PSK Error Performance with Impulsive Noise and Cochannel Interference

(임펄스 雜音 및 同一 채널 干涉下의 PSK 信號의 誤率 特性)

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(Byung Ock Kong and Sung Joon Cho)

要 約

임펄스 잡음과 동일 채널 간섭 환경하에서 PSK 시스템의 오율 특성을 반송파대 잡음 전력비, 반송파대 간섭파 전력비, 임펄스 지수 및 신호와 간섭파간의 위상차의 함수로써 究明했다. 동기 위상 검출기의 출력에 대한 확률 밀도 함수(p.d.f.)의 일반식을 도출하여 2상 PSK 신호의 경우에 대한 수치 계산의 결과를 그래프로 나타냈다. 해석 결과, 신호와 간섭파 간의 상대 위상차가 90° 일때 오율 특성이 가장 좋게됨을알 수 있었다.

Abstract

The error rate performance of phase shift keyed(PSK) signal has been evaluated in terms of carrier-to-noise ratio(CNR), carrier-to-interferer ratio(CIR), impulsive index, and the phase difference between signal and interferer in the environment of cochannel PSK interference and impulsive noise. We have derived a general equation of the probability density function (p.d.f.) of output of coherent phase detector. And the error rate of the received binary PSK(BPSK) signal has been numerically evaluated.

The graphic results show us that the best case is the situation of the signal and the interferer meet with orthogonal phase.

I. Introduction

Coherent phase shift keying(CPSK) scheme is the best efficient technique used in digital transmission system as an effective process. Accordingly, this technique has become the subject of interest by many authors. However the ever increasing demand and supply for communication channel in the radio frequency (RF) bands causes a serious problem of electromagnetic interference(EMI).[1] And in urban area, impulsive noise, which is generated by many electromechanical devices and the ignition spark of automobile, etc., has also become a serious degradation factor to the receiving system.[2]

Consequently, as the RF band is limited, the reuse of the existing band which is in use has been considered by many investigators.[1-2]

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And the statistical treatment of impulsive random noise has been treated by many authors.[3-8]

In this paper, first, we will adopt one interferer which influences PSK receiver and clarify the statistical characteristics of impulsive noise. Thus, we shall study the influence of interferer and impulsive noise in the CPSK receiving system in a point of the bit error probabilities.

II. Analysis Model

The analysis model is presented in Fig. 1, where s(t) is the desired PSK signal, n(t) the impulsive noise, and i(t) the other interfering PSK signal.

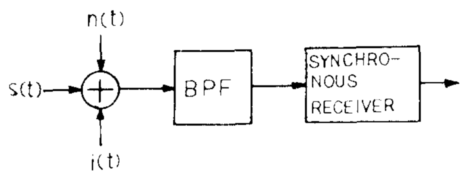

Fig. 1. Analysis model.

Here the receiver is assumed to be perfectly synchronized with the transmitter. The statistical models of the signal, the noise, and the interferer are given as follows.

1. PSK Signal

PSK signal can be represented as

$$S(t) = S\cos(2\pi f_0 t + 2\pi\lambda/M), \tag{1}$$

where S is the amplitude of signal, f_0 the carrier frequency, M the number of levels, and λ (=0,1,$\cdots$,M-1) the M-ary information. The probability of occurrence of each information is assumed to be same.

2. Impulsive Noise Model

In a number of digital transmission systems, in addition to Gaussian noise, impulsive noise interference is also present. It might be generated in two cases. The one is man-made radio noise, and the other is natural impulsive random noise.[3]

In this paper, as an analytically tractable model of man-made and natural radio noise, Middleton's recently developed cannonical statistical-physical model of impulsive noise[4] is introduced. Middleton classified impulsive noise into three types.

1) Class A; Its frequency components are constrained to a spectral width that is less than the bandwidth of the narrowest element of a linear receiving system (narrow band vis-à-vis the receiver).
2) Class B; It has wider bandwidth in comparison with that of the receiving system (broad band vis-à-vis the receiver).
3) Class C; Class A + Class B

Only the type of class A is considered as an impulsive noise model in this paper, for no significant transient impulses are produced in the receiver by the noise signal of class A.

