大韓交通學會誌 第一卷 第一號, pp.64～81. TRSK JOURNAL VOL. 1 NO. 1

□論 文□

An Assessment of Trip-maker's Behavior Under Uncertainty for Value of Travel Time.

時間價値의 不確實性 아래 通行者行態에 관한 研究.

文 東 周 *
(國土開發硏究院 首席硏究員)

目 次

〈要 約〉

消費者 行態에 관한 新古典 經濟理論을 通行需要의 分析에 그대로 適用하는데 問題가 되는 특기 事項으로는 運賃이외에 通行時間과 같은 서어비스 質도 通行手段의 選擇에 影響을 미친다는 점과 特定 두지점간의 通行에서 同一한 消費者도 여러 輸送手段을 利用한다는 점을 들수 있다.

本 研究의 主題는 이와같은 通行需要의 독특한 特性을 반영할 수 있는 消費者의 效用最大化模型의 設定과 이 模型에서 誘導된 需要函數의 構造를 分析함에 있다.

詳述하면, 通行時間 價値의 不確實性 아래에서 效用을 極大化하는 意思決定問題를 Stochastic Programming 模型으로 表現하였다. 또한 이 模型에서 誘導된 特性 通行手段의 需要函數는 이 手段이 가장 有利한 時間價値의 範圍에 對한 不定積分으로 表現되며 이 積分範圍와 被積分函數는 모든 競爭手段의 運賃과 通行手段의 函數로 定義됨을 증명하였다.

또한 需要函數는 通行需要에 관한 統計分析模型에서 묵시적으로 假定되고 있는 通行手段間 代替性(Property of gross substitute)과 對角方向優勢性(Property of diagonal dominance)등의 特性을 가지고 있음을 보여 주었다.

I. Introduction

The demand for available trip alternatives has peculiar characteristics that have made it difficult to apply the neoclassical consumer theory for proper understanding of trip-maker's behavior. Firstly, quality of service such as travel time and service reliability significantly influences the consumer's choice of trip alternatives. Secondly, the consumer usually utilizes more than one alternative, in

* 本學會理事 · 工學博士

travelling from one point to another.

The main theme of this paper is to introduce a new approach to the trip demand analysis which can accommodate the two peculiar aspects mentioned above. This approach is on the same line with the household production theory which includes value of travel time in estimating the real price of trip, similar to Becker (1965), Moses and Williamson (1968), and Willig (1978). Its departure from the household production theory is to consider value of travel time as a random variable having a certain distribution.

The adequancy of the approach mentioned above could be ascribed to the fact that value of travel time refers to the implicit price of travel time perceived by a consumer, and that his perception is affected by many uncertain factors. To accommodate this point in the conventional demand analysis, this paper specifies the decision problem of the consumer having multiple trip alternatives, as a stochastic programming problem. This stochastic model, unlike the traditional utility maximization problem, is designed to find the optimal consumption choice, including the trip demand, which maximizes the expected value of his satisfaction level within his budget ocnstraint.

Like the demand function of the traditional consumer theory, the trip demand function of the stochastic decision problem has the image corresponding to the optimal trip frequencies for the given real price of the available trip alternatives. As to be shown later, this demand function is expressed as indefinite integral with respect to the random variable of value of travel time, where the integrand and domain are the functions of the real price. It is also verified that the function can properly characterize the two peculiar aspects of the trip demand introduced earlier. Furthermore, it is proved that the comparative static of the function has the property of a gross substitute and diagonal dominance among available trip alternatives, which has been implicitly assumed in the various methods for the trip demand estimation.

This paper proceeds as follows, Section 2 presents the analysis result for the demand function derived from the preference ordering problem of a consumer with perfect information on the real price of available trip alternatives. Section 3 characterizes the trip demand function for the consumer's problem of a stochastic programming model. This stochastic model has the identical structure with the deterministic problem of the previous section, except with the difference that value of travel time is specified as random variable. Subsequently, Section 4 introduces the analysis result for the continuity and comparative static of the demand function defined in the previous section.

II. Trip Demand Function under Certainty for Value of Travel Time

The demand for a trip is commonly called as a derived demand. As one can infer from this term of the derived demand, the trip is not an ultimate ingredient determining a consumer's level of satisfaction, but means to go to a destination apart from an origin for a certain activity. This statement, in turn, implies that utility of the consumer is determined by the total number of trips between a pair of points, regardless of the trip alternative chosen by him.

A relevant question is how to assess the consumer's choice of a trip alternative from a set of the available one, which cannot be comprehended through the simple comparision of their service charges. One approach to assess it would be to apply the concept of real price introduced in Becker (1965). It assumes that the real price of a trip alternative is a sum of the service charge and value of his resource consumption associated with its travel time, which is usually termed as value of travel time.

A problem is how to valuate the shadow price of the consumed human resources that are not traded at the market. One approach is to take as a deterministic quantity, another one is to hypothesize it as a random variable. This question, which has not been carefully assessed in the previous studies, will be the main issue of the demand analysis presented in this paper.

In this section, it is assumed that value of travel time is a deterministic quantity. In addition, it is also assumed that the trip between a different pair of points is a different commodity. Then, without loss of generality, the decision model of a neoclassical consumer could be formulated as a preference ordering problem stated below.

Assumption 1 utility index of a consumer is determined by the total number of trips $\sum_i x_i$, $i \in <1, I>$, and a numeraire y, where x_i is the number of trips made via trip alternative i, serving between a particular pair of points, and $<1, I>$ refers to the index set representing the available trip alterna tives.

Assumption 2 The Utility function of a consumer is neoclassical, i.e., strictly quasi-concave and twice differenctiable in Σx_i and y.

Assumption 3 The real price of i is $r_i + wt_i$ where r_i is service charge of i, t_i is travel time of i, and w is a positive real number representing value of travel time.

Under Assumptions 1, 2, and 3, the decision problem of a consumer can be expressed as below:

$$\text{maximize } u\left(\sum_i x_i, y\right) \quad \cdots\cdots \quad (1)$$

$$\text{subject to } \sum_i^I (r_i + wt_i)\, x_i + y \leqslant m,\ x_i \geqslant 0, \forall i,\ y \geqslant 0$$

where m refers to an intial enaowment, compriced of monetary imcome and other human resources.

