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Analysis for Measuring the Thermal Diffusivity of Multilayered Composites in Flash Method

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다층복합재료의 열확산계수를 섬광법으로 측정하기 위한 해석

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초 록

섬광법의 응용범위를 증대하기 위하여 시편 전, 후면에서 복사와 대류 열손실이 있고, 전면에 임의 열원이 가해지는 3층 복합재료의 열확산 방정식을 Green 함수를 도입하여 해석하였다.

본 해석결과는 고체 재료를 1층 재료로 표면처리를 실시한 얇은 층 또는 코오팅 재료를 2층재료로, 용기내에 들어있는 액체나 기체를 3층 재료로 하여 저온으로부터 고온에 이르기까지 광범위한 온도에 걸쳐 열확산 계수를 구하는데 응용될 수 있다.

Nomenclature

a : Cross sectional area of sample (m^2)

c : Specific heat at constant pressure (J/kg·K)

G : Green's function

h_c : Heat transfer coefficient combined by convection and radiation ($W/m^2 \cdot K$)

h_r : Radiation heat transfer coefficient as defined $h_r=4\sigma\varepsilon T_\infty^3$ ($W/m^2 \cdot K$)

h_f : Convection heat transfer coefficient ($W/m^2 \cdot K$)

H : Volumetric heat capacity ($J/m^3 \cdot K$)

k : Thermal conductivity (W/m·K)

l : Thickness of homogeneous sample(m)

l_i : Thickness of i-th layer (m)

$K_{\frac{1}{2}}, K_{\frac{2}{3}}$: Value defined in equation (13)

L : Distance from reference point (m)

N_n : Norm

$q(z,t)$: Instantaneous volumetric heat source generated by the heat pulse(W/m^3)

Q : Heat energy supplied per unit area at the front surface of the sample from the heat source (J/m^2)

t : Time (sec)

$t_{\frac{1}{2}}$: The time required for the back surface to reach hal fof the maximum temperature(sec)

t' : Dummy variable for time

t_p : Peak time for exponential type heat pulse produced by a Xenon flash lamp (sec)

T : Temperature in the layer at location z and time t (K)

T : Temperature of surrounding fluid (K)

z : Axial coordinate (m)

z' : Dummy variable for axial coordinate

α : Thermal diffusivity (m^2/sec)

β : Fraction between zero and one for the triangular heat pulse produced by a laser

σ : Stefan-Boltzmann constant(5.669×10^{-8}W/

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$m^2 \cdot K^4$)

ε : Surface emissivity

λ_n : Eigenvalue

ψ_{in} : Eigenfunction

ρ : Density (kg/m^3)

δ : Kronecker delta function

$\phi(t)$: Normalized heat pulse

Subscripts

i : Value of i-th layer ($i=1, 2, 3$)

max : Value at maximum time

1. Introduction

With the development of the industries, the use of layered composities, especially surface treated or coated materials on the substrates, has been rapidly increased in a number of applications such as thermal barriers, emissivity control, electrical insulation, wear and tear, and erosion or corrosion resistance protection. Thus the layered composites have become one of the most important engineering materials in practical use. It is therefore a very interesting and important problem to predict and measure the thermophysical properties of the layered composites.

Larson and Koyama[1], Gilchrist[2], Schriempf[3] Bulmer and Taylor[4], and Lee[5] measured the thermal diffusivity of the layered composites by the flash method for measuring thermal diffusivity of homogeneous materials. But they neglected the heat loss from the front and rear surfaces of the sample. Lee[5] measured the thermal diffusivity by the flash method, using the vessel of the three-layer composites with distilled water. From the measured results of the thermal diffusivity of distilled water, he demonstrated that the flash method can be also applied to measure the thermal diffusivity of fluids. But his results can be applied only in the limited ranges because of the neglected heat loss.