The impulsive noise belonged to class A is written as

$$n(t) = E'\cos(2\pi f_0 t + \varphi_e), \tag{2}$$

where E' and φ_e, which are independent random variables, are the envelope and the phase. The probability density function (p.d.f.) of the normalized envelope of n(t) has been formed by Middleton as[4]

$$P(E) = e^{-A}\sum_{i=0}^{\infty}\frac{A^i}{i!}\frac{2E}{\sigma_i^2}e^{\frac{E^2}{\sigma_i^2}}, \tag{3}$$

where

$E(=E'/\sqrt{2(\sigma_G^2+\Omega_A)}$); normalized noise envelope

$\Gamma'(=\sigma_G^2/\Omega_A)$; the ratio of the intensity of the independent Gaussian component (σ_G^2) to the intensity of the "impulsive" component (Ω_A) of the impulsive noise

$\sigma_i^2 = (i/A+\Gamma')/(1+\Gamma')$

A; impulsive index, which is the product of received average numbers of the impulse per unit second and the burst's emission duration.

The initial phase φ_e is assumed to be uniformly distributed in the interval between 0 and 2π. It has been shown the assumption is reasonable.[5] We can extend this model to the Gaussian noise by setting the parameters of A=1 and relatively large Γ' (about 10).

3. Interferer

Degradation in bit transmission quality, an increase in the probability of error, can be arison from many different sources. In designing the communication systems it should be considered that the transmitter or receiver selectivity, equipment shielding, antenna directivity, etc. And in planning the communication it should be thought over degradation factors, such as site considerations, foreground reflections, unwanted couplings between channels, and inadequate angular separation between crossing routes.[2] In addition, it should be decided the acceptable level of interference.

The main interest is to get the adequate interference level for the allowable probability of bit error. Interference can be classified by two ways, the one is by the occurrence source, the other is by the spectrum. The most important sources of interference are cochannel interference (CCI) and adjacent channel interference (ACI).

But now, in this paper, cochannel interference by the other PSK signal of eq. (4), which is generated by the reuse of existing band which is in use, is considered. For example, in the satellite communication, one factor of limiting the FDMA capacity is the interference between signals reusing the same frequencies. Intelsat-IV, in practical case, is interference limited by cochannel signals rather than by intermodulation distortion.[9] The cochannel interferer may be written as

$$i(t) = I\cos(2\pi f_0 t + 2\pi\nu/M + \phi)\,, \tag{4}$$

where I is the amplitude of the interferer, ν (=0,1,···,M-1) the information, M the number of levels, and f_0 the carrier frequency of the interferer. And ϕ is the phase difference between carriers of the signal and the interferer.

III. Statistical Characteristics of the Composite Wave

1. The Analysis of the Composite Wave

For a M-ary CPSK system the received PSK signal of the zero-phase in the presence of noise and interferer is shown in Fig. 2. The noise and the interferer corrupting the desired signal distort the signal both in amplitude and in phase. Assuming that the probability of occurrence of each information is equally likely, the analysis of only zero-phase signal (correspond to λ=0) is sufficient.[10-13] Zero-phase signal disturbed by noise and single interferer is also shown in Fig. 2. Where E is the sum of Gaussian noise and impulsive noise, φ_e the relative phase to the signal, and Z, θ are the envelope and the phase of received signal. The phase difference between the signal and the interferer is denoted by ψ, which is written as

$$\psi = 2\pi\nu/M + \phi\,. \tag{5}$$

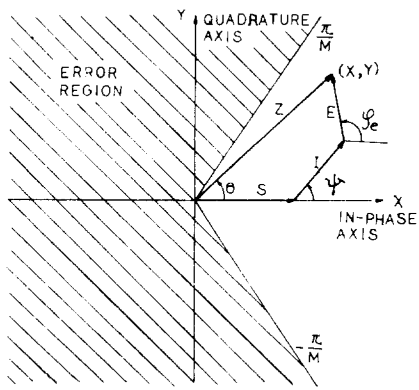

Fig. 2. Phasor representation of the received PSK signals corrupted by the impulsive noise and the interferer.

