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Numerical Analysis of Natural Convection from a Vertical Ice Circular Cylinder in Pure Water

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純粹물속에 있는 垂直얼음 圓기둥에 의한 自然對流의 數値解析

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抄　　錄

比較的 低溫의 純粹물 속에 있는 垂直얼음 원기둥 周圍에서 일어나는 自然對流 현상을 數値計算방법을 使用하여 理論的인 解를 求하였으며 기존 實驗結果와 比較하였다. 流動의 形態는 주위유체의 溫度에 따라서 上向流動, 下向流動, 이 양자가 同時에 存在하는 流動의 형태로 구분되었으며 解를 구할 수 있는 영역內에서는 上向流動과 下向流動이 동시에 존재하는 구역에서 熱傳達率이 最少가 됨을 알 수가 있었다.

Nomenclature

C_p : Constant pressure specific heat
g : Acceleration of gravity
h : Local heat transfer coefficient
$\bar{h}$: Mean heat transfer coefficient
K : Thermal conductivity
l : Length of cylinder
L : Specific internal energy per unit mass
Nu_z : Local Nusselt number
$\overline{Nu}$: Mean Nusselt number
P : Pressure
Pr : Prandtl number
r, z : Cylinderical coordinates
T : Local temperature of fluid
u, v : Velocities in z and r directions
X, Y : Body forces in z and r directions
ρ : Local density of fluid
μ : Dynamic viscosity
ψ : Stream function
ω : Vorticity
ϕ : Dependent variable

Subscripts

∞ : Free stream condition
W : Wall condition

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1. Introduction

Heat transfer problems around the maximum density point have a practical importance in designing the utility systems for northern districts, for example, freezing of water, engineering ice making, etc.

By the dependence of density on temperature and pressure, density extrema may occur across a given flow region. A general analysis of a particular flow geometry seems to be very complicated, even if the Boussinesq approximation is applied to the extent of neglecting density variations in continuity considerations. And the analysis is more complicated when a density extremum condition arises, since the thermal expansion coefficient may be positive, zero, and negative within such a flow. Through one portion of the thermal boundary layer the buoyancy forces will be upwards, and through the other portion such forces will be downwards. Consequently a complicated flow structure may result.

In case where an ice surface is in contact with a surrounding pure water, the analysis becomes more complicated by the melting or fusion process which occurs at the ice-water interface. In such a case the melting or fusion process can be approximated by applying a blowing or suction boundary condition at the interface.

The phenomena related to the free convective melting of an ice surface into a pure water medium have been investigated to some extent in the past. Merk[1] is the first to analyze convective reversals around the density extremum. Using an integral method, he predicted that around $T_\infty=5.31°C$, the Nusselt number has a minimum and the direction of the flow changed from upward at lower temperatures to downward at higher temperatures.

Dumore et al[2] performed an experiment by melting an ice sphere immersed in still water. They demonstrated that a convective inversion occurred at 4.8°C.

Tkachev[3] obtained a minimum Nusselt number at 5.5°C in his experiments with vertical ice cylinders.

The observations of Schenk & Schenkels[5] for ice spheres in cold water were in fair agreement with the previous experimental results[1,2]. Their results showed that a dualflow exists in the range of $4°C \leq T_\infty \leq 6°C$ and a minimum value in heat transfer parameter occurs at $T_\infty=5.3°C$.

Heat transfer characteristics passing through the maximum density point around a horizontal ice cylinder immersed in water was studied theoretically and experimentally by Takeo-Saitoh[7]. It was found that at about 6°C of water temperature the stagnation Nusselt number takes its minimum value, and the instability of the flow was observed.

Gebhart and Mollendorf[10] also carried out numerical calculations for thermally induced buoyant flows using their new density relation[9]. Analysis of flows arising from thermally induced buoyancy and from combined buoyancy effects showed the simplicity of the formulation and closer agreement with the previous experiments[8].

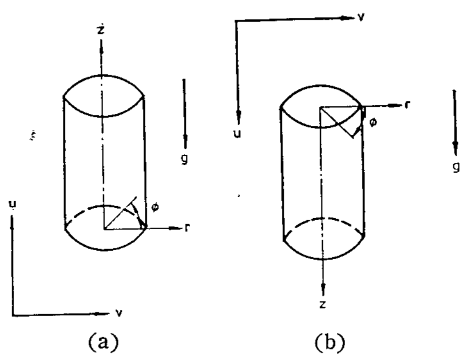
A steady state finite difference analysis for a vertical ice wall has been reported in the ranges of free stream temperatures from 0°C to 24°C by Wilson and Lee[13]. The calculated stream function showed that near the wall there existed a noted upward flow region for $5.73°C \leq T_\infty \leq 6.0°C$ in pure water.

