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Analysis of a 2-Unit Standby Redundant System of Reparable 3-State Devices

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ABSTRACT

A device is said to have three states if it has one good state and two mutually exclusive failure modes ; e. g., in one failure mode, it operates when it should not, in the other it doesn't operate when it should. Some examples of such devices include a fluid flow valve, an automatic machine, and an explosive. A Markov model is developed to obtain the availability Function of a 2-unit standby redundant system of such devices

I. INTRODUCTION

The instantaneous and steady-state availabilities of 2-state devices have been widely studied in textbooks (3, 4). Multistate components can assume three or more sta states. An automatic machine used on an assembly line represents a good exampie of 3-state system ; it can either operate satisfactorily in its normal mode or it can fail. Failure can occur either by carrying out some unwanted operations on the assembly line items or by ceasing to operate. Literature review shows that little work has been carried out on multistate devices-specially 3-state devices (1,2). The purpose of this paper is to extend the work of (2) for standby redundant system to develop the availability function of such a standby system.

II. THE RELIABILTY MODEL

To formulate the Markov model, the following assumptions were made ;

1. The states are 0-good; 1-failed in mode 1; 2-failed in mode 2. Transitions are possible between states 0 and 1 or 0 and 2, but not between 1 and 2.
2. The Markov transition rates are constant. λ_1 is failure rate for mode 1 and λ_2 for mode 2, but repair rate is the same μ for both failure modes.
3. Both units (active and standby) are good at time 0.
4. Both units are identical and standby unit never fails during the waiting period.
5. The probability of simultaneous events in an infinitesimal interval of time is 0.
6. When active unit fails, the unit undergo service immediately and the standby unit takes over the operations.
7. There exists only 1 repair facility.

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The transition diagram is shown in Figure 1.

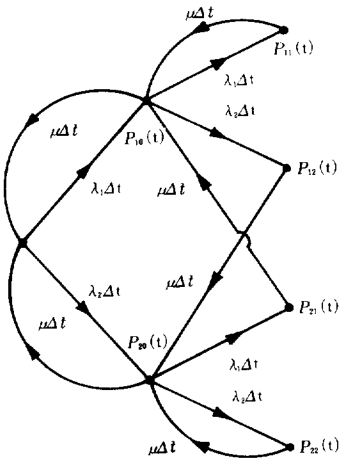

Figure1. A standby redundant system of reparable 3-state devices

Ⅲ. ANALYSIS OF THE MODEL

Notation

t time

i s-independent failure mode, $i=1,2$

$P_{jk}(t)$ probability that active unit is in state j and standby unit is in state k at time t $(j=0,1,2 \quad k=0,1,2)$

λ_i constant failure rate to failure mode i

μ constant repair rate for both failure modes

$\boldsymbol{P}^T$ transpose of $\boldsymbol{P}$

$\boldsymbol{P}^*(s)$ Laplace transform of $\boldsymbol{P}(t)$

The differential probability equations for the model can be written in the form.

$$\dot{\boldsymbol{P}}(t)=\boldsymbol{A}\cdot\boldsymbol{P}(t) \cdots\cdots (1)$$

where $\dot{\boldsymbol{P}}(t)=[P_{00}(t),\ P_{10}(t),\ P_{20}(t),\ P_{11}(t),\ P_{12}(t),\ P_{21}(t),\ P_{22}(t)]$

$\boldsymbol{A}\equiv$

$-(\lambda_1+\lambda_2)$	μ	μ				
λ_1	$-(\lambda_1+\lambda_2+\mu)$		μ		μ	
λ_2		$-(\lambda_1+\lambda_2+\mu)$		μ		μ
	λ_1		$-\mu$			
	λ_2			$-\mu$		
		λ_1			$-\mu$	
		λ_2				$-\mu$

Now, suppose we have initial conditions $P_{ij}(0)=0$ except for $i=j=0$ and $P_{00}(0)=1$, then taking Laplace transform of (1), we have the following system :

$$\boldsymbol{B}=\boldsymbol{A}_S\boldsymbol{P}^*(s) \cdots\cdots (2)$$

where $\boldsymbol{B}^T=(1,0,0,0,0,0,0)$

$\boldsymbol{A}_S\equiv$

$(s+\lambda_1+\lambda_2)$	$-\mu$	$-\mu$				
λ_1	$-(s+\lambda_1+\lambda_2+\mu)$		μ		μ	
λ_2		$-(s+\lambda_1+\lambda_2+\mu)$		μ		μ
	λ_1		$-(s+\mu)$			
	λ_2			$-(s+\mu)$		
		λ_1			$-(s+\mu)$	
		λ_2				$-(s+\mu)$