Definition 1. For the decision problem in (1), the trip demand function for any $j \in <1, I>$, denoted by $\bar{x}_j$, is defined by

$$\text{(a) } \bar{x}_j(p) = \{ x_j \mid u(\sum_i x_i, y) - \bar{m}(p) \geqslant 0 \}$$

where $p = (r, t, 1, m)$, $r = (r_1, ..., r_I)$, $t = (t_1, ..., t_I)$, and

$$\text{(b) } \bar{m}(p) = \max \{ u(\sum_i x_i, y) \mid \Sigma (r_i + wt_i)\, x_i + y \leqslant m,\ x_i \geqslant 0,\ \forall_i,\ y \geqslant 0 \}$$

A good starting point to analyze the mathematical stucture of $\bar{x}_i$ would be to identify the trip alternative chosen by the consumer. The answer to this question is that the trip-maker chooses the least cost alternative, in terms of the real cost, as formally stated below.

Proposition 1. Let the index set of the least cost alternatives, S, be defined as

$$S(r,t) = \{ j \in \langle 1, I \rangle \mid r_j + wt_j \leqslant h(r,t) \}, \qquad \text{where } h(r,t) = \inf_i (r_i + wt_j).$$

Then, under Assumptions 1, 2, and 3,]

(a) $\bar{x}_i(p) \geqslant 0$, if $i \in S(r,t)$

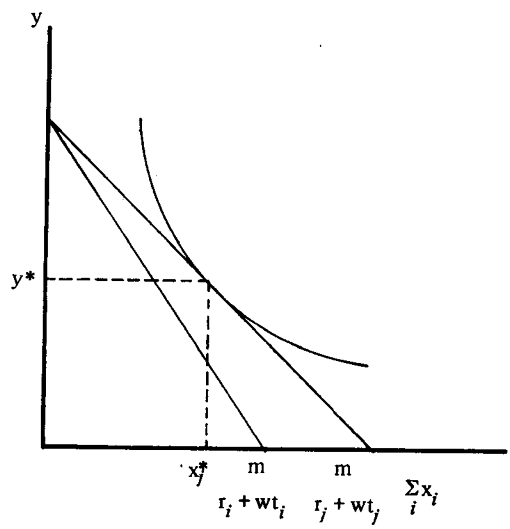
(b) $\bar{x}_i(p) = 0$, if $i \notin S(r,t)$

(c) $\sum_{i \in S(\cdot)} \bar{x}_i(p) = \sum_{\forall i} \bar{x}_i(p) > 0$.

(Proof) (a) and (b) are self-evident. The equality in (c) is the consequence of the assumption that u is strictly quasi-concave, i.e., such an objective function of u in (1) yieds a unique value of $\sum_i x_i$ together with $\bar{m}(p)$. ☒

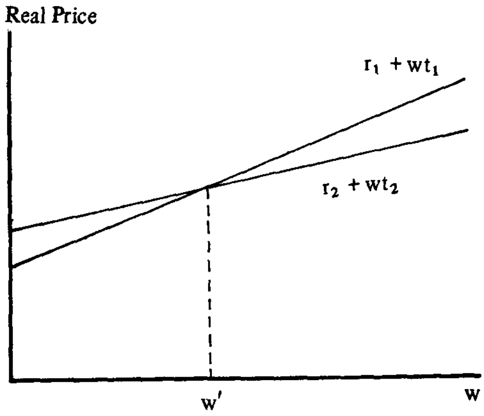
The statments of Proposition 1 can graphically be explained with use of Figure 1. In the figure, j is the least cost option, and the real cost of i is more expensive than j. From the figure, it can be inferred that the highest level of the consumer's satisfaction is achieved by choosing x^*_j and y^*. Furthermore, it can be inferred that his satisfaction is indifferent from the availability of i.

Figure 1



Another aspect of Proposition 1 can be explained by using Figure 2, which represents the real price of two available alternatives, as the function of w, value of travel time. This figure depicts, if value of travel time perceived by the consumer is w', then $r_1 + wt_1 = r_2 + wt_2$. From the figure, it can be inferred that, if w is smaller than w', Alternative 1 is the least cost alternative and $x_1(p) > 0$, where $p = (r_1, r_2, t_1, t_2, 1, m)$. On the other hand, if w is larger than w', Alternative 2 is the least cost alternative. However, if w is equal to w', the two alternatives are the least cost ones simultaneously. Then, $x_1(p)$ and $x_2(p)$ are degenerate, but $x_1,(p) + x_2(p)$ is unique, by Proposition 1.

Figure 2



At this point, it should be mentioned that the neoclassical consumer problem in (1) can be converted to a preference ordering problem of the household production approach. Also, it should be noted that Propostion 1 can be applied in showing that the two problems yield the same optimal solution for any p. This issue is discussed below.

The first step problem of the new consumer theory is to estimate the minimum production cost of $\sum_i x_i$, an ingradient of the utility function. The optimal solution to this special problem of the household production is that the unit cost is h(r, t) defined in Propostion 1. The next step problem is to maximize his utility, and can be expressed as:

$$\text{maximize } u(x, y) \quad \cdots\cdots \quad (2)$$
$$\text{subject to } h(r, t)\, x + y \leqslant m, \quad x, y \geqslant 0$$

where $x = \sum_i x_i$.

The decision problem in (2) has the identical structure with the neoclassical consumer problem for the two commodities of x and y, except that the price of x is a function of two exogeneous vectors r and t. Hence, it is possible to evaluate the mathematical structure of the trip demand function for the problem in (2), by using the well-known result of the neoclassical demand analysis.

Definition 2 For the preference ording problem in (2), the trip demand function, x', is defined by

(a) $x'(p') = \{ x \mid u(x, y) - m'(p') \geqslant 0 \}$,
where $p' = (h(r, t), 1, m)$, and
(b) $m'(p') = \max \{ u(x, y) \mid h(r, t)\, x + y \leqslant m, x, y \geqslant o \}$;

Proposition 2 Under Assumptions 1, 2, and 3,

(a) h is continuous in r and t.
(b) x' is continuous in r ant t.