In the present work, in order to extend the application of the flash method to the fluids as well as the solids, an arbitrary heat pulse is applied to the front surfaces of the sample made by three-layer composites, and the heat diffusion equation is analyzed by Green's function with assumption of the boundary conditions which closely corresponds with the actual experimental conditions. Special emphasis is on the heat loss from the front and rear surfaces of the sample.

2. Formulation of the Problem

In the flash method of measuring thermal diffusivity, a small sample of cylindrical shape is subjected to a short pulse of radiant energy on the front face and the resulting temperature-vs-time of the back surface is recorded. From this history, thermal diffusivity of the sample is obtained by digital computer data reduction procedure with analytical solution.

In order to apply the flash method for measuring the thermal diffusivities of fluids as well as those of oxidized or nitrided films, the heat diffusion equation must be established for the three-layer composites shown in Fig. 1 and solved with the closely corresponding assumption as the measuring conditions. Therefore, in the present analysis, the following assumptions are made to solve the heat diffusion equation of the flash method.

(1) Heat flow is one dimensional.

(2) Each layer is homogeneous.

(3) There is no interfacial thermal contact resistance.

(4) Instantaneous heat pulse acted on the front surface of the sample is a function of the time and is uniformly absorbed on the front surface.

(5) All of the thermophysical properties are constant in measuring temperature range but the thermophysical properties of each layer have

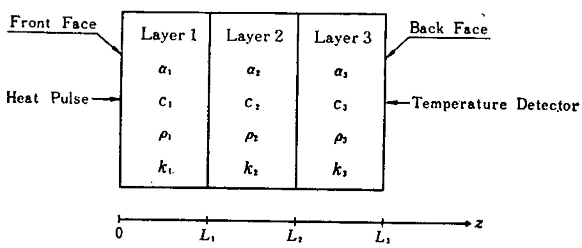

Fig. 1 Diagram of three-layer sample

different values.

(6) There is heat loss from the front and rear surfaces of the sample.

In the above condition (6), the temperature difference between the sample and the surrounding fluid is assumed to be small. So the usual fourth-power law for radiation heat loss may be approximated by the linear formula as follows;

$$\sigma\varepsilon(T^4-T_\infty^4)\simeq h_r(T-T_\infty),\ \text{where } h_r=4\sigma\varepsilon T^3$$

Therefore the heat loss on the surface of the sample by the combined heat transfer of convection and radiation is

$$h_f(T-T_\infty)+h_r(T-T_\infty)=h_c(T-T_\infty),\ \text{where}$$
$$h_c=h_f+h_r \tag{1}$$

Under the above assumptions, the heat diffusion equation for each layer is mathematically described in the following way:

$$\alpha_1\frac{\partial^2 u_1}{\partial z^2}+\frac{\alpha_1}{k_1}q(z,t)=\frac{\partial u_1}{\partial t}\ \text{ in } 0<z<L_1, t>0 \tag{2}$$

$$\alpha_2\frac{\partial^2 u_2}{\partial z^2}=\frac{\partial u_2}{\partial t}\ \text{ in } L_1<z<L_2,\ t>0 \tag{3}$$

$$\alpha_3\frac{\partial^2 u_3}{\partial z^2}=\frac{\partial u_3}{\partial t}\ \text{ in } L_2<z<L_3,\ t>0 \tag{4}$$

The $q(z,t)$ in equations (2) is the heat pulse radiated on the front surface of the sample. In order to consider the finite-pulse-time effect, the $q(z,t)$ can be described in the form

$$q(z,t)=Q\delta(z)\phi(t)$$

The boundary conditions and initial conditions are

$$-k_1\frac{\partial u_1}{\partial z}+h_{c_1}u_1=0\ \text{ at } z=0 \tag{5a}$$

$$u_1=u_2\ \text{ at } z=L_1 \tag{5b}$$

$$k_1\frac{\partial u_1}{\partial z}=k_2\frac{\partial u_2}{\partial z}\ \text{ at } z=L_1 \tag{5c}$$

$$u_2=u_3\ \text{ at } z=L_2 \tag{5d}$$

$$k_2\frac{\partial u_2}{\partial z}=k_3\frac{\partial u_3}{\partial z}\ \text{ at } z=L_2 \tag{5e}$$

$$k_3\frac{\partial u_3}{\partial z}+h_{c_3}u_3=0\ \text{ at } z=L_3 \tag{5f}$$

The initial conditions are

$$u_1=u_2=u_3=0\ \text{ for } t=0 \tag{6}$$

where

$$u_i=T_i-T_\infty\ \ i=1,2,3 \tag{7}$$

3. Analysis

In order to analyze heat diffusion equation by Green's function, the following eigenvalue problems must be solved.