Provided that the terminal point is (x, y), the probability density functions (p.d.f.) of x, y are written as

$$P(x\,|\,I,E,\psi,\varphi_e) = \delta\{x-(S+I\cos\psi + E\cos\psi_e)\}\,, \tag{6}$$

and

$$P(y|I,E,\psi,\varphi_e) = \delta\{y-(I\sin\psi + E\sin\varphi_e)\} \quad . \quad (7)$$

The characteristic function of eq.(6) and eq.(7) would be written as[14, p.213]

$$M(u,\upsilon|I,E,\psi,\varphi_e) = \exp\{ju(S+I\cos\psi+E\cos\varphi_e) + j\upsilon(I\sin\psi+E\sin\varphi_e)\}. \quad (8)$$

The joint p.d.f. of (x, y) is carried out by the inverse Fourier transform. Thus,

$$P(x,y|I,E,\psi,\varphi_e) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\{ju(S+I\cos\psi+E\cos\varphi_e-x) + j\upsilon(I\sin\psi+E\sin\varphi_e-y)\}\,du\,d\upsilon . \quad (9)$$

In order to obtain the polar form, the transformation of the coordinates is introduced as follows

$$x = Z\cos\theta,\ \ y = Z\sin\theta, \quad (10)$$

$$u = \rho\cos\xi,\ \ \upsilon = \rho\sin\xi,\ du dv = \rho d\rho d\xi, \quad (11)$$

where ρ and ξ are dummy variables. Then, eq. (9) is easily transformed into the polar form of eq. (12) by introducing eq. (10), eq. (11), and the Jacobian.

$$P(Z,\theta|I,E,\psi,\varphi_e) = \frac{1}{(2\pi)^2}\int_0^{\infty}\int_0^{2\pi}\rho d\rho d\xi \cdot Z \cdot \exp\{j[\rho S\cdot\cos\xi+\rho I\cdot\cos(\xi-\psi) - \rho Z\cdot\cos(\xi-\theta)+\rho E\cdot\cos(\xi-\varphi_e)]\} \quad (12)$$

The phase φ_e can be averaged out by using the formula (F.1), for the φ_e is uniformly distributed in $[0, 2\pi]$. After the averaging, we can integrate above equation with respect to ξ by using the formula (F.2). Then, eq. (12) becomes

$$P(Z,\theta|E) = \sum_{n=0}^{\infty}\frac{\epsilon_n}{2\pi}\cdot\cos n(\theta-r)\cdot Z\cdot\int_0^{\infty}\rho \cdot J_0(\rho E)\, J_n(\rho K)\cdot J_n(\rho Z)\, d\rho, \quad (13)$$

where

$J_n(\cdot)$; Bessel function

$$r = \tan^{-1}\frac{\sin\psi}{S/I + \cos\psi}$$

$$K = (S^2 + 2S\, I\cdot\cos\psi + I^2)^{1/2}$$

and

ϵ_n; Neumann coefficient

$$= \begin{cases}\epsilon_0 = 1, & n = 0\\ \epsilon_n = 2, & n \neq 0.\end{cases}$$

Next, using the p.d.f. of the normalized envelope of n(t), eq.(13) can be averaged with E by adopting the relation of (F.3) and the formula of (F.4).

$$P(Z,\theta) = \sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\frac{\epsilon_n}{2\pi}\cdot\cos\{n(\theta-r)\}\frac{e^{-A}A^i}{i!}\cdot Z\cdot\int_0^{\infty}\rho\cdot J_n(\rho K)\cdot J_n(\rho Z)\cdot\exp(-N\sigma_i^2\rho^2/2)\,d\rho, \quad (14)$$

where N is the total noise power. Adopting the series expansion of $J_n(\rho K)$ of (F.5) to ease the eq.(14), and carrying out the integration of eq.(14) with ρ by using the formula (F.6), $P(Z,\theta)$ becomes

$$P(Z,\theta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{\epsilon_n}{2\pi}\cdot\cos\{n(\theta-r)\}\cdot\frac{e^{-A}A^i}{i!}\frac{Z^{n+1}}{\Gamma(n+1)}\frac{(-1)^k(1/2)^{k+n}K^{2k+n}}{k!(N\sigma_i^2)^{1+k+n}}\cdot {}_1F_1[1+k+n;n+1;-Z^2/2N\sigma_i^2], \quad (15)$$

where the ${}_1F_1[\cdot;\cdot;\cdot]$ is a confluent hypergeometric function and $\Gamma(\cdot)$ the gamma function. Lastly, eq.(15) is integrated from zero to infinite with respect to Z as

$$P(\theta)\Big|_{n\neq 0} = \sum_{n=1}^{\infty}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{n}{2\pi}\cos\{n(\theta-r)\}\cdot\frac{e^{-A}A^i}{i!}\frac{\Gamma(k+n/2)}{\Gamma(1+k+n)}\cdot\frac{(-1)^k(K^2)^{k+n/2}}{k!\,(\sigma_i^2)^{k+n/2}(2N)^{k+n/2}} \quad (16)$$

