jammed or unjammed), n_T, is also Gaussian and its spectral density may be given by N_0 with probability $1 - \gamma$ and $N_0 + N_J/\gamma$ with probability γ, where $N_J = J/W$ is the effective jamming spectral density.

We now define an event J_e, where $J_e = 0$ or 1, such that

$$J_e = \begin{cases} 0\text{ ; unjammed with } Pr(J_e{=}0) = 1-\gamma, & \text{(2a)} \\ 1\text{; jammed with } Pr(J_e{=}1) = \gamma. & \text{(2b)} \end{cases}$$

Then, the noisy baseband signal r(t), which is recovered after dehopping, may be represented as

$$r(t) = s(t) + n_T(t)\ , \qquad (3)$$

where the spectral density of $n_T(t)$, N_T, is given as

$$N_T = \begin{cases} \frac{1}{2}N_o\ ;\ J_e = 0 \text{ with } Pr(J_e{=}0) = 1-\gamma, & \text{(4a)} \\ \frac{1}{2}(N_o + N_J/\gamma)\ ;\ J_e = 1 \text{ with } Pr(J_e{=}1) = \gamma. & \text{(4b)} \end{cases}$$

General Expression for the Probability of Error

From Figure 1 the decision statistic is given by

$$x = \sum_{k=1}^{L} (r_{1k} - r_{2k}) = \sum_{k=1}^{L} z_k\ , \qquad (5a)$$

where r_{1k} and r_{2k} are samples of the squared envelopes at channel 1 and channel 2, respectively, taken at $t = k\tau$; $k = 1, 2, \ldots, L$, where τ and L are the hop interval and the number of hops per bit, respectively, and

$$z_k \triangleq r_{1k} - r_{2k}\ . \qquad (5b)$$

The decision rule based on the statistic x is to choose "space" if $x > 0$ and "mark" if $x < 0$. Thus, assuming the "space" and "mark" symbols are equally likely with probability 1/2 we may write the probability of error

$$P(e) = \tfrac{1}{2} P(e|\text{space}) + \tfrac{1}{2} P(e|\text{mark})$$
$$= P(e|\text{space})$$

$$= \Pr[\, x < 0 \mid \text{space} \,] = \int_{-\infty}^{0} p_x(\alpha \mid \text{space})\, d\alpha, \tag{6}$$

where $p_x(\alpha|\text{space})$ is the probability density function of x given that a "space" is transmitted.

To proceed further we require the conditional probability density function $p_x(\alpha|\text{space})$. We will first obtain the conditional characteristic function of x and then take the inverse Fourier transform of this characteristic function to obtain $p_x(\alpha|\text{space})$.

The Characteristic Function

We assume the successive hops are independent as a result of frequency hopping. The random variables z_k, k = 1,2, ..., L are then statistically independent and may be assumed identically distributed. Let $C_z(\nu)$ denote the characteristic function of any one of L identically distributed variables $\{z_k\}$. Then, using (5a), we may write the characteristic function of x as

$$\begin{aligned} C_x(\nu) &= E[e^{j\nu x}] \\ &= E[\exp\{j\nu \sum_{k=1}^{L} z_k\}] = E[\prod_{k=1}^{L} e^{j\nu z_k}] \\ &= \prod_{k=1}^{L} E[e^{j\nu z_k}] = \{E[e^{j\nu z}]\}^L \\ &= [C_z(\nu)]^L, \end{aligned} \tag{7}$$

where z represents any one of the variables $\{z_k, k = 1, 2, ..., L\}$ and

$$C_z(\nu) = E[e^{j\nu z}] . \tag{8}$$

The averaging procedure of the characteristic function $C_z(\nu)$ of (8) may first be performed with respect to the event J_e with the help of (2),

$$\begin{aligned} C_z(\nu) &= E[e^{j\nu z}] = \Pr(J_e = 0)\, E[e^{j\nu z}|J_e=0] \\ &\quad + \Pr(J_e=1)\, E[e^{j\nu z}|J_e=1] \\ &= (1-\gamma)\, C_z(\nu|J_e=0) + \gamma\, C_z(\nu|J_e=1), \end{aligned} \tag{9}$$

where

$$C_z(\nu|J_e = 0) = E[e^{j\nu z}|J_e = 0] \tag{10a}$$

$$C_z(\nu|J_e = 1) = E[e^{j\nu z}|J_e = 1] . \tag{10b}$$

Substituting (9) into (7) and using the binomial theorem we obtain

$$\begin{aligned} C_x(\nu) &= [(1-\gamma)\, C_z(\nu|J_e=0) + \gamma\, C_z(\nu|J_e=1)]^L \\ &= \sum_{\ell=0}^{L} \binom{L}{\ell} (1-\gamma)^{\ell} \gamma^{L-\ell} [C_z(\nu|J_e=0)]^{\ell} \\ &\quad [C_z(\nu|J_e=1)]^{L-\ell} . \end{aligned} \tag{11}$$