$$\frac{d^2\psi_{in}}{dz^2}+\frac{\lambda_n^2}{\alpha_i}\psi_{1n}=0\qquad i=1,2,3 \tag{8}$$

$$-k_1\frac{d\psi_{1n}}{dz}+h_{c_1}\psi_{1n}=0\ \text{ at } z=0 \tag{9a}$$

$$\psi_{1n}=\psi_{2n}\ \text{ at } z=L_1 \tag{9b}$$

$$k_1\frac{d\psi_{1n}}{dz}=k_2\frac{d\psi_{2n}}{dz}\ \text{ at } z=L_1 \tag{9c}$$

$$\psi_{2n}=\psi_{3n}\ \text{ at } z=L_2 \tag{9d}$$

$$k_2\frac{d\psi_{2n}}{dz}=k_3\frac{d\psi_{3n}}{dz}\ \text{ at } z=L_2 \tag{9e}$$

$$k_3\frac{d\psi_{3n}}{dz}+h_{c3}\psi_{3n}=0\ \text{ at } z=L_3 \tag{9f}$$

The general solution of equation(8) is taken as

$$\psi_{in}(z)=A_{in}\sin\left(\frac{\lambda_n}{\sqrt{\alpha_i}}z\right)+B_{in}\cos\left(\frac{\lambda_n}{\sqrt{\alpha_i}}z\right),\quad i=1,2,3 \tag{10}$$

In order to determine six coefficients A_{in}, B_{in} with $i=1,2,3$, the eigenfunctions $\psi_{in}(z)$ given by equation(10) with $B_{1n}=1$ without loss of generality are inserted into the equation(9)

The resulting system of equations can be represented as a following matrix notation

$$[a][x]=[0] \tag{11}$$

where

$$[a]=\begin{pmatrix} 1 & -B_1/\gamma_n & 0 & 0 & 0 & 0 \\ \sin\gamma_n & \cos\gamma_n & -\sin\left(\frac{L_1}{L_2}\eta_n\right) & -\cos\left(\frac{L_1}{L_2}\eta_n\right) & 0 & 0 \\ K_{1/2}\cos\gamma_n & -K_{1/2}\sin\gamma_n & -\cos\left(\frac{L_1}{L_2}\eta_n\right) & \sin\left(\frac{L_1}{L_2}\eta_n\right) & 0 & 0 \\ 0 & 0 & \sin\eta_n & \cos\eta_n & -\sin\left(\frac{L_2}{L_3}\sigma_n\right) & -\cos\left(\frac{L_2}{L_3}\sigma_n\right) \\ 0 & 0 & K_{2/3}\cos\eta_n & -K_{2/3}\sin\eta_n & -\cos\left(\frac{L_2}{L_3}\sigma_n\right) & \sin\left(\frac{L_2}{L_3}\sigma_n\right) \\ 0 & 0 & 0 & 0 & \cos\sigma_n+\frac{B_3}{\sigma_n}\sin\sigma_n & \frac{B_3}{\sigma_n}\cos\sigma_n-\sin\sigma_n \end{pmatrix} \tag{12a}$$

$$[x]=\begin{pmatrix} A_{1n} \\ 1 \\ A_{2n} \\ B_{2n} \\ A_{3n} \\ B_{3n} \end{pmatrix} \tag{12b}$$