(Proof) The continuity of h is a direct consequence of the fact that the pointwise infimum of concave functions is also a concave function, which is continuous. The assertion that x' is continuous in

p', if u is strictly quasi-concave, is a well-known fact in the neoclassical demand analysis [See Arrow and Hahn (1971)]. Hence, x' is continuous in r and t, by (a).

Proposition 3 Under Assumptions, 1, 2, and 3,

(a) $x'(p') = \sum_i \bar{x}_i(p)$

Suppose $j \in <1, I>$ is only one element of S(r, t). Then, the above expression is simplified as

(b) $x'(p') = \bar{x}_j(p)$

(Proof) By Proposition 1, if $j \in S(r,t)$, it is clear that $h(r, t) = r_j + wt_j$. Then, the optimal value of $\sum_i x_i$ for the problem in (1) is equal to the optimal value of x for the problem in (2). This implies (a) and (b).

From Propositions 1, 2, and 3, it can readily be stated that $\bar{x}_i$ is continuous in r and t, except at points where the image of $\bar{x}_i$ is degenerate. This conjecture is formally stated below.

Proposition 4 Under Assumptions 1, 2, and 3, for every $i \in <1, I>$
(a) $\bar{x}_i$ is upper semi-continuous in r and t.
More precisely,
(b) $\bar{x}_i$ is not continuous at (r, t) such that $i \in S(r, t)$ and $r_i + wt_i = r_j + wt_j$, $i \neq j$, but $\bar{x}_i$ is continuous at other points of (r, t).
(Proof). (b) implies (a), only if the image of $\bar{x}_i$ at (r, t), where $\bar{x}_i$ is not continuous, belongs to a compact set. Hence, it is sufficient to prove that the image of $\bar{x}_i$ belongs to a compact set at (r, t) such that $i \in S(r, t)$ and $r_i + wt_i = r_j + wt_j$, $i \neq j$, and that $\bar{x}_i$ is continuous at other points of (r, t). The first part that the image of $\bar{x}_i$ belongs to a compact set, whose lower bound is zero and the upper bound is $x'(p')$, is obvious by Proposition 3. The proof of the second part that $\bar{x}_i$ is continuous is to be worked for two cases separately. If $i \notin S(r, t)$, then the image of $\bar{x}_i$ is zero at the neighborhood of (r, t), by Propostion 1. Hence, it is continuous at that (r, t). On the other hand, if i is only one element of S (r, t), then the image of $\bar{x}_i$ is equal to that of x' at the neighborhood of that (r, t), by Proposition 3. Therefore, it is continuous at that point, by Proposition 2. ☒

The comparative static of $\bar{x}_i$ can also be asessed by identifying the relationship between X_i and x', since that of x' can readily be characterized by using the analysis result of the neoclassical consumer theory.

Proposition 5 Suppose x' is a normal good. Suppose further $i \in H < 1, I>$ is only one element of S(r, t). Then, under Assumptions 1, 2, and 3,

(a) $\dfrac{\partial x'(p')}{\partial r_i} = \dfrac{\partial \bar{x}_i(p)}{\partial r_i} < 0$

(b) $\dfrac{\partial x'(p')}{\partial t_i} = \dfrac{\partial \bar{x}_i(p)}{\partial t_i} < 0$

(Proof) This proposition is obvious by Proposition 3 (b) and the assumption that x' is a normal good. ☒

Proposition 6 Suppose x_i, for every $i \in \langle 1, I \rangle$, is a normal good. Then, under Assumptions, 1, 2, and 3,

$$\text{(a)}\ \frac{\partial \bar{x}_i(p)}{\partial r_i} = \frac{1}{w}\frac{\partial \bar{x}_i(p)}{\partial t_i} < 0, \text{ if } i \text{ is only one element of } S(r, t).$$

$$\text{(b)}\ \frac{\partial \bar{x}_j(p)}{\partial r_i} = \frac{\partial \bar{x}_j(p)}{\partial t_i} = 0, \qquad \text{if } i \text{ and/or } j \notin S(r, t).$$

(Proof) Show first (a). If i is only one element of S (r, t), $\bar{x}_i(p) = x'(p')$, by Proposition 3. Hence, it can be said that (a) is nothing but an alternative statement of Proposition 5. The proof of (b) is to be worked out for three cases separately. If $i \in S$ and $j \notin S$, the assertion follows from the fact that the image of $\bar{x}_j$ at the neighborhood of p is zero. By the same token, if i and $j \notin S$, the assertion is valid. On the other hand, if $i \notin S$ but $j \in S$. the equality follows from the fact that $\bar{x}_j(p) = x'(p)$ and that $\partial h(\cdot)/\partial r_i = \partial h(\cdot)/\partial t_i = 0$. ☒

The trip demand function examined so far has been derived from the preference ordering problem of a consumer constructed under the assumption that value of travel time is constant. This demand function can yield a plausible result that the consumer's level of utility is indifferent from the availability of trip alternatives other than a chosen one, as one can infer from Proposition 1. This function, however, fails to explain a number of important aspects related to behavior of trip-makers, Firstly, the function cannot properly characterize the cross-elasticities among available trip alternative, as one can infer from Proposition 1, Secondly, it cannot accommodate the fact that the consumer usually utilizes more than one trip alternative in travelling from one point to another, as one can infer from Proposition 3 implies.

III. Travel Demand Function under Uncertainty for Value of Travel Time

The implicit price of travel time refers to the perceived cost for human resources consumed in making a trip. Therefore, this price can hardly be quantified in an objective manner. Moreover, perception of the consumer is affected by many uncertain factors, such as his physical condition. For this reason, it does not appear to be feasible to find a fixed price of travel time, which can be applicable to all the trips made by the consumer during a certain period.

A plausible approach to assess the choice of trip alternatives made by a consumer for a certain period could be to hypothesize that the implicit price is a random variable. Alternatively, it may be assumed that the consumer has his own subjective distribution of the price, which reflects his previous experience. This uncertain decision environment could be expressed as follows.