We note here that the term $C_z(\nu|J_e = 0)$ represents the characteristic function of the random variable z (any one of z_k) in the absence of jamming, whereas $C_z(\nu|J_e = 1)$ represents that for he case of jamming present.

We now assume without loss of generality that a space frequency (f_1) is transmitted. Then the r_{1k} and r_{2k} in (5b) (see Fig. 1) may be given by [3, chapter 7]

$$r_{1k} = (\sqrt{E_s} \cos\theta_{1k} + a_{c1})^2 + (\sqrt{E_s} \sin\theta_{1k} + a_{s1})^2, \tag{12a}$$

$$r_{2k} = a_{c2}^2 + a_{s2}^2 , \tag{12b}$$

where E_s is the signal energy associated with kth hop, the θ_{1k} is the phase corresponding to kth hop, and a_{c1}, a_{s1}, a_{c2}, a_{s2} are statistically independent Gaussian random variables with Variance σ^2, where σ^2 is given by, in accordance with (4),

$$\sigma^2 = \begin{cases} \sigma_N^2 = N_o/2 \;;\; J_e=0,\ \Pr(J_e=0) = 1-\gamma, & (13a) \\ \sigma_T^2 = \frac{1}{2}(N_o+N_J/\gamma) \;;\; J_e=1,\ \Pr(J_e=1) = \gamma. & (13b) \end{cases}$$

We can immediately see that the variables r_{1k} and r_{2k} in (12) are noncentral and central x^2 variables, with two degrees of freedom, respectively, and their characteristic functions are given by [7, chapter 4]

$$C_z(\nu | J_e = 0) = E\,[e^{j\nu(r_{1k}-r_{2k})} \,|\, J_e = 0\,] = \frac{e^{j\nu E_s/(1-j2\sigma_N^2\nu)}}{1-(j2\sigma_N^2\,\nu)^2}, \quad (14a)$$

and

$$C_z(\nu | J_e=1) = E\,[e^{j\nu(r_{1k}-r_{2k})} \,|\, J_e = 1] = \frac{e^{j\nu E_s/(1-j2\sigma_T^2\,\nu)}}{1-(j2\sigma_T^2\,\nu)^2}, \quad (14b)$$

where σ_N^2 and σ_T^2 are as given in (13). Substituting (14) into (11) and expanding the exponential functions which include $j\nu$ in the exponents as power series we obtain

$$C_x(\nu) = \sum_{\ell=0}^{L} \binom{L}{\ell} (1-\gamma)^{\ell} \gamma^{L-\ell} e^{-\ell\rho_N} e^{-(L-\ell)\rho_T} \cdot \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\ell\rho_N)^m}{m!} \frac{[(L-\ell)\rho_T]^n}{n!} \phi(\nu), \quad (15a)$$

where

$$\phi(\nu) = \left(\frac{1}{1-j\,2\sigma_N^2\nu}\right)^{\ell+m} \left(\frac{1}{1+j\,2\sigma_N^2\nu}\right)^{\ell} \cdot \left(\frac{1}{1-j\,2\sigma_T^2\nu}\right)^{L-\ell+n} \left(\frac{1}{1+j\,2\sigma_T^2\nu}\right)^{L-\ell}, \quad (15b)$$

where we defined

$$\rho_N \triangleq E_s/(2\sigma_N^2) = E_s/N_o, \quad (16a)$$

$$\rho_T \triangleq E_s/(2\sigma_T^2) = E_s/(N_o + N_J/\gamma). \quad (16b)$$

The Probability Density Function

The conditional probability density function of x, $p_x(\alpha/\text{space})$, may now be obtained by the inverse Fourier transform of (15), which requires the inverse Fourier transform of $\phi(\nu)$. An expedient method in doing this is the use of partial fraction expansion by means of the calculus of residues. The partial fraction expansion of (15b) may be written