Next, $P(\theta)$, in the case of k=n=0, can be derived from eq.(14). After the substitution of k=n=0 into eq.(14), we can also perform the double integration with ρ and Z from zero to infinite. Then eq. (14) becomes

$$P(\theta)\big|_{k=n=0} = \sum_{i=0}^{\infty} \frac{1}{2\pi} \frac{e^{-A}A^i}{i!} \quad . \tag{17}$$

Here, the following identity[3] is adopted.

$$e^{-A} \sum_{i=0}^{\infty} \frac{A^i}{i!} = 1 \tag{18}$$

Then,

$$P(\theta)\big|_{k=n=0} = \frac{1}{2\pi} \quad . \tag{19}$$

The p.d.f. of the phase θ of the composite vector is the sum of eq. (16) and eq.(19). Thus,

$$P(\theta) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \frac{n}{2\pi} \cdot \cos\{n(\theta-r)\} \cdot \frac{e^{-A}A^i}{i!} \frac{(-1)^k}{k!(\sigma_i^2)^{k+n/2}} \frac{\Gamma(k+n/2)}{\Gamma(1+k+n)} \cdot (\alpha + 2\alpha\cdot\cos\psi/\sqrt{\beta} + \alpha/\beta)^{k+n/2} , \tag{20}$$

where

$\alpha(=S^2/2N)$; carrier to noise power ratio (CNR)

$\beta(=S^2/I^2)$; carrier to interferer power ratio (CIR).

Modifying above equation to simplify the calculation, the eq.(20) becomes

$$P(\theta) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} \frac{n}{2\pi} \cdot \cos\{n(\theta-r)\} \cdot \frac{e^{-A}A^i}{i!} \frac{1}{(\sigma_i^2)^{n/2}} \frac{\Gamma(n/2)}{\Gamma(n+1)} \cdot (\alpha+2\alpha \cos\psi/\sqrt{\beta} + \alpha/\beta)^{n/2} \cdot {}_1F_1[n/2;n+1;-(\sigma_i^2)^{-1} (\alpha+2\alpha \cos\psi/\sqrt{\beta} + \alpha/\beta)] \ . \tag{21}$$

2. The Error Rate of PSK Signal

When the terminal of composite vector, in the M-ary PSK system, lies in the error region (the shaded portion of Fig. 3), an error is made in the receiver. In Fig. 3, if the phase θ of the composite vector lies in $[\pi/M, \pi]$ and $[-\pi, -\pi/M]$, an error will be made.

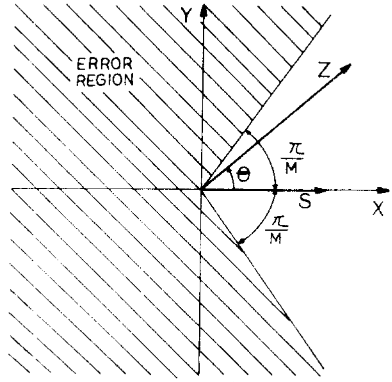

Fig. 3. The phasor diagram of received PSK signal.

The error rate of the received PSK signal can be derived from $P(\theta)$ of eq.(21) as

$$P_e = \int_{\pi/M}^{\pi} P(\theta)\, d\theta + \int_{-\pi}^{-\pi/M} P(\theta)\, d\theta$$

$$= (1-\frac{1}{M}) - \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} \frac{1}{\pi} \sin\frac{n\pi}{M} \cdot \cos nr \cdot \frac{e^{-A}A^i}{i!} \frac{1}{(\sigma_i^2)^{n/2}} \frac{\Gamma(n/2)}{\Gamma(n+1)} \cdot (\alpha+2\alpha \cos\psi/\sqrt{\beta} + \alpha/\beta)^{n/2} \cdot {}_1F_1[n/2;n+1;-(\sigma_i^2)^{-1} (\alpha+2\alpha \cos\psi/\sqrt{\beta} + \alpha/\beta)] . \tag{22}$$