Assumption 4 The implicit price per unit travel time, w, has an integrable density function, g, such that $\int_W g(w)\,dw = 1$ (or $\int_{w'}^{w''} g(w)\,dw = 1$), defined on the δ-compact set W which refers to a closed interval $[w', w'']$.

Uner Assumptions 1, 2, and 4, the decision problem of a consumer can be expressed as a stochastic programming problem stated below:

$$\begin{aligned}&\text{maximize } E[u(\sum_i x_i(w), y(w))]\\ &\text{subject to } \sum_i (r_i + wt_i)\, x_i(w) + y(w) \leqslant m,\ x_i(w) \geqslant 0, \forall_i, y(w) \geqslant 0,\end{aligned} \tag{3}$$

for every $w \in W$, where E refers to the expectation operator. Here, it should be noted that the choice variable of x_i and y are to be expressed as the function of w [Refer to Rockafella (1978)].

Definition 3 For the decision problem in (3), the stochastic trip demand function $\hat{x}_j$, $j \in \langle 1, I \rangle$, is defined by

(a) $\hat{x}_j(p) = \{ E[x_j(w)] \mid E[u(\sum_i x_i(w), y(w))] - \hat{m}(p) \geqslant 0, \sum_i (r_i + wt_i)\, x_i(w) + y(w) \leqslant m,$
$x_i(w) \geqslant 0.\ \forall_i, y(w) \geqslant 0 \mid \forall\, w \in W \}$ where $p = (r_1, \ldots, r_I, t_1, \ldots t_I, 1, m)$, and

(b) $\hat{m}(p) = \sup \{ E[u(\sum_i x_i(w), y(w))] \mid \sum_i (r_i + wt_i)\, x_i(w) + y(w) \leqslant m, x_i(w) \geqslant 0, \forall_i, y(w) \geqslant 0 \mid$
$w \in W \}$

In the above definition, the function $\hat{m}(p)$ corresponds to the optimal value of the objective function for the problem in (3). This function, representing the expected value of the consumer's optimal utility level, is a Lebesgue-Stieltjes integral which can be expressed as:

$$\hat{m}(p) = \int_W u(\sum_i x_i(w), y(w))\, g(w)\, dw, \tag{4}$$

where $x_i(w)$ and $y(w)$ refer to the optimal value of x_i and y, respectively, for a given p. The functions $x_i(w), \forall_i$, and $y(w)$ should satisfy the budget constraint for each w, as indicated in Definition 3 (b).

Using $x_i(w)$ in (4), the value of the stochastic trip demand function $\hat{x}_i(p)$ can be estimated. More precisely, the function $\hat{x}_i(p)$ is a Lesbesgue-Stieltjes integral, whose integrand is $x_i(w)$ in (4), and can be expressed as:

$$\hat{x}_i(p) = \int_W x_i(w)\, g(w)\, dw \tag{5}$$

Therefore, the key to understand the structure of $\hat{x}_i$ is to derive the specific expression of $x_i(w)$ and $y(w)$ in (4). This question is discussed below.

Proposition 7* Let $\bar{\bar{x}}_i$, $i \in \langle 1, I \rangle$, be an arbtrary function satisfying the condition such that

(a) $\int_W (\bar{\bar{x}}_i(w) - \bar{x}_i(w))\, dw = 0$

whre $\bar{\bar{x}}_i$ is the demand function of Definition 1. Let $\bar{\bar{y}}$ be an arbitrary function such that

(b) $\int_W (\bar{\bar{y}}(w) - \bar{y}(w))\, dw = 0$

where $\bar{y}$ is the demand function of y, which can be defined in the same manner as $\bar{x}_i$. Then, under Assumptions 1, 2, and 4,

$$\text{(c)}\quad \hat{m}(p) = E[u(\sum_i \bar{\bar{x}}_i(p,w), \bar{\bar{y}}(w))]. \quad \cdots\cdots (6)$$

$$\text{(d)}\quad \hat{x}_i(p) = E[\bar{\bar{x}}_i(p, w)].$$

(Proof) By definition, (c) implies (d). On the other hand, the definition of $\bar{\bar{x}}_i$ and $\bar{\bar{y}}$ implies

$$E[u(\sum_i \bar{x}_i(p, w), \bar{y}(p, w))] = E[u(\sum_i \bar{\bar{x}}_i(p, w), \bar{\bar{y}}(p,w))].$$

Hence, the proof can be completed by showing that

$$\hat{m}(p) = E[u(\sum_i \bar{x}_i(p,w), \bar{y}(p,w))] \quad \cdots\cdots (7)$$

Prove first the integrability of $u(\sum_i \bar{x}_i(p,w), \bar{y}(p,w))$, by showing that it is continuous and bounded in $w \in W$ for any (r,t). The objective function u is continuous and bounded, by assumption. Therefore, it is sufficient to prove the integrability condition of $\bar{x}_i$ and $\bar{y}$, since the composite function of two integrable functions are also integrable. As shown in Proposition 2, $\sum_i \bar{x}_i$ is equal to x' which is a function of h, and x'. and h are continuous in h and w, respectively. Also, x' and h are bounded, by assumption. Hence, $\bar{x}_i$ is integrable. The integrability of $\bar{y}$ can be proved in the exactly same manner.

Show next the equality in (7). By definition, it follows that

$$\hat{m}(p) \geqslant E[u(\sum_i \bar{x}_i(p,w), \bar{y}(p,w))]. \quad \cdots\cdots (8)$$

On the other hand, by Proposition 2, $u(\Sigma\bar{x}_i(p,w), \bar{y}(p,w))$ is unique and satisfies the condition such that

$$u(\sum_i \bar{x}_i(p,w), \bar{y}(p, w)) \geqslant u(\sum_i x_i(w), y(w)) \quad \cdots\cdots (9)$$

for any $w \in W$, where x_i and y are arbitrary integrable functions which satisfy the constraint of the consumer's decision problem in (3). Hence, it follows that

$$E[u(\sum_i \bar{x}_i(p, w), \bar{y}(p, w))] \geqslant \hat{m}(p) \quad \cdots\cdots (10)$$

From (8) and (10), the equality (c) follows. ⊠

Proposition 7 implies that $\hat{x}_i(p)$ can be expressed differently as:

$$\hat{x}_i(p) = \int_W \bar{x}_i(p, w)\, g(w)\, dw \quad \cdots\cdots (11)$$

The structure of $\hat{x}_i(p)$ in the above equation can be explained easily with the geometry of $\bar{x}_i(p,w)$ examined in the previous section. To facilitate the forthcoming discussion about this issue, a number of terms need to be introduced.