$$\phi(\nu) = \prod_{i=1}^{4} (1-j2a_i\nu)^{-q_i} = \sum_{i=1}^{4} \sum_{r=1}^{q_i} A_{ir} (1-j\,2a_i\,\nu)^{-r}, \quad (17a)$$

where

$$a_1 = \sigma_N^2,\ a_2 = -\sigma_N^2,\ a_3 = \sigma_T^2,\ a_4 = -\sigma_T^2, \quad (17b)$$

$$q_1 = \ell+m,\ q_2 = \ell,\ q_3 = L-\ell+n,\ q_4 = L-\ell, \quad (17c)$$

and the A_{ir} are the partial fraction coefficients not containing ν. For the error probability expression, only A_{2r} and A_{4r} will be required, as will be shown shortly.

By taking the inverse Fourier transform of (15) in accordance with (17) we thus have the conditional probability density function of x:

$$p_x(\alpha | \text{space}) = \sum_{\ell=0}^{L} \binom{L}{\ell} (1-\gamma)^{\ell} \gamma^{L-\ell} e^{-\ell\rho_N} e^{-(L-\ell)\rho_T} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\ell\rho_N)^m}{m!} \frac{[(L-\ell)\rho_T]^n}{n!}$$

$$\cdot \begin{cases} \left[\sum_{r=0}^{\ell+m} A_{1r} \frac{1}{\sigma_N^2} p_{x^2}\left(\frac{\alpha}{\sigma_N^2}; 2r\right) + \sum_{r=0}^{L-\ell+n} A_{3r} \frac{1}{\sigma_T^2} p_{x^2}\left(\frac{\alpha}{\sigma_T^2}; 2r\right)\right]; \alpha \geqslant 0 & (18a) \\ \left[\sum_{r=0}^{\ell} A_{2r} \frac{1}{\sigma_N^2} p_{x^2}\left(\frac{-\alpha}{\sigma_N^2}; 2r\right) + \sum_{r=0}^{L-\ell} A_{4r} \frac{1}{\sigma_T^2} p_{x^2}\left(\frac{-\alpha}{\sigma_T^2}; 2r\right)\right]; \alpha \leqslant 0, & (18b) \end{cases}$$

where $p_{x^2}(u;2r)$ is the x^2 density function with 2r degrees of freedom.

Probability of Error Expression

Substituting (18b) into (6) and noting that $(1/b)\int_0^\infty p_{x^2}(\alpha/b;2r)\,d\alpha=1$, we obtain for the probability of error

$$P(e)=\sum_{\ell=0}^{L}\binom{L}{\ell}(1-\gamma)^{\ell}\gamma^{L-\ell}e^{-\ell\rho_N}e^{-(L-\ell)\rho_T}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\ell\rho_N)^m}{m!}\frac{[(L-\ell)\rho_T]^n}{n!}\cdot\left(\sum_{r=0}^{\ell}A_{2r}+\sum_{r=0}^{L-\ell}A_{4r}\right);\ A_{20}=A_{40}=0. \tag{19}$$

Assuming that signal energy per bit, say E_b, is equally divided among L hops, $E_s = E_b/L$, we may write ρ_N and ρ_T as

$$\rho_N = E_s/N_o = E_b/(N_oL), \tag{20a}$$

$$\rho_T=\frac{E_s}{N_o+N_J/\gamma}=\frac{1}{1/\rho_N+1/(\gamma\rho_J)}, \tag{20b}$$

$$\rho_J = E_s/N_J = E_b/(N_JL). \tag{20c}$$

The coefficients A_{2r} and A_{4r} are obtained to be (see the Appendix)

$$A_{2r}=\left(\frac{1}{2}\right)^{\ell+m}\left(\frac{1}{1+\delta}\right)^{L-\ell+n}\left(\frac{1}{1-\delta}\right)^{L-\ell}\cdot\frac{\mu_2(\ell-r)}{(\ell-r)!};\ r=1,2,\ldots,\ell, \tag{21a}$$

$$A_{4r}=\left(\frac{\delta}{1+\delta}\right)^{\ell+m}\left(\frac{\delta}{\delta-1}\right)^{\ell}\left(\frac{1}{2}\right)^{L-\ell+n}\cdot\frac{\mu_4(L-\ell-r)}{(L-\ell-r)!};\ r=1,2,\ldots,L-\ell, \tag{21b}$$

where μ_{ih} (moments about the origin), i = 2, 4, are related to the κ_{ih} (cumulants), i = 2, 4, through the well-known relations,