Substituting ψ and r, which are defined previously, into eq. (22), we can rewrite eq. (22) as

$$P_e = (1-\frac{1}{M}) - \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} \frac{1}{\pi} \sin\frac{n\pi}{M} \cdot \cos\{n[\tan^{-1} \frac{\sin(\phi+2\pi\nu/M)}{\sqrt{\beta} + \cos(\phi+2\pi\nu/M)}]\} \cdot \frac{e^{-A}A^i}{i!} \frac{1}{(\sigma_i^2)^{n/2}} \frac{\Gamma(n/2)}{\Gamma(n+1)} \cdot [\alpha+2\alpha\cdot\cos(\phi+2\pi\nu/M)/\sqrt{\beta}+\alpha/\beta]^{n/2} \cdot {}_1F_1[n/2;n+1;-(\sigma_i^2)^{-1} \cdot \{\alpha+2\alpha\cdot\cos(\phi+2\pi\nu/M)/\sqrt{\beta} + \alpha/\beta\}] \tag{23}$$

For the calculation of bit error rate, in eq.(23), M cases of $\nu=0$, $\nu=1$, $\cdots$, and ν=M-1 must be considered. We can also assume that the probability of occurrence of the message of the interferer is equally likely. Then, the total probability of error is the average of M cases. Thus,

$$P_E = \frac{1}{M}(P_e|_{\nu=0} + P_e|_{\nu=1} + \cdots + P_e|_{\nu=M-1}). \tag{24}$$

IV. Numerical Calculation and Discussions

The bit error rate (BER) of the received binary PSK(BPSK) signal has been shown in Fig.4. The noteworthy points from the graphs are summarized as follows.

1) In Fig. 4-2, in the case of A=1, which is the maximum value, the graph approaches to Gaussian case. This phenomenon bases on the reason that the impulsiveness becomes weaker to Gaussian with respect to the change of A to unity.

   Additionally, the reversion occurs with the variation of A. The reason of this phenomenon is regarded as the error probability, in the case of low CNR, becomes lower, for the instantaneous value of the noise seldomly exceeds the level of the signal. At high CNR, the major factor causing the bit error is the impulsiveness. Therefore, at high CNR, the impulsive noise causes more errors than Gaussian noise.

2) The bit error rates for a fixed value of A(=0.01) and with the variaion of Γ' are shown in Fig. 4-3 and Fig. 4-4. There occurred reversion near the CNR of 10 dB in the case of CIR of 5 dB and did near the CNR of 7 dB in the case of CIR of 10 dB. At high CNR, the BER is nearly independent with the variation of Γ'. Comparing Fig. 4-3 and Fig. 4-4, at high CNR, we can find the fact that the error rate is nearly independent on CIR.

3) The most interesting result, however, is shown in Fig. 4-5. This graph is the case of normalization with the worst case ($\phi=0°$, and $\phi=180°$ in BPSK) with the variation of the phase difference between the carriers of the signal and the interferer from 0° to 180°. As shown in the figure, the variation of bit error probability with ϕ is great at low CIR or at high CNR. It is the best case that the carriers of the signal and the interferer meet with orthogonal phase.

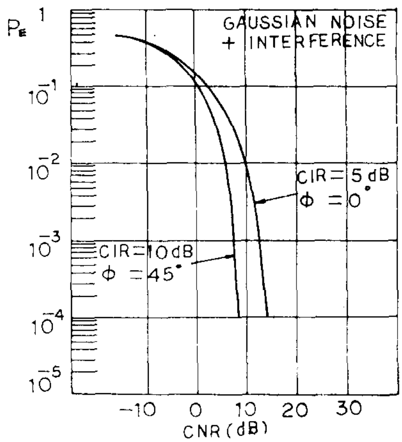

Fig. 4. Bit error rate (BER) of BPSK signal interfered by Gaussian noise and one interferer.