Definition 4. The effective domain of trip alternative $i \in < 1, I >$, denoted by W_i, is defined by

$$W_i(r, t) = [\, w \in W \mid r_i + wt_i \leqslant h(r, t)$$

Definition 5 Let μ be a positive measure defined on W. Then, for $i \in <1, I>$,

(a) Alternative i is relevant, if $\mu\, [W_i(r, t)\,] > 0.$

(b) Alternative i is irrelevant, if $\mu[W_i\,(r, t)\,] = 0.$

The definition of terms can easily be understood by examining Figure 3, which depicts the real price of available alternatives and the probability denisty function of g(w) for varying w values. In the figure, the effective domains of $i-1, i, i+1$ have non-empty interiors. Hence, they are releveant alternatives whose probabilities to be chosen by the consumer are $\int_{W_k(\cdot)} g(w)\, dw$, $k = i\text{-}1, i, i\text{+}1$. On the other hand, the effective domains of j and j+1, as depicted in the figure, are characterized by an empty set and an empty interior, respectively. Therefore, they are irrelevarnt alternatives which have zero probabilities.

Figure 3

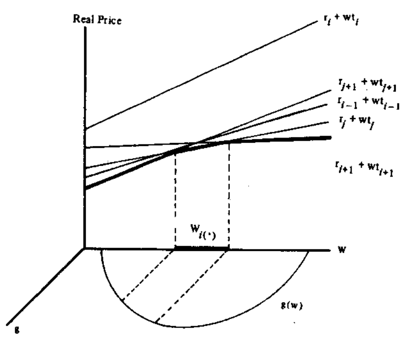
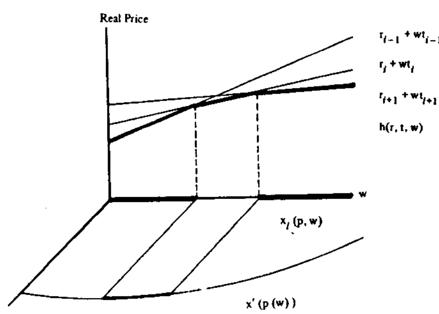


Figure 4



*In this section, the functions x_i, x', and h were defined by taking w as a constant. In this section, the notations of x_i, x', and h refer to the functions which are defined in the same manner with the respective expressions of the previous section, except with a difference that w is taken as a random variable.

Another diagram of Figure 4 illustrates a possible configuration of $\bar{x}_i$ (p,w), by using its relationship to x$'$ (p$'$(w)), where $p'(w)=(h(r, t, w), 1, m)$ and $h(r, t, w)= \inf_k (r_k+wt_k)$. The bold curve in the upper part of the figure shows the geometry of h (r, t, w) for a given (r, t). This curve illustrates that

$$h(r, t, w) = r_i + wt_i, \text{if } w \in W_i(r, t), \qquad (12)$$

$$h(r, t, w) \neq r_i + wt_i, \text{if } w \notin W_i(r, t), \qquad (13)$$

On the other hand, the bold line in the lower part schematically represents the geometry of $\bar{x}_i$(p,w). This curve is developed by identifying the relationship between $\bar{x}_i$(p,w) and $x'_i(p'(w))$ such that

$$x'(p'(w)) = \bar{x}_i(p, w) > 0, \text{if } w \in \text{interior of } W_i(r, t), \qquad (14)$$

$$x'(p'(w)) \neq \bar{x}_i(p,w) = 0, \text{if } w \notin W_i(r, t), \qquad (15)$$

where (14) and (15) are alternative expressions of Proposition 3 (b), and Porposition 1 (b), respectively. Finally, it should be noted that the representation that $x'(p'(w'))$ is downward with respect to w reflects the assumption that x$'$ is a normal good, since h(r, t, w), a variable of x$'$, is an inceasing function with respect to w.

Proposition 8 Suppose $(r_i, t_i) \neq (r_j, t_j)$, for every $i, j \in \langle 1, I \rangle$, $i \neq j$. Then, under Assumptions 1, 2, and 4,

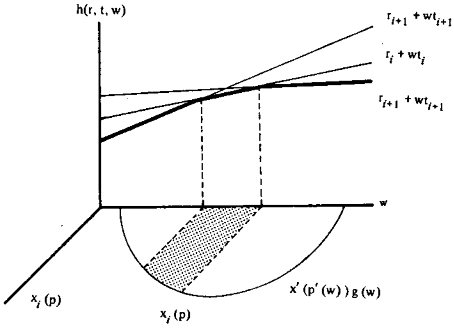
$$\hat{x}_i(p) = \int_{W(\cdot)} x'(p'(w))\, g(w)\, dw,$$

where $W_i(\cdot)$stands for $W_i(r, t)$.

(Remark) This is obious by the above discussion. ☒

Using the above proposition, the value of $\hat{x}_i(p)$ is schematically represented in Figure 5, as a shaded area. The integrand of $\hat{x}_i(p)$ correpsonds to the multiple of $x'(p(w))$ depicted in Figure 4, and g(w) illustrated in Figure 3.

Figure 5



Ⅳ. Comparative Static of Stochastic Trip Demand Functions

This section presents the result of analyses on the continuity and differentiability of the trip demand function $\hat{x}_i$ derived from the stochastic programming problem of a consumer. As to be shown later, $\hat{x}_i$ is continuous and differentiable in r and t, except at a set of points such that $W_i(r, t)$ is not an empty set and $(r_i, t_i) = (r_j, t_j)$. This exception corresponds to the situation that two relevant alternatives of i and j have the identical service characteristics.