$$\mu_{i0}=1,\ \mu_{i1}=\kappa_{i1},\ \mu_{i2}=\kappa_{i2}+\kappa_{i2}^2,\ \ldots;\quad i=2,4 \tag{22}$$

and

$$\kappa_{2h}=(h-1)!\left[(\ell+m)\left(\frac{1}{2}\right)^h+(L-\ell+n)\left(\frac{\delta}{1+\delta}\right)^h+(L-\ell)\left(\frac{\delta}{\delta-1}\right)^h\right], \tag{23a}$$

$$\kappa_{4h}=(h-1)!\left[(\ell+m)\left(\frac{1}{1+\delta}\right)^h+\ell\left(\frac{1}{1-\delta}\right)^h+(L-\ell+n)\left(\frac{1}{2}\right)^h\right], \tag{23b}$$

with

$$\delta=\frac{N_0+N_J/\gamma}{N_0}=1+\frac{1}{\gamma}\frac{N_J}{N_0}=1+\frac{1}{\gamma}\frac{\rho_N}{\rho_J}$$

The probability of error expression (19) will be computed and plotted as a function of E_b/N_J with E_b/N_o and L as parameters in conjunction with the optimum jamming strategy (worst case from communicator's viewpoint) to be discussed shortly.

Special Case 1: Single Hop/Bit (L=1)

In this case, the probability of error (19) reduces to

$$P(e)=(1-\gamma)e^{-\rho_N}\sum_{m=0}^{\infty}\frac{1}{m!}\frac{1}{2}\left(\frac{1}{2}\rho_N\right)^m+\gamma e^{-\rho_T}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{1}{2}\left(\frac{1}{2}\rho_T\right)^n=\frac{1}{2}(1-\gamma)e^{-\frac{1}{2}\rho_N}+\frac{1}{2}\gamma e^{-\frac{1}{2}\rho_T}, \tag{24}$$

where ρ_N and ρ_T are as given in (20). We observe that (24) reduces to the conventional BFSK result for $\gamma = 0$ or 1.

Special Case 2: No Thermal Noise

The case of no thermal noise ($\sigma_N^2 = N_0/2 = 0$) is of particular interest since most of work related to the present type of problem employ this assumption.[4],[8],[9],[10] In our formulation it should be possible to obtain the special case expression by taking the limit of (19) with $\rho_N \to \infty$. The procedure is cumbersome, however, due to the complexity of the error probability expression as a function of ρ_N. Instead, we repeated the derivation by imposing the condition of $\sigma_N^2 = N_0/2 = 0$ in (14). The result is

$$P(e)=\sum_{\ell=0}^{L}\binom{L}{\ell}(1-\gamma)^{\ell}\gamma^{L-\ell}e^{-(L-\ell)\gamma\rho_J}$$

$$\cdot \sum_{n=0}^{\infty} \frac{[(L-\ell)\gamma\rho_J]^n}{n!} \cdot \sum_{r=0}^{L-\ell} A_{4r} \cdot \left[\sum_{k=0}^{r-1} e^{-\ell\gamma\rho_J} \frac{(\ell\gamma\rho_J)^k}{k!} \right], \qquad (25)$$

whree

$$\rho_J = E_s/N_J = E_b/(N_J L)$$

$$A_{4r} = (\tfrac{1}{2})^{L-\ell+n} \frac{\mu_4(L-\ell-r)}{(L-\ell-r)} \;; r=1, 2, ..., L-\ell,$$

$$\mu_{40} = 1, \mu_{41} = \kappa_{41}, \kappa_{42} = \kappa_{42} + \kappa_{42}^2, ...,$$

$$\kappa_{4h} = (h-1)!\,(L-\ell+n)\,(\tfrac{1}{2})^h .$$

Equation (25) is an exact expression for the error probability in the absence of thermal noise. We note here that Viterbi and Jacobs[4] treated a problem similar to the above and obtained an upper bound on the error probability using the Chernoff bounding technique. However, their analysis assumes a smart receiver, which knows at each hop whether the jamming signal is present or not.[11]