The present study is intended to relate the experimental results of Tkachev which were not enough to analyze heat transfer characteristics. In the present work, more detailed heat transfer characteristics and flow patterns are studied for vertical ice circular cylinder.

2. Analysis

The physical model and the coordinate systems are schematically shown in Fig. 1. The problem is to analyze numerically the convective flow around a vertical ice circular cylinder (radius $r_w=2.5$cm, height $l=30$cm) which is located in quiescent pure water of constant temperature T_∞. The temperature of the cylinder wall was 0°C. And cylindrical coordinate system is employed.

For simplicity, the following assumptions and restrictions are made.



(a) For dominant upflow
(b) For dominant downflow
Fig. 1 Coordinate systems

a) Flow is laminar

b) The shape of an ice surface does not change during melting.

c) All fluid properties remain constant with an exception of density in the body force term.

d) The flow is symmetrical with respect to the vertical plane passing through the axis.

Under these postulations, the governing equations for this problem can be described as follows.

$$\frac{\partial u}{\partial z}+\frac{\partial v}{\partial r}+\frac{v}{r}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}\right)=\pm\rho g-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$\rho\left(u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}\right)=\frac{K}{C_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \tag{4}$$

Here, the $\pm\rho g$ term in equation (2) is the body force per unit volume, and the sign of the term will be negative for dominant upward flow and positive for dominant downward flow. The approach of Lafound[13] was employed as density equation in terms of temperature as shown in Fig. 2.

By introducing vorticity and stream function, and

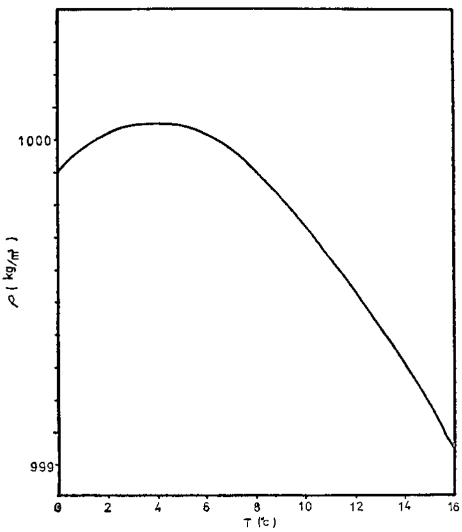


Fig. 2 Density distribution of pure water

by eliminating the pressure term from equations (2) and (3), the governing equations can be written as:

$$\frac{1}{\rho}\left\{\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial \psi}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)\right\}+\omega=0 \tag{5}$$

$$r^2\left\{\frac{\partial}{\partial z}\left(\frac{\omega}{r}\frac{\partial \psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\frac{\omega}{r}\frac{\partial \psi}{\partial z}\right)\right\}-\mu\left[\frac{\partial}{\partial z}\left\{r^3\frac{\partial}{\partial z}\left(\frac{\omega}{r}\right)\right\}+\frac{\partial}{\partial r}\left\{r^3\frac{\partial}{\partial r}\left(\frac{\omega}{r}\right)\right\}\right]+r^2\left\{\frac{\partial}{\partial z}\left(\frac{u^2+v^2}{2}\right)\frac{\partial \rho}{\partial r}-\frac{\partial}{\partial r}\left(\frac{u^2+v^2}{2}\right)\frac{\partial \rho}{\partial z}\right\}\pm gr\frac{\partial \rho}{\partial r}=0 \tag{6}$$

$$\frac{\partial}{\partial z}\left(T\frac{\partial \psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(T\frac{\partial \psi}{\partial z}\right)-\frac{K}{C_p}\left\{\frac{\partial}{\partial z}\left(r\frac{\partial T}{\partial z}\right)+\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right\}=0 \tag{7}$$

where the stream function, ψ, is defined as:

$$\rho u=\frac{1}{r}\frac{\partial \psi}{\partial r},\quad \rho v=-\frac{1}{r}\frac{\partial \psi}{\partial z} \tag{8}$$

and vorticity, ω, is defined as:

$$\omega=\frac{\partial v}{\partial z}-\frac{\partial u}{\partial r} \tag{9}$$

Boundary conditions are listed as follows.