here

$$\gamma_n=\frac{\lambda_n L_1}{\sqrt{\alpha_1}} \quad \eta_n=\frac{\lambda_n L_2}{\sqrt{\alpha_2}} \quad \sigma_n=\frac{\lambda_n L_3}{\sqrt{\alpha_3}}$$

$$B_1=\frac{h_{c1}L_1}{k_1} \quad B_3=\frac{h_{c3}L_3}{k_3} \quad K_{1/2}=\frac{k_1}{k_2}\sqrt{\frac{\alpha_2}{\alpha_1}}$$

$$K_{2/3}=\frac{k_2}{k_3}\sqrt{\frac{\alpha_3}{\alpha_2}} \tag{13}$$

The first five equations of equation(11) can be used to determine the coefficients. The coefficients are

$$A_{1n}=B_1/\gamma_n \tag{14a}$$

$$A_{2n}=\left(\frac{B_1}{\gamma_n}\sin\gamma_n+\cos\gamma_n\right)\sin\left(\frac{L_1}{L_2}\eta_n\right)+K_{1/2}\left(\frac{B_1}{\gamma_n}\cdot\cos\gamma_n-\sin\gamma_n\right)\cos\left(\frac{L_1}{L_2}\cdot\eta_n\right) \tag{14b}$$

$$B_{2n}=\left(\frac{B_1}{\gamma_n}\cdot\sin\gamma_n+\cos\gamma_n\right)\cos\left(\frac{L_1}{L_2}\eta_n\right)-K_{1/2}\left(\frac{B_1}{\gamma_n}\cdot\cos\gamma_n-\sin\gamma_n\right)\sin\left(\frac{L_1}{L_2}\cdot\eta_n\right) \tag{14c}$$

$$A_{3n}=\left(\frac{B_1}{\gamma_n}\cdot\sin\gamma_n+\cos\gamma_n\right)\left\{\sin\left(\frac{L_2}{L_3}\sigma_n\right)\cos\left(\frac{L_2-L_1}{L_2}\eta_n\right)-K_{2/3}\cos\left(\frac{L_2}{L_3}\sigma_n\right)\sin\left(\frac{L_2-L_1}{L_2}\eta_n\right)\right\}+K_{1/2}\left(\frac{B_1}{\gamma_n}\cdot\cos\gamma_n-\sin\gamma_n\right)\left\{\sin\left(\frac{L_2}{L_3}\sigma_n\right)\sin\left(\frac{L_2-L_1}{L_2}\right)\eta_n+K_{2/3}\cos\left(\frac{L_2}{L_3}\sigma_n\right)\cos\left(\frac{L_2-L_1}{L_2}\eta_n\right)\right\} \tag{14d}$$

$$B_{3n}=\left(\frac{B_1}{\gamma_n}\cdot\sin\gamma_n+\cos\gamma_n\right)\left\{\cos\left(\frac{L_2}{L_3}\sigma_n\right)\cos\left(\frac{L_2-L_1}{L_2}\eta_n\right)+K_{2/3}\sin\left(\frac{L_2}{L_3}\sigma_n\right)\sin\left(\frac{L_2-L_1}{L_2}\eta_n\right)\right\}+K_{1/2}\left(\frac{B_1}{\gamma_n}\cdot\cos\gamma_n-\sin\gamma_n\right)\left\{\cos\left(\frac{L_2}{L_3}\sigma_n\right)\sin\left(\frac{L_2-L_1}{L_2}\eta_n\right)-K_{2/3}\sin\left(\frac{L_2}{L_3}\sigma_n\right)\cos\left(\frac{L_2-L_1}{L_2}\eta_n\right)\right\} \tag{14e}$$

