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A NOTE ON "GENERALIZED ITERATION PROCESS" BY T. HU AND G.-S. YANG

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In [1], T. Hu and G.-S. Yang obtained the following

THEOREM. *Suppose f is a continuous mapping which maps the closed interval $[0, 1]$ into itself, and $A=(a_{nk})$ is a stable iteration matrix, then for any $x_1 \in [0, 1]$, the generalized iteration sequence $\{v_n\}$ converges to a fixed point of f on $[0, 1]$.*

Here, a stable iteration matrix $A=(a_{nk})$ is an infinite lower triangular matrix such that

(1) $$a_{nk} \geq 0, \quad \sum_{k=1}^{n} a_{nk} = 1,$$

(2) $$\lim_{n\to\infty} a_{nn} = 0, \quad \lim_{n\to\infty} a_{nk} = 0, \quad k = 1, 2, \cdots, \text{ and}$$

(3) $$a_{n+1,k} = (1-a_{n+1,n+1}) a_{nk} \text{ for } k = 1, 2, \cdots, n$$

and $\{x_n\}$ and $\{v_n\}$ are defined inductively by

$$v_n = \sum_{k=1}^{n} a_{nk} x_k, \; x_{n+1} = f v_n, \; n=1, 2, \cdots \quad [2].$$

In this note, Theorem of Hu and Yang is actually a simple consequence of the following

PROPOSITION ([3], Corollary 3.1). *Let f be a continuous selfmap of a compact interval I, and $\{x_n\}$ a sequence in I such that $v_{n+1} \in \overline{v_n(fv_n)}$ for all $n \in \omega$. Then*
(1) *$v_n - v_{n+1} \to 0$ iff $\{v_n\}$ converges, and*
(2) *$v_n - f v_n \to 0$ iff $\{v_n\}$ converges to a fixed point of f.*

Here, — denotes the closed interval joining two points.
Note that the stable iteration matrix $A=(a_{nk})$ in Theorem is regular. Hence, A maps every convergent sequence into a convergent sequence with invariant limit in the sense that if $x_k \to l$, then $\sum_{k=1}^{\infty} a_{nk} x_k \to l$ (cf. [4]).

Proof of Theorem. Since $|v_{n+1} - v_n| \leq a_{n+1, n+1} \to 0$, $\{v_n\}$ converges to some $v_0 \in [0, 1]$ by Proposition (1). Therefore, $x_k = f(v_{k-1}) \to f v_0$ from the continuity of f. Since A is regular, $v_n = \sum_{k=1}^{n} a_{nk} x_k \to f v_0$. Hence, $v_0 = f v_0$.

References

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