<Research Paper>

Preventive Maintenance Policies for a System with Two Types of Units Subject to Deterioration

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Abstract

This paper considers prevertive maintenance policies for a system with two types of units which is subject to deterioration. Two generalized models are investigated ; a preventive maintenance policy based on the cumulative operating time and a policy based on the number of minimal repairs performed. Optimal preventive maintenance policies which minimize the expected average cost per unit time including the earning loss due to the deterioration are discussed and some numerical examples are given .

I. Introduction

Consider a machine in an industrial plant which is expected to maintain a high rate of return on the investment. In general the earning rate of a machine is high in the beginning and decreases gradually due to the deterioration of the machine. And unpredictable failure of an operating machine is costly and/or dangerous. Hence, it is important to determine when to perform preventive maintenances.

Barlow and Hunter [1] discuss two age replacement policies ; one is most useful in maintaining simple equipments and the other for a large and complex system. Scheaffer [6], and Cleroux and Hanscom [3] generalize Barlow - Hunter's models to include the cases where the cost of keeping an individual unit in operation increases with the age of the unit. Beichelt and Fischer [2] propose a model with two types of failures which includes Barlow - Hunter 's models as special cases. Park [5] proposes a preventive maintenance policy based on the number of minimal repairs performed, which is extended by Nakagawa [4] to a system with two types of failures and a system with two types of units.

This paper develops a model for a system with two types of units which is subject to deterioration; when type 1 unit fails the system undergoes minimal repair, and when type 2 unit fails the system must be replaced. The decision variables considered are the cumulative operating time and the number of minimal repairs performed. The optimal policies which minimize the expected average costs per unit time including the earning loss due to the declining efficiency of the system are discussed and some numerical examples are given.

II. Basic Assumptions

The following 7 assumptions are made for

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the modeling.

1. The earning rate of a unit decreases with age.
2. The planning horizon is infinite.
3. All failure events are independent.
4. All maintenance actions take only negligible time.
5. In case of preventive maintenance or replacement a unit is renewed.
6. Failure rate is not disturbed by minimal repairs.
7. The hazard rate of each unit is nondecreasing.

Ⅱ. Preventive Maintenance Policy Based on the Cumulative Operating Time

In this section, a preventive maintenance policy based on the cumulative operating time is developed. Let T be the prescribed preventive maintenance time. Maintenance of the system is performed as follows.

When the system reaches age T without any type 2 unit failures, preventive maintenance is performed and if a type 2 unit failure occurs prior to T, the system is replaced on failure. Whenever type 1 unit fails, minimal repair is performed.

Let $W(t)$, $L(t)$, $N_2(t)$ and $N_3(t)$ be the total minimal repair cost, the total earning loss, the number of type 2 unit failures and the number of preventive maintenances performed up to time t, respectively. The total cost at time t is given by

$$C(t)=W(t)+c_2N_2(t)+c_3N_3(t)+L(t) \quad (1)$$

where c_2 and c_3, $c_3 \leq c_2$, are the replacement cost and the preventive maintenance cost, respectively.

Let Y be the random variable denoting the time to the first type 2 unit failure and U be the random variable denoting min (Y, T). Then it is obvious that

$$\lim_{t\to\infty} E(W(t))/t=E(c_1N_1(U))/E(U),$$

where c_1 is the minimal repair cost and $N_1(U)$ is the number of type 1 unit failures during U. We also have

$$\lim_{t\to\infty} E(L(t))/t=\int_0^T q(t)(1-G(t))\,dt\,/E(U),$$

where $q(t)$ is the earning loss rate of the system at age t which is continuous and nondecreasing and $G(\cdot)$ is the cumulative distribution function of Y. It can also be shown that

$$\lim_{t\to\infty} E(N_2(t))/t=G(T)/E(U)$$

and

$$\lim_{t\to\infty} E(N_3(t))/t=(1-G(T))/E(U).$$

Since $E(N_1(U))=\int_0^T r_1(t)(1-G(t))\,dt$ and $E(U)=\int_0^T(1-G(t))\,dt$, the expected average cost per unit time is

$$\begin{aligned}A(T)&=\lim_{t\to\infty} E(C(t))/t\\&=[(c_2-c_3)\int_0^T G(t)\,dt+c_3+c_1\int_0^T r_1(t)(1-G(t))\,dt+\int_0^T q(t)(1-G(t))\,dt]/\int_0^T(1-G(t))\,dt, \quad (2)\end{aligned}$$

where $r_1(t)$ is the nondecreasing hazard rate of type 1 unit. A(T) is a continuous function of T, $\lim_{T\to 0} A(T)=\infty$ and

$$\begin{aligned}A(\infty)&=\lim_{T\to\infty} A(T)\\&=[c_2+c_1E(N_1(Y))+E(L')]/E(Y),\end{aligned}$$

where L' is the total earning loss during Y and A(∞) is the expected average cost when preventive maintenances are not performed.