$z=0$(except for the wall) ; $\psi=\omega=0$, $T=T_\infty$

$r=r_\infty$; $\frac{\partial \psi}{\partial r}=\frac{\partial^2 \psi}{\partial r^2}=\omega=0$, $T=T_\infty$

$r=r_w$; $u=T=0$, $v=-\frac{K}{\rho L}\left.\frac{\partial T}{\partial r}\right|_{r_w}$, $\psi=-\int_0^z \rho r v\, dz$

To solve above equations (5), (6), and (7), the finite difference scheme developed by Gosman et al[15] was employed.

3. Results and Discussion

Figure 3 shows the velocity profiles at a distance of $z=0.0515$m along the cylinder for various selected free stream temperatures. The positive velocity indicates upflow and the negative velocity means downflow. It is shown that the flows below 4°C which is approximately the temperature of the maximum density point in pure water are wholly upward. Up to 2.5°C, rather vigorous flow pattern is obse-

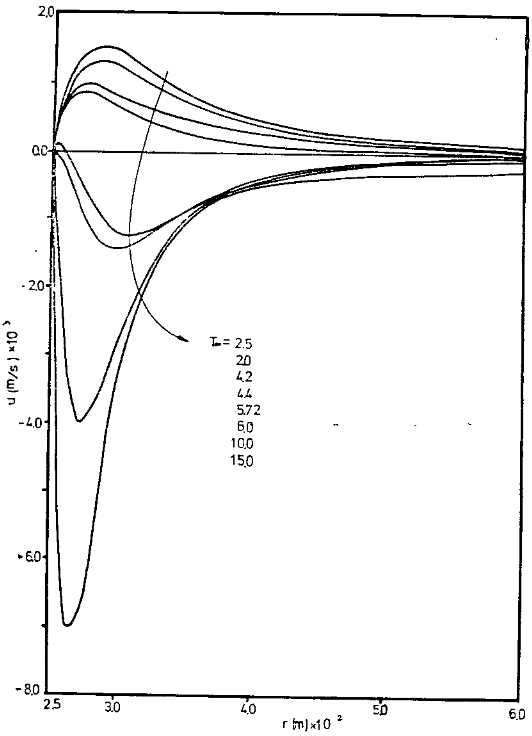


Fig. 3 Velocity profiles for various free stream temperatures at $z=0.0515$m

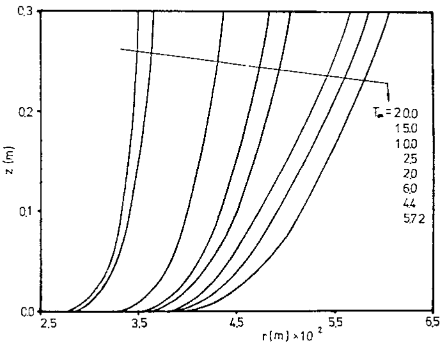


Fig. 4 Thermal boundary layer thickness for various free stream temperatures

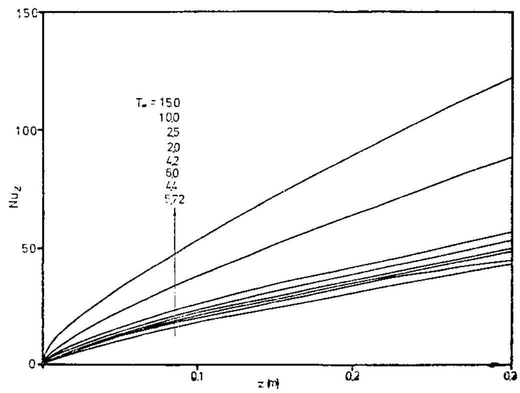


Fig. 5 Local Nusselt numbers for various free stream temperatures

rved with increasing free stream temperature. But the fluid velocities are decreasing with increasing T_∞ for $2.5°C<T_\infty\leq4.4°C$. This results from the relatively lower buoyancy forces which exist in the outer portion of the thermal boundary layer for values of T_∞ approaching to or surpassing 4°C.

For $4.4°C<T_\infty<5.72°C$, converging solutions could not be obtained for either upflow or downflow. And an oscillation was observed during computational procedure. Various grid configurations and a great number of factors of under-relaxation were employed to obviate this difficulty but all attempts were failed.