The continuity of $\hat{x}_i$ can be characterized by using the formula in Proposion 7, where the integrand is $\bar{\bar{x}}_i$. On the other hand, the differentiability of $\hat{x}_i$ can only be characterized by applying the equation in Proposition 8, where the integrand is x' or $\bar{x}_i$. This difference implies that, unlike the former, the latter can only be characterized by applying a restrictive definition of $\hat{x}_i$ which can be obtained from the alternative expression of Defintion 3, such that the suprimum in (b) of that definition is replaced by the maximum. For this alternative definition, the integrand x' is absolutely continuous on W, by Proposition 2. This absolute continuity of x', is a necessary condition for the diffentiability. However, the integrand $\bar{\bar{x}}_i$ is an arbitary element belonging to the collection of functions, such that $E[\bar{\bar{x}}_i(p, w) - x_i(p, w)] = 0$, and this function is generally not absolutely continuous on W.

Proposition 9. Under Assumptions 1, 2, and 4, for every $i \in \langle 1, I \rangle$,

(a) $\hat{x}_i$ of Definition 3 is upper semi-continuous in r and t.

(b) $\hat{x}_i$ of Definition 3 is continuous at every (r, t), except at the point such that Wi(r, t) is not empty and $(r_i, t_i) = (r_j, t_j), i \neq j$.

(Proof) The proof of (b) is presented Appendix. If (b) is valid, the proof of (a) can be completed by showing that the image of $\hat{x}_i$ at (r, t), where it is not continuous, belongs to a compact set. This fact is illustrated with an example below.

By trite calculation, it can be shown that the set $W_i(r, t)$, depicted in Figure 5, is computed by using the following:

$$W_i(r, t) = [\theta_i'(r, t), \theta_i''(r, t)] \quad \cdots\cdots (16)$$

$$= \left[\frac{r_{i-1} - r_i}{t_i - t_{t-1}} , \frac{r_i - r_{i+1}}{t_{i+1} - t_i} \right]$$

where θ_i and θ'' are functions estimatiing the lower and upper bounds of W_i, respectively.

Using the above equation, $\hat{x}_i(p)$ can alternatively be specified as indefinite integral such that

$$\hat{x}_i(p) = \int_{\theta_i'(\cdot)}^{\theta_i''(\cdot)} x'(p'(w))\, g(w)\, dw \quad \cdots\cdots (17)$$

From the above epxression, it can be inferred that, if $\acute{\theta}_i(r, t)$ or $\theta_i''(r, t)$ is degenerate, $\hat{x}_i(p)$ cannot be defined.

For example, if $(r_i, t_i) = (r_{i+1}, t_{i+1})$, then, $\theta_i'(r, t)$ and $\theta_i''(r, t)$ are indefinite. Then, the indefinite integral $\hat{x}i(p)$ cannot be estimated. However, from Proposition 3, one can deduce that

$$\hat{x}_i(p) + \hat{x}_{i+1}(p) = \int_{\theta''_i(\cdot)}^{\theta''_i(\cdot)} x'(p'(w))\, g(w)\, dw \quad \cdots\cdots (18)$$

Hence, it can be concluded that $\hat{x}i(p)$ belongs to a compact set, whose lower and upper bounds are zero and $\hat{x}_i(p) + \hat{x}_{i+1}(p)$, respectively. ⊠

Proposition 10 Let x_i, $i \in \langle 1, D \rangle$, be expressed by

(a) $\hat{x}_i(p) = \int_{\theta'_i(\cdot)}^{\theta''_i(\cdot)} x'(p'(w))\, g(w)\, dw,$

as introduced in Proposition 8. Suppose also that $\hat{x}_i$ is continuous at a certain (r, t), then

(b) $\hat{x}_i(p)$ is differentiable at that (r, t)

(c) $$\frac{\partial \hat{x}_j(p)}{\partial r_i} = \int_{\theta'_i(\cdot)}^{\theta''_i(\cdot)} \frac{\partial x'(\cdot)}{\partial r_i} g(\cdot)\, dw + x'(\cdot)\, g(\cdot) \frac{\partial \theta''(\cdot)}{\partial r_i} \Bigg|_{w = \theta''_j(\cdot)} - x'(\cdot)\, g(\cdot) \frac{\partial \theta'(\cdot)}{\partial r_i} \Bigg|_{w = \theta'_j(\cdot)}$$

(d) $$\frac{\partial \hat{x}_j(p)}{\partial t_i} = \int_{\theta'_i(\cdot)}^{\theta''_i(\cdot)} \frac{\partial x'(\cdot)}{\partial r_i} g(\cdot)\, dw + x'(\cdot)\, g(\cdot) \frac{\partial \theta''_i(\cdot)}{\partial t_i} \Bigg|_{w = \theta''_i(\cdot)} - x'(\cdot)\, g(\cdot) \frac{\partial \theta'_j(\cdot)}{\partial t_i} \Bigg|_{w = \theta'_j(\cdot)}$$

(Remark) The proof is listed is Appendix. ⊠

Proposition 11 (Continuation of Proposition 10) Suppose further that x_i, $i \in \langle 1, D \rangle$, is a normal good. Then,

(a) $\dfrac{\partial \hat{x}_i(p)}{\partial r_i} \leqslant 0, \quad \dfrac{\partial \hat{x}_i(p)}{\partial t_i} \leqslant 0.$

(b) $\dfrac{\partial \hat{x}_j(p)}{\partial r_i} \geqslant 0, \quad \dfrac{\partial \hat{x}_j(p)}{\partial t_i} \geqslant 0, \text{ if } i \neq j.$

(c) $\left|\dfrac{\partial \hat{x}_i(p)}{\partial r_i}\right| \geqslant \sum_{j \neq i} \dfrac{\partial \hat{x}_j(p)}{\partial r_i}, \quad \left|\dfrac{\partial \hat{x}_i(p)}{\partial t_i}\right| \geqslant \sum_{j \neq i} \dfrac{\partial \hat{x}_j(p)}{\partial t_i}$

(Remark) The signs shown above can be verified by using (c) and (d) of Proposition 10, More precisely, it can be proved by checking the three terms in the left side of the two equations. The sign of the first terms can be evaluated by examining the integrand of the terms, which can readily be determined with use of Proposition 5. The second and third terms can be identified by using the specifiic expressions of θ''_i (r, t) and θ'_j in (16), respectively. The evaluation of the sign of those terms is straightforward, but complex. Hence, the detailed computation is omitted. Instead, an informal proof of those signs are presented below. ☒

A set of inequalites in (a) and (b) of Proposition 11 indicates that a set of avaiable trip alternatives are gross substitutes. The two inequalities in (c) imply that the trip alternatives are diagnoal dominances each other. Using Figures 6 and 7, those properties are verified below.