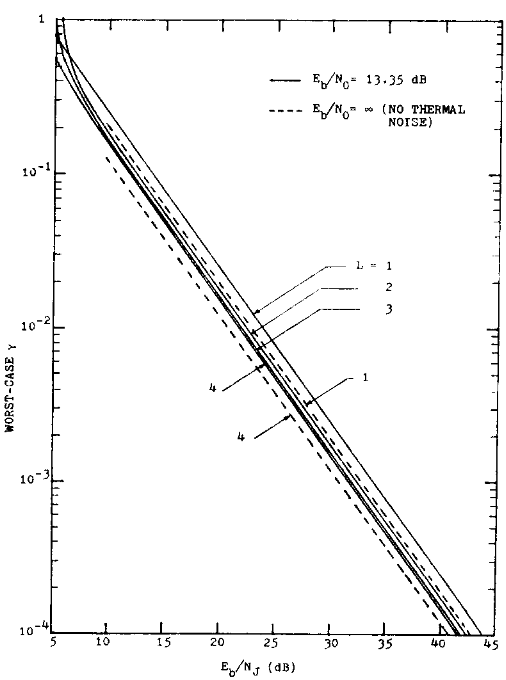

Fig. 2. Worst-case γ vs. E_b/N_J.

IV. Computed Performance Results

The most effective jamming strategy is to distribute the total jamming power J (choose γ) in such a way as to cause the communicator to have maximum probability of error. There will be an optimum value of γ(worst-case γ from the communicator's viewpoint) which will give the maximum value of error probability (worst-case error probability). We carried out the maximization procedure numerically, selecting E_b/N_o = 13.35 dB and ∞, for which the probability of error becomes 10^{-5} and zero, respectively, in the absence of jamming.

Fig. 2 shows the worst-case γ as a function of E_b/N_J with L as a parameter. As seen, the worst-case γ decreases (almost linearly) with increasing E_b/N_J. While the worst-case γ's for $L \geqslant 2$ are slightly less than those for L = 1 for most of the E_b/N_J values shown, there occur some crossovers in the neighborhood of $E_b/N_J \simeq$ 5dB. It seems that in this range of E_b/N_J ($\simeq$ 5dB) signal bit energy and jamming density are quite competitive and thus splitting the bit energy into small pieces results in small hop energies relative to the jamming density, causing the crossover phenomena.

We note that the worst-case γ, γ_w, for the case of $E_b/N_o = \infty$ and L = 1 may be given, analytically, by $\gamma_w = 2/(E_b/N_J)$. This can be shown by differentiating the second term of (24) with respect to γ and setting the result equal to zero to find the root for $\gamma = \gamma_w$. Although the worst-case γ's for other cases ($L \neq 1$, $E_b/N_o \neq \infty$) were numerically obtained, they all seem to behave in a manner similar to the case of L = 1 and $E_b/N_o = \infty$. We also observe that the worst-case γ is insensitive to L and remains virtually unchanged for $L \geqslant 4$. The worst-case probability of error for $L > 4$ may thus simply be calculated using the worst-case γ's for L = 4.

The worst-case error probabilities corresponding to the worst-case γ's are shown in Fig. 3. We readily see that the probability of error increases, for a given value of E_b/N_J, as L increases. Note that the curves for E_b/N_o =

13.35 dB (solid curves) approach asymptotically certain values as E_b/N_J approaches ∞ (P(e) $\rightarrow$ 10^{-5}, 3.8×10^{-5}, 8.8×10^{-5}, 1.7×10^{-4}, 4.3×10^{-4} for L=1, 2, 3, 4, 6, respectively). The reason that all the curves do not approach 10^{-5} is due to the "combining loss". The asymptotic values in Fig. 3 coincide with those values of P(e) predicted by combining loss under the thermal noise only case.[12] It seems that the combining loss applies to all the range of E_b/N_J, as shown.

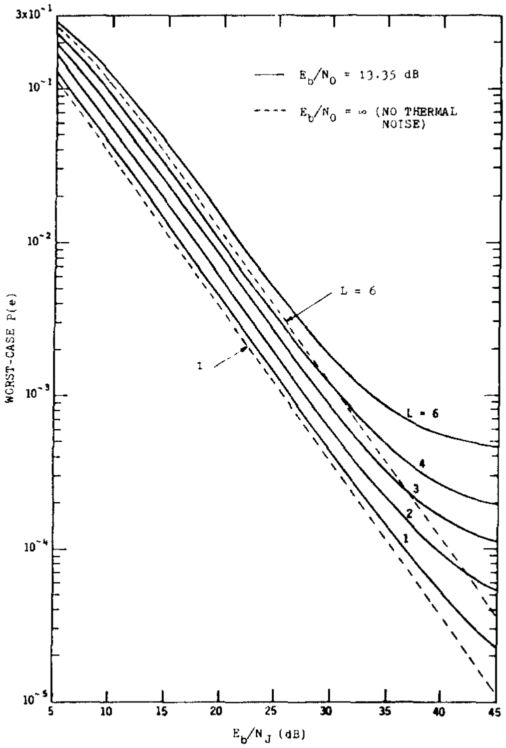

Fig. 3. Worst-case P(e) vs. E_b/N_J.