Equation(11) has a non-trivial solution when the determinant of $[a]$ is equal to zero. After setting this determinant equal to zero, an expanded form of this determinant is then given as

$$\left(\frac{B_1}{\gamma_n}\sin\gamma_n+\cos\gamma_n\right)\left[\left\{\frac{B_3}{\sigma_n}\cos\left(\frac{L_3-L_2}{L_3}\sigma_n\right)-\sin\left(\frac{L_3-L_2}{L_3}\sigma_n\right)\right\}\cos\left(\frac{L_2-L_1}{L_2}\eta_n\right)-K_{2/3}\left\{\frac{B_3}{\sigma_n}\sin\left(\frac{L_3-L_2}{L_3}\sigma_n\right)+\cos\left(\frac{L_3-L_2}{L_3}\sigma_n\right)\right\}\sin\left(\frac{L_2-L_1}{L_2}\eta_n\right)\right]$$
$$+K_{1/2}\left(\frac{B_1}{\gamma_n}\cos\gamma_n-\sin\gamma_n\right)\left[\left\{\frac{B_3}{\sigma_n}\cos\left(\frac{L_3-L_2}{L_3}\sigma_n\right)-\sin\left(\frac{L_3-L_2}{L_3}\sigma_n\right)\right\}\sin\left(\frac{L_2-L_1}{L_2}\eta_n\right)+K_{2/3}\left\{\frac{B_3}{\sigma_n}\sin\left(\frac{L_3-L_2}{L_3}\sigma_n\right)+\cos\left(\frac{L_3-L_2}{L_3}\sigma_n\right)\right\}\cos\left(\frac{L_2-L_1}{L_2}\eta_n\right)\right]=0 \tag{15}$$

for determining the eigenvalue λ_n.

The corresponding Green's function $G_i(z,t\,|\,z',t')$ of the problem[6] is

$$G_i(z,t\,|\,z',t')=\sum_{n=1}^{\infty}\frac{1}{N_n}\frac{k_1}{\alpha_1}e^{-\lambda_n^2(t-t')}\psi_{in}(z)\psi_{1n}(z'),$$
$$i=1,2,3 \tag{16}$$

where norm N_n is given as

$$N_n=\frac{k_1}{\alpha_1}\int_0^{L_1}\psi_{1n}^2(z)dz+\frac{k_2}{\alpha_2}\int_{L_1}^{L_2}\psi_{2n}^2(z)dz+\frac{k_3}{\alpha_3}\int_{L_2}^{L_3}\psi_{3n}^2(z)dz \tag{17}$$

here

$$\int_0^{L_1}\psi_{1n}^2(z)dz=\frac{L_1}{2}\left\{\frac{B_1^2}{\gamma_n^2}+1+\frac{1}{2\gamma_n}\sin(2\gamma_n)\left(1-\frac{B_1^2}{\gamma_n^2}\right)+2\frac{B_1}{\gamma_n^2}\cdot\sin^2\gamma_n\right\} \tag{18a}$$

$$\int_{L_1}^{L_2}\psi_{2n}^2(z)dz=\frac{1}{2}(L_2-L_1)(A_{2n}^2+B_{2n}^2)-\frac{L_2}{4\eta_n}(A_{2n}^2-B_{2n}^2)\left\{\sin(2\eta_n)-\sin\left(2\frac{L_1}{L_2}\eta_n\right)\right\}+\frac{L_2}{2\eta_n}\cdot A_{2n}\cdot B_{2n}\left\{\cos\left(2\frac{L_1}{L_2}\eta_n\right)-\cos(2\eta_n)\right\} \tag{18b}$$

$$\int_{L_2}^{L_3}\psi_{3n}^2(z)dz=\frac{1}{2}(L_3-L_2)(A_{3n}^2+B_{3n}^2)-\frac{L_3}{4\sigma_n}(A_{3n}^2-B_{3n}^2)\left\{\sin(2\sigma_n)-\sin\left(2\frac{L_2}{L_3}\sigma_n\right)\right\}+\frac{L_3}{2\sigma_n}\cdot A_{3n}B_{3n}\left\{\cos\left(2\frac{L_2}{L_3}\sigma_n\right)-\cos(2\sigma_n)\right\} \tag{18c}$$