For $T_\infty=5.72°C$, because of high free stream temperature, the fluid far from ice wall is significantly heavier than that near the wall. The fluid near the wall flows upwards and downflow is observed far from the wall. Thus, a dualflow pattern can be observed at this free stream temperature. This trend was found to be valid for $5.72°C\leq T_\infty\leq6.0°C$.

If T_∞ is increased beyond this temperature range, downflows are observed and the flows become vigorous with increasing free stream temperature in all the velocity fields.

Thermal boundary layer thickness is shown in Figure 4. For $T_\infty\leq2.5°C$, the thermal boundary layer thickness decreases progressively and then increases for $2.5°C<T_\infty\leq4.4°C$ with increasing temperature. For $T_\infty>5.72°C$, the thermal boundary layer thickness decreases with increasing free stream temperature. This trend is very similar to that of the velocity boundary layer.

The local Nusselt number, Nu_z, can be defined as:

$$Nu_z=\frac{hz}{K}=-\frac{z}{T_w-T_\infty}\frac{\partial T}{\partial r}\Big|_{r_w}$$

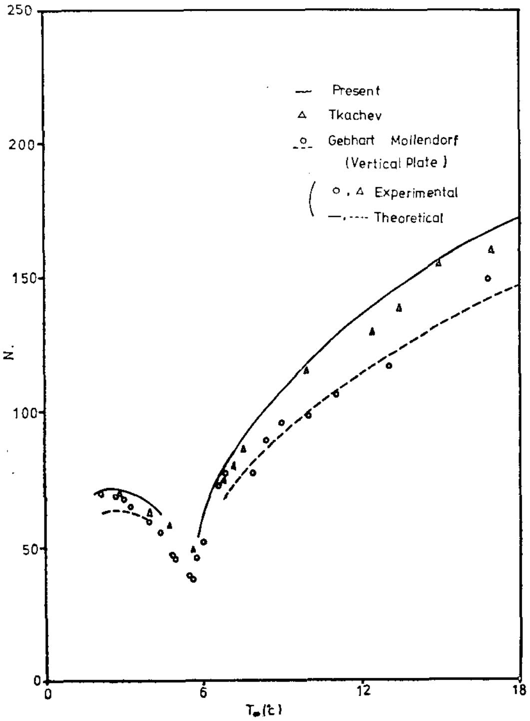


Fig. 6 Comparison of mean Nusselt numbers over a length, 0.3m

If $\left.\frac{\partial T}{\partial r}\right|_{r_w}$ is evaluated numerically, the Nusselt number variation along the cylinder can be determined for various free stream temperatures. Local Nusselt numbers for various T_∞ appear in Figure 5. The local Nusselt number increases with increasing T_∞ for the range of $T_\infty \leqq 2.5°C$ and also with increasing z, but it decreases with increasing T_∞ for the range of $2.5°C < T_\infty \leqq 4.4°C$. Above $T_\infty = 5.72°C$, this increases again with increasing T_∞. And it also increases with increasing z. These trends correspond to convective flow velocity patterns.

From the local heat transfer coefficient, the mean heat transfer coefficient, $\bar{h}$, and mean Nusselt number, $\overline{Nu}$, can be obtained as follows;

$$\bar{h} = \frac{1}{l}\int_0^l h dz$$

$$\overline{Nu} = \frac{\bar{h} l}{K}$$

In Figure 6, the mean Nusselt number is plotted for various free stream temperatures. Qualitatively, the present results show a good agreement with the experimental results of Tkachev[3]. As shown in Figure 6, the computed results have greater values of Nusselt number than those of the experimental results in all the temperature range. It was found that the mean Nusselt number distribution of a vertical ice cylinder is higher than that of a vertical ice plate for whole range of free stream temperatures. This trend appears because the ratio, $\frac{r_w}{l}$, is relatively small[17].

4. Conclusion

The results show three distinct flow regimes: dominant upward flow, dominant downward flow, and dualflow region. The range of upward flow is $T_\infty \leqq 4.4°C$ and that of downflow is $T_\infty \geqq 6.0°C$. Dualflow is obtained in the range of $5.72°C \leqq T_\infty < 6.0°C$. No solution has been obtained in the range of $4.4°C < T_\infty < 5.72°C$.

The computed results of mean Nusselt number are slightly greater than the experimental results.

The heat transfer rate of a vertical ice cylinder is higher than that of a vertical ice plate for the whole ranges of free stream temperatures.

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