On the other hand, the curves for the no thermal noise case (shown by dotted curves for L = 1 and 6 only) are slightly lower than the solid curves (with corresponding L's), serving as good lower bounds to the case of thermal noise present for lower values of E_b/N_J ($E_b/N_J \lesssim 35$ dB). As E_b/N_J approaches ∞, however, the error probabilities go to zero, without any bottoming-out effects.

Finally, Fig. 4 shows the error rate performance comparisons for the cases of worst-case γ (optimum jamming from jammer's point of view) and wideband jamming for L=2. As a reference, ideal BFSK performance is plotted with a dotted curve. (For the ideal BFSK curve the abscissa reads E_b/N_o). As seen, the worst-case γ is clearly much more damaging than the wideband jamming strategy.

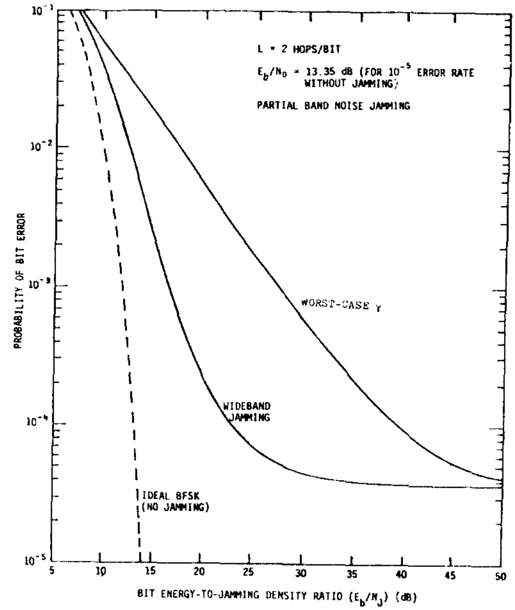

Fig. 4. Optimum jamming and wideband jamming perforamnces for BFSK/FH with L=2 HOPS/BIT when E_b/N_o = 13.35 dB. (For ideal BFSK curve the abscissa reads E_b/N_o.)

V. Conclusion

An analysis has been presented for the derivation of the probability of error for a fast hopping (multi-hops per bit) spread spectrum system employing binary FSK modulation in a partial-band jamming environment. The main contribution of this paper is the complete derivation of the error probability expression in the presence of both partial-band jamming and thermal noise. Most of workers dealing with these types of problems have simplified the analysis by making certain assumptions such as no thermal noise, which is hardly applicable in real situations. We have obtained the solution without the simplifying assumptions and presented graphical results for both wideband and optimum jamming strategies.

Regarding the variety of prameters and variables that affect the system performance, the designers of a multi-hops/bit BFSK/FH system must take various factors into consideration. Our results will be very useful in providing necessary information to the designers of such system.

Appendix
Evaluation of Coefficients A_{ir} in (17a)

The partial fraction expansion for the type of (17a) was studied in [13]. The coefficients A_{ir} are given by [13, Eqs. 2.26, 2.25, and 2.24]

$$A_{i(q_i-h)} = B_{iq_i}\,\mu_{ih}/h!\ ;\ h = 0, 1, 2, \ldots, q_i-1 \tag{A.1a}$$

with

$$B_{iq_i} = \prod_{\substack{k=1 \\ (k\neq i)}}^{4} \left(\frac{a_i}{a_i-a_k}\right)^{q_k}, \tag{A.1b}$$

where the moments (about the origin) μ_{ih} can be expressed in terms of the cumulants κ_{ih} as in (22) and the κ_{ih} are given by

$$\kappa_{ih} = (h-1)! \sum_{\substack{k=1 \\ (k\neq i)}}^{4} \left[q_k \left(\frac{-a_k}{a_i-a_k}\right)^h \right]. \tag{A.1c}$$

Rewriting (A.1) for i = 2, 4 with the substitution of (17b) and (17c) and making the change of variables, $h = \ell-r$ for $i = 2$ and $h = L-\ell-r$ for $i = 4$ we obtain (21) and (23).

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