Figure 6

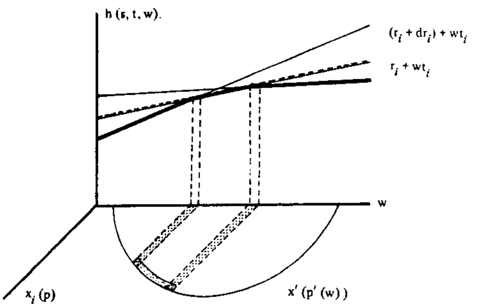
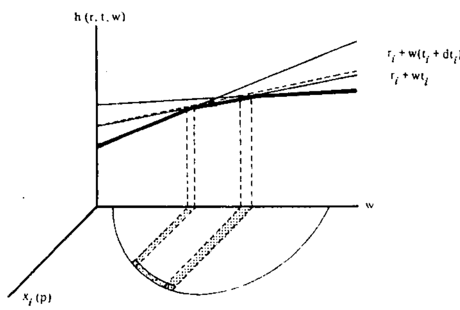


Figure 7



Suppose the service charge of a relevant alternative i is changed from r_i to r_i+dr_i. This change increases the image of h by dr_i at the effective domain of i, $W_i(r, t)$, as illustrated in Figure 6. This increase, in turn, decreases the image of x', the integrand of x_i' at $W_i(r, t)$, since x_i is a normal good. The decrease in the image of x', in turn, would result a decrease in the value of x_i, by the dotted area shown in the figure. On the other hand, the change of r_i also causes a chang in the inage of W_i. In other words, the domain of the integral is shrinked from $W_i(r, t)$ to a smaller set, as illustrated in the figure. The shrink of the effective domain, in turn, results in an decrease in the value of x_i by the two shaded areas shown in the figure.

In summary, an increase in r_i would result a decrease of the demand for i, which is equal to the sum of the dotted and shaded areas. This implies the inequality in (a) of the proposition. On the other hand, the shaded areas correspond to the modal shift of the trip demand among the competing alternatives. This modal shift constitutes the inequality in (b). Finally, it should be noted that this change in r_i results in the decrease in the total number of trips, which is equal to the dotted area, and which consequences the inequality in (c).

Figure 7, which shematically illustrates the impact of an increase in travel time, can be interpreted in the exactly same manner as Figure 6. Therefore, the explanation will be omitted.

V. Concluding Remarks

A new approach is introduced to assess the trip-making behavior of a consumer to whom multiple trip alternatives are available. The essence of this approach is to analyze the decision of a trip-maker for a certain period under the assumption that the implicit price of travel time perceived by him is a random variable. To accommodate such an uncertain decision environment, this paper formulates the consumer's decision model as a stochastic programming problem.

The advantage of the approach can be ascribed to a number of findings obtained through the analysis of the stochastic trip demand function, which refers to the demand functions derived from the decision model mentioned above. Firstly, the demand function can characterize the possibility that the consumer may utilize more than one trip alternative for his trip. Secondly the demand function for a certain trip alternative is sensitive to the service charge and travel time of the competing alternatives as well as the corresponding one. Thirdly, the function has the property of a gross substitute and diagonal dominance among the available alternatives.

REFERENCE

1. Arrow, K.J. and F.H. Hahn, "General Competitive Analysis", Holden-Day Inc., 1971.
2. Barnett, W.A., "Pollak and Watcher on the Household Production Function Approach Comment", Journal of Political Economy, 1977.
3. Becker, G.S., "A Theory of Allocation of Time", the Economic Journal, 1965.
4. Berge, C., "Topological Spaces: Multi-valued Functions, Vector Spaces, and Convexity", translated by E.M. Patterson, the McMillan Co., 1963.
5. Daughety, A.F. and F.S. Inaha, "Estimation of Service-differentiated Transport Demand Functions", TRR 668, 1978.
6. Debreu, G., "Theory of Value: An Aximatic Analysis of Economic Equilibrium", Yale University Press, 1959.
7. Domencichi, T.A. and D. McFadden, "Urban Travel Demand: A Behaviroal Analysis", North-Holland Publishing Co., 1975.
8. McFadden, D., "Conditional Logit Analysis of Qualitative Choice Behavior", In Frontiers in Econometrics, edited by P. Zarembka, Academic Press, 1973.
9. Moses, L. N. and H.F. Williamson, "Value of Time, Choice of Mode, and Subsidy Issue in Urban Transportation", Journal of Political Economy, 1968.
10. Muth, R.F., "Household Production and Consumer Demand Function", Econometrica, 1966.
11. Pollak, R.A. and M.L. Watcher, "The Relevance of the Household Production Function and its Implications for the Allocation of Time, "Journal of Political Economy, 1975.
12. Quant, R.E. and W.J. Baumol, "The Demand for Abstract Transport Modes: Theory and Measurement", Journal of Regional Scrience, 1966.
13. Quant, R.E. and K.H. Young, "Cross-Sectional Travel Demand Models: Estimates and Tests", Journal of Regional Science, 1969.
14. Rochafellar, R.T., "Convex Analysis", Princeton University Press, 1970.
15. Rockafellar, R.T. and J.B. Wets, "The Optimal Resource Problem in Discrete Time: L^1-Multipliers for Inequality Constraints", SIAM Journal of Controland Optimization, 1978.
16. Rudin, W., "Real and Complex Analysis", McGraw-Hill, Inc., 1974.
17. Willig, R.D., "Incremental Consumer's Surplus and Hedonic Price Adjustment, "Journal of Economic Theory, 1978.