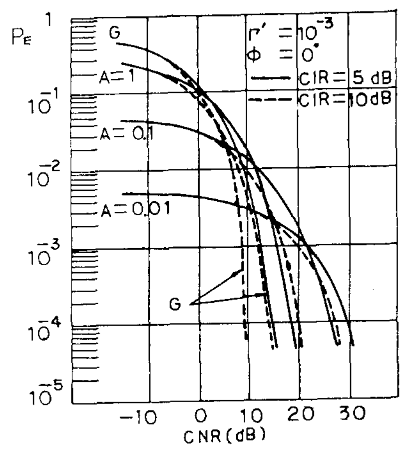

Fig. 5. BER of BPSK signal interfered by one interferer and impulsive noise (with respect to the change of A).

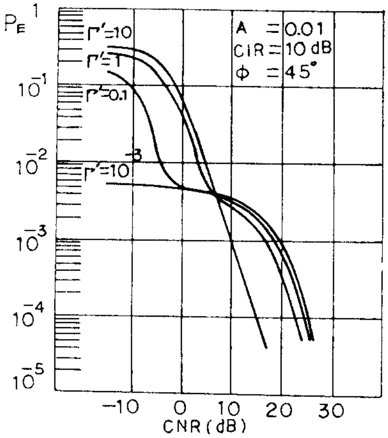

Fig. 6. BER of BPSK signal interfered by one interferer and impulsive noise (with respect to the change of Γ', CIR=5 dB).

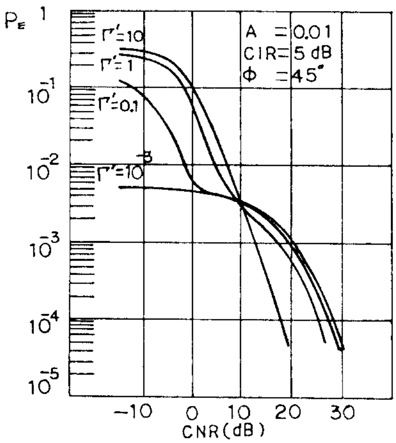

Fig. 7. BER of BPSK signal interfered by one interferer and impulsive noise (with respect to the change of Γ', CIR=10 dB).

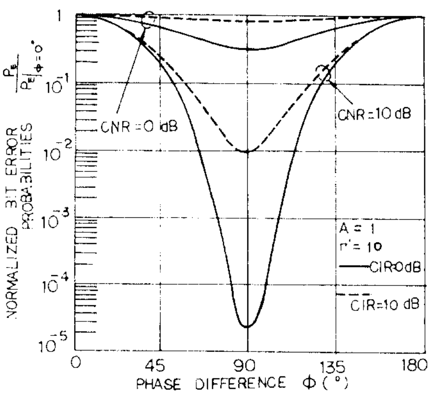

Fig. 8. Bit error probabilities normalized by the worst case (ϕ=0°, and ϕ=180°).

Appendix[15]

$$J_0(Z) = \frac{1}{2\pi}\int_0^{2\pi} e^{iZ\cos\theta}\, d\theta \qquad \text{(F.1)}$$

$$e^{iZ\cos\theta} = \sum_{n=0}^{\infty} \epsilon_n (-1)^n J_{2n}(Z)\cos 2n\theta + 2j\sum_{n=0}^{\infty} (-1)^n J_{2n+1}(Z)\cos(2n+1)\theta \qquad \text{(F.2)}$$

$$P(Z,\theta) = \int_0^{\infty} P(Z,\theta \mid E)\, P(E)\, dE \qquad \text{(F.3)}$$

$$\int_0^{\infty} J_0(at)\cdot \exp(-p^2t^2)\cdot t\cdot dt = \frac{1}{2p^2}\cdot \exp\left(-\frac{a^2}{4p^2}\right) \qquad \text{(F.3)}$$

$$J_\nu(Z) = \sum_{k=0}^{\infty} \frac{(-1)^k (Z/2)^{\nu+2k}}{k!\,\Gamma(\nu+k+1)} \qquad \text{(F.4)}$$

$$\int_0^{\infty} t^{\mu-1} J_\nu(at)\, e^{-p^2t^2}\, dt = \frac{\Gamma(\frac{\nu}{2}+\frac{\mu}{2})\cdot(\frac{a}{2p})^\nu}{2p^\mu\,\Gamma(\nu+1)} \cdot {}_1F_1\left[\frac{\nu}{2}+\frac{\mu}{2};\nu+1;-\frac{a^2}{4p^2}\right] \qquad \text{(F.5)}$$

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