Therefore the temperature distribution of each layer described by Green's function is

$$u_i(z,t)=\int_{t'=0}^{t}\int_{z'=0}^{L_1}G_i(z,t\,|\,z',t')\frac{\alpha_1}{k_1}q(z',t')dz'dt'$$
$$i=1,2,3 \tag{19}$$

By inserting equation (16) and $q(z',t')=Q\delta(z')\phi(t')$ into equation(19), the temperature distribution of each layer is

$$u_1(z,t)=Q\sum_{n=1}^{\infty}\frac{1}{N_n}\left\{\frac{B_1}{\gamma_n}\cdot\sin\left(\frac{\lambda_n}{\sqrt{\alpha_1}}z\right)+\cos\left(\frac{\lambda_n}{\sqrt{\alpha_1}}z\right)\right\}\int_0^t e^{-\lambda_n^2(t-t')}\phi(t')dt' \tag{20a}$$

$$u_2(z,t)=Q\sum_{n=1}^{\infty}\frac{1}{N_n}\left\{A_{2n}\cdot\sin\left(\frac{\lambda_n}{\sqrt{\alpha_2}}z\right)+B_{2n}\cdot\cos\left(\frac{\lambda_n}{\sqrt{\alpha_2}}z\right)\right\}\int_0^t e^{-\lambda_n^2(t-t')}\phi(t')dt' \quad (20b)$$

$$u_3(z,t)=Q\sum_{n=1}^{\infty}\frac{1}{N_n}\left\{A_{3n}\cdot\sin\left(\frac{\lambda_n}{\sqrt{\alpha_3}}z\right)+B_{3n}\cdot\cos\left(\frac{\lambda_n}{\sqrt{\alpha_3}}z\right)\right\}\int_0^t e^{-\lambda_n^2(t-t')}\phi(t')dt' \quad (20c)$$

By letting $z=L_3$ in equation (20c), the rear surface temperature of the sample is

$$u_3(L_3,t)=Q\sum_{n=1}^{\infty}\frac{1}{N_n}\left\{A_{3n}\cdot\sin\sigma_n+B_{3n}\cdot\cos\sigma_n\right\}\int_0^t e^{-\lambda_n^2(t-t')}\phi(t')dt' \quad (21)$$

When there is no heat loss, the maximum temperature rise of the sample, u_{max}, is obtained by dividing the intergration of heat pulse by the volumetric heat capacity $(\rho_1c_1l_1a+\rho c_2l_2a+\rho c_3l_3a)$

Then

$$u_{max}=\frac{aQ\int_0^t\phi(t')dt'}{\sum_{i=1}^{3}H_i},\ \text{where } H_i=a\rho_ic_il_i \quad (22)$$

By taking the dimensionless form of equation(21) with equation(22) the virtual dimensionless temperature W is

$$W(L_3,t)=\frac{u(L_3,t)}{u_{max}}=\frac{1}{a}(H_1+H_2+H_3)\sum_{n=1}^{\infty}\frac{1}{N_n}\left\{A_{3n}\cdot\sin\sigma_n+B_{3n}\cdot\cos\sigma_n\right\}S(\lambda_n,t) \quad (23)$$

where

$$S(\lambda_n,t)=\frac{\int_0^t e^{-\lambda_n^2(t-t')}\ \phi(t')dt'}{\int_0^t\phi(t')dt'} \quad (24)$$

By dividing equation(23) with the maximum virtual dimensionless temperature $W(L_3,t_{max})$, actual dimensionless temperature V is

$$V=\frac{W(L_3,t)}{W(L_3,t_{max})} \quad (25)$$

where t_{max} is the required time to reach maximum temperature at $z=L_3$ when there is heat loss.

In order to include the finite pulse time effect, the function $S(\lambda_n,t)$ can be represented by three different ways depending on the heat pulse commonly used in flash method.