APPENDIX

A.1 Proof of Proposition 9 (b)

The proof can be completed by showing that $f: R^n \rightarrow R$ defined below is Lipschitzian in $t \in R^n$,

$$f(t) = \int_{\theta'(t)}^{\theta''(t)} x(t, w)\, g(w)\, dw \cdots\cdots (A.1)$$

where f, x, t stands for $\hat{x}_i$, $\bar{\bar{x}}_i$, (r, t), respectively. (For simplicity of discussion, the proof is worked out by using (A. 1))

Since, θ' (t) and θ''(t) are continuous at the point t, where the continuity of f is evaluated, it is possible to pick the positive numbers α_1 and α_2, such that

$$\| \theta'(t+s) - \theta'(t) \| \leqslant \alpha_1 \| S \|, \cdots\cdots (A.2)$$
$$\| \theta''(t+s) - \theta''(t) \| \leqslant \alpha_2 \| S \|.$$

where θ' and θ'' are continuous on [t, t+s]. On the other hand, since x and g are Lebesgue integrable, there are the continuous functions y and z such that

$$\int [\, x(t, w) - y(t, w)\,]\, dw = 0,$$
$$\int [\, g(w) - z(w)\,]\, dw = 0, \cdots\cdots (A.3)$$

for every t. Then, it is possible to pick β_1 and β_2, such that

$$\max \{\, y(u, \theta'(u))\, z(\theta'(u)) \mid t \leqslant u \leqslant t+s \,\} \leqslant \beta_1$$
$$\max \{\, y(u, \theta''(u))\, z(\theta''(u)) \mid t \leqslant u \leqslant t+s \,\} \leqslant \beta_2 \cdots\cdots (A.4)$$

Furthermore, it is possible to pick the integrable function h such that

$$\| [\, x(t+s, w) - x(t)\,]\, g(w) \| \leqslant h(w) \| S \|, \cdots\cdots (A.5)$$

for every s. Thus, it can be said that

$$\| f(t+s) - f(t) \| \leqslant \alpha_1 \beta_1 + \alpha_1 \beta_2 + \| S \int_{\theta'(t)}^{\theta''(t)} h(w)\, dw \| \cdots\cdots (A.6)$$

On the other hand, since h is integrable, there is a positive number λ such that

$$\lambda = \int_{\theta'(t)}^{\theta''(t)} h(w)\, dw \cdots\cdots (A.7)$$

Let $a = \alpha_1\beta_1 + \alpha_2\beta_2 + \lambda$. Then

$$\| f(t+s) \| \leqslant a \| S \| . \quad \text{(A. 8)}$$

Therefore, f is Lipschitizian, as claimed.

A. 2 Proof of Proposition 10

The proof will be worked in three steps. The first step will be to evaluate the differential of h defined as

$$h(r, t) = \int_{w'}^{w''} x(r + wt)\, g(w)\, dw, \quad \text{(A. 9)}$$

where x is absolutely continuous in r, t, w∈R (r and t are not vectors), and w′ and w″ are positive. The second step will be to estimate the differential of f difined by

$$f(r, t) = \int_{\theta'(r,t)}^{\theta''(r,t)} x(r + wt)\, g(w)\, dw \quad \text{(A. 10)}$$

where the structure of the integrand is identical with the of h, and θ' and θ'' are continuously differentiable in r and t. The final step will be to prove the proposition by using the analysis result of the previous steps.

(a) Evaluate the partial derivative $\partial h(\cdot)/\partial r$ (and $\partial h(\cdot)/\partial t$). Since x is $\partial h(\cdot)/\partial r$ is integrable, and

$$x(r + wt) - x(a + wt) = \int_a^r \frac{\partial x(r + wt)}{\partial r}\, dr, \quad \text{(A. 11)}$$

for any a in the neighborhood of h. And, since g(w) is integrable in [w′, w″], it follows that

$$h(r, t) = \int_a^r dr \left[\int_{w'}^{w''} \frac{\partial x(r + wt)}{\partial r}\, g(w)\, dw \right] + \int_{w''}^{w''} x(a + wt)\, g(w)\, dw \quad \text{(A. 12)}$$

Differentiating the both sides of the above equation, it follows that

$$\frac{\partial h(r, t)}{\partial r} = \int_{w'}^{w''} \frac{\partial x(r+wt)}{\partial r}\, g(w)\, dw \quad \text{(A. 13)}$$

Finally, it should be noted that $\partial h(')/\partial t$ can be evaluated in the same manner.

(b) Evaluate the partial derivative $\partial f(')/\partial r$ (and $\partial f(')/\partial t$). Set

$$G(r, t, u, v) = \int_v^u x(r + wt)\, g(w)\, dw$$

By the chain rule, it follows

$$\frac{\partial f(r,t)}{\partial r} = \frac{\partial G(\cdot)}{\partial r} + \frac{\partial G(\cdot)}{\partial u}\frac{\partial u}{\partial r} + \frac{\partial G(\cdot)}{\partial v}\frac{\partial u}{\partial r} \qquad \text{(A. 14)}$$

The first term of the left side can be estimated by using (A. 13). The second and third terms can be evaluated by applying the relationship in (A. 11). Then,

$$\frac{\partial f(r,t)}{\partial r} = \int_{\theta'(\cdot)}^{\theta''(\cdot)} \frac{\partial x(\cdot)}{\partial r} dw + x(\cdot)\frac{\partial \theta''(\cdot)}{\partial r}\Bigg|_{w=\theta''(\cdot)} - x(\cdot)\frac{\theta'(\cdot)}{\partial r}\Bigg|_{w=\theta'(\cdot)} \qquad \text{(A. 15)}$$

C. Prove Proposition 10 by using (A. 16). To this end, it suffices to introduce the alternative expression of $x_i(p)$. That is

$$x_i(p) = \int_{\theta'(\cdot)}^{\theta''(\cdot)} x'(r, + wt_i)\, f(w)\, dw, \quad \text{if } i \text{ is relevant}, \quad \text{if } i \text{ is irrelevant}, \qquad \text{(A. 16)}$$

since $p'(w) = \inf_k (r_k + wt_k)$.