From (2),

$$dA(T)/dT=[1-G(T)][\int_0^T B(t)\int_t^T(1-G(s))\,ds\,dt-c_3]/[\int_0^T(1-G(t))\,dt]^2, \quad (3)$$

where $B(t)=d[c_1r_1(t)+(c_2-c_3)r_2(t)+q(t)]/dt$ and $r_2(t)$ is the nondecreasing hazard rate of type 2 unit. If there exists a solution T* satisfying

$$\int_{0}^{T^*} B(t) \int_{0}^{t} (1-G(s))\, ds\,dt = c_3,$$

it is the unique optimal solution since $B(t)$ is a nondecreasing function. Otherwise, the optimal solution is $T^* = \infty$, i. e., no preventive maintenance is to be performed.

Notice that the results obtained are identical with those by Beichelt and Fischer [2] if we let $q(t)$ be zero for all t.

Example

Maintenance problem of an electronic furnace is considered. A vital part of the furnace has a life time distribution $g(t) = t^4 e^{-t}/4!$. The life distribution of other components of which failures can be corrected by minimal repairs is $f(t) = e^{-t}$. The earning loss rate reflecting the loss in efficiency is given by $q(t) = 400(1-e^{-t/10})$. If $c_1 = \$10$, $c_2 = \$300$ and $c_3 = \$100$, we have $T^* = 2.0$ and $A(T^*) = \$103.3$. If the earning loss is ignored, the optimal policy is $T^{**} = 3.3$ and $A(T^{**}) = \$114.7$.

Ⅳ. Preventive Maintenance Policy Based on the Number of Minimal Repairs Performed

In this section, a preventive maintenance policy based on the number of minimal repairs is discussed. Let n be the number of type 1 unit failures when preventive maintenance is to be performed. Maintenance of the system is carried out as follows.

When the system reaches n^{th} type 1 unit failure without any intervening type 2 unit failures, preventive maintenance is made. And if type 2 unit fails prior to the n^{th} type 1 unit failure, the system is replaced on failure. For all other type 1 unit failures, minimal repairs are performed.

Let T_n be the random variable denoting the time to the n^{th} type 1 unit failure without any intervening type 2 unit failures and Z be the random variable denoting $\min(Y, T_n)$.

Since the hazard rate of the system is not disturbed by minimal repairs,

$$p_j(t) = P(N_1(t) = j) = (R_1(t))^j \exp(-R_1(t))/j!,$$

where $R_1(t) = \int_0^t r_1(s)\, ds$.

Let L be the total earning loss and $N_1(Z)$ be the number of type 1 unit failures during Z. Then

$$E(L) = \sum_{j=0}^{n-1} \int_0^\infty q(t)\,(1-G(t))\, p_j(t)\,dt,$$

$$E(Z) = \sum_{j=0}^{n-1} \int_0^\infty (1-G(t))\, p_j(t)\,dt,$$

and

$$E(N_1(Z)) = \sum_{j=0}^{n-1} \int_0^\infty r_1(t)\,(1-G(t))\, p_j(t)\,dt.$$

Hence the expected average cost per unit time is given by

$$A(n) = \Big[c_3 + (c_2 - c_3)\int_0^\infty G(t)\, p_{n-1}(t)\, r_1(t)\,dt + c_1 \sum_{j=0}^{n-2} \int_0^\infty r_1(t)\,(1-G(t))\,p_j(t)\,dt + \sum_{j=0}^{n-1} \int_0^\infty q(t)\,(1-G(t))\,p_j(t)\,dt\Big] \Big/ \sum_{j=0}^{n-1} \int_0^\infty (1-G(t)\, p_j(t)\,dt. \quad (4)$$

The optimal policy n^* which minimizes $A(n)$ cannot be obtained in a closed from. However, some numerical studies suggest that there is indeed a unique optimal solution. And if we let $q(t)$ be zero for all t, the optimal solution is the same as that suggested by Nakagawa [4].

Example

A small heating system in a factory is made up of a major nonrepairable subsystem and some minor components. If the major subsystem fails, the system must be replaced However the failures of minor components can be corrected by minimal repairs. The life distribution of the major subsystem is

$G(t)=1-\exp(-t^2/25)$ and that of minor components is $F(t)=1-\exp(-t^2)$. The earning loss rate of the system is $q(t)=100(1-\exp(-t^2/9))$. If $c_1=\$10$, $c_2=\$300$ and $c_3=\$220$, we have $n^*=9$ and $A=(n^*)=\$241.76$. Moreover, it is found that n^* is the unique optimal solution. If the earning loss is ignored, the optimal policy is $n^{**}=18$ and $A(n^{**})=\$247.34$.

V. Remarks

In preventive maintenance preblems with minimal repairs, one can also consider a system with two types of failures instead of two types of units. However, optimal policies for this case can be obtained by the methods presented in this paper.

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