(a) For an instantaneous heat pulse used by Parker, et al[7]

$$\phi(t)=\delta(t) \quad (26a)$$

$$S(\lambda_n,t)=e^{-\lambda_n^2t} \quad (26b)$$

(b) For the triangular heat pulse of a laser discussed by Taylor[8]

$$\phi(t)=\begin{cases}\frac{2}{\tau}\left(\frac{t}{\beta\tau}\right) & 0<t<\beta\tau\\ \frac{2}{\tau}\left(\frac{\tau-t}{\tau-\beta\tau}\right) & \beta\tau<t<\tau\\ 0 & t>\tau\end{cases} \quad (27a)$$

$$S(\lambda_n,t)=\frac{2e^{-\lambda_n^2t}}{\lambda_n^4\beta\tau(\tau-\beta\tau)}\left[\beta(e^{\lambda_n^2\tau}-1)+1-e^{\lambda_n^2\beta\tau}\right] \quad (28b)$$

where β is a fraction between zero to one and τ is the pulse time,

(c) For an exponential type heat pulse function represented by Larson and Koyama[1]

$$\phi(t)=\frac{t}{t_p^2}e^{-t/t_p} \quad (29a)$$

$$S(\lambda_n,t)=\frac{e^{-\lambda_n^2t}\left[e^{(\lambda_n^2-\frac{1}{t_p})t}\left\{\left(\lambda_n^2-\frac{1}{t_p}\right)t-1\right\}+1\right]}{\left[t_p-e^{t/t_p}(t+t_p)\right]t_p\left(\lambda_n^2-\frac{1}{t_p}\right)^2} \quad (29b)$$

In the present analysis when $\rho_1=\rho_2=\rho_3=\rho$, $c_1=c_2=c_3=c$, $\alpha_1=\alpha_2=\alpha_3=\alpha$, $L_1=L_2=L_3=l$ and no heat loss from the sample surface, actual dimensionless temperature of back surface is

$$V=1+2\sum_{n=1}^{\infty}(-1)^nS(\lambda_n,t) \quad (30a)$$

where

$$\lambda_n=\frac{n\pi\sqrt{\alpha}}{l} \quad (30b)$$

For an instantaneous heat pulse represented by a Kronecker delta function equation(30a) becomes

$$V=1+2\sum_{n=1}^{\infty}(-1)^n\exp\left(-\frac{n^2\pi^2\alpha t}{l^2}\right) \quad (31)$$

This is equivalent to the well-known equation for the flash method by Parker, et al[7]. From equation (31), the thermal diffusivity can be evaluated directly by the following well-publicized formula

$$\alpha=\frac{0.139l^2}{t_{1/2}} \quad (32)$$

4. Discussion

In the present work, the mathematical analysis of layered composites without interfacial thermal contact resistance allows the measurement of the unknown one thermophysical properties for one of the composite layers if those for other layers are known. Three-layer composites are emphasized in the analysis because two-layer composite or one-layer homogeneous materials can be treated as a special case of the three-layer composites by letting the thermophysical property values of contiguity layers be equal. From this analysis the unknown property can be determined more accurately than before because of the consideration of the finite pulse time effects and of the heat loss effects from the three-layer sample surfaces in the analysis. The layered sample can be made of three-layer, two-layer or one-layer with solid materials. The application of the three-layer composite analysis can be adapted to the fluid sample, such as liquid or gas, contained in a cell composed of two thin plates consisted of the high heat conducting solid materials.

The required unknown thermal property in the flash method may be extended from the thermal diffusivity, conductivity, thermal capacity to the total emissivity by using this method. The total emissivity may be determined by the establishment of experimental set up in the flash diffusivity apparatus. If the sample is placed in high vacuum system, the emissivity may be calculated from this analysis at the given temperature and roughness of the sample by using the half time and the maximum time measured in the experiment when the remained properties of each layer are known. Therefore the emissivity may be determined as a function of temperature and the roughness of the sample surfaces.

5. Conclusion

For the extension of application in the flash method, the heat diffusion equation, with the radiation and convection heat transfer from the front and rear surfaces of three-layer composites and with an arbitrary heat pulse applied over the front face of them, is mathematically analyzed with an appropriate Green's function.

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