Solving the system (2), the followings were obtained:

$$P^*_{00}(s)=\frac{s^2+(\lambda_1+\lambda_2+2\mu)\,s+\mu^2}{s(s-s_1)\,(s-s_2)} \quad \cdots\cdots \quad (3\cdot 1)$$

$$P^*_{10}(s)=\frac{\lambda_1 s+\lambda_1\mu}{s(s-s_1)\,(s-s_2)} \quad \cdots\cdots\cdots\cdots \quad (3\cdot 2)$$

$$P^*_{20}(s)=\frac{\lambda_2 s+\lambda_2\mu}{s(s-s_1)\,(s-s_2)} \quad \cdots\cdots\cdots\cdots \quad (3\cdot 3)$$

$$P^*_{11}(s)=\frac{\lambda_1^2}{s(s-s_1)\,(s-s_2)} \quad \cdots\cdots\cdots\cdots \quad (3\cdot 4)$$

$$P^*_{12}(s)=\frac{\lambda_1\,\lambda_2}{s(s-s_1)\,(s-s_2)} \quad \cdots\cdots\cdots\cdots \quad (3\cdot 5)$$

$$P^*_{21}(s)=P^*_{12}(s) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3\cdot 6)$$

$$P^*_{22}(s)=\frac{\lambda_2^2}{s(s-s_1)\,(s-s_2)} \quad \cdots\cdots\cdots\cdots \quad (3\cdot 7)$$

where $s_1=-(\lambda_1+\lambda_2+\mu)+\sqrt{(\lambda_1+\lambda_2)\mu}$

$s_2=-(\lambda_1+\lambda_2+\mu)-\sqrt{(\lambda_1+\lambda_2)\mu}$

Taking inverse Laplace transform of (3. 1)~(3. 7) the following results were obtained:

$$P_{ij}(t)=A_{ij}+B_{ij}\exp(s_1 t)+C_{ij}\exp(s_2 t)$$

where

$$A_{00}=\frac{\mu^2}{s_1 s_2},\qquad B_{00}=\frac{-(\lambda_1+\lambda_2)}{2s_1},$$

$$C_{00}=\frac{-(\lambda_1+\lambda_2)}{2s_2}$$

$$A_{i0}=\frac{\lambda_i\,\mu}{s_1 s_2}$$

$$B_{i0}=\frac{\lambda_i\,[(\lambda_1+\lambda_2)^2-\mu\sqrt{(\lambda_1+\lambda_2)\mu}]}{s_1 s_2(s_1-s_2)}$$

$$C_{i0}=\frac{-\lambda_i\,[(\lambda_1+\lambda_2)^2+\mu\sqrt{(\lambda_1+\lambda_2)\mu}]}{s_1 s_2(s_1-s_2)}$$

$(i=1,\ 2)$

$$A_{ij}=\frac{\lambda_i\,\lambda_j}{s_1 s_2},\qquad B_{ij}=\frac{\lambda_i\,\lambda_j}{s_1(s_1-s_2)}$$

$$C_{ij}=\frac{-\lambda_i\,\lambda_j}{s_2(s_1-s_2)}$$

$(i=1,2;\quad j=1,2)$

$s_1=-(\lambda_1+\lambda_2+\mu)+\sqrt{(\lambda_1+\lambda_2)\,\mu}$

$s_2=-(\lambda_1+\lambda_2+\mu)-\sqrt{(\lambda_1+\lambda_2)\,\mu}$

Therefore, the availability function can be expressed as followings:

$$A(t)=1-[P_{11}(t)+P_{12}(t)+P_{21}(t)+P_{22}(t)]$$

$$=1-(\lambda_1+\lambda_2)^2\left\{\frac{1}{s_1 s_2}+\frac{1}{s\ s_1-s_2}\exp(s_1 t)-\frac{1}{s_2(s_1-s_2)}\exp(s_2 t)\right\}$$

And steady-state probabilities are as follows:

$$P_{ij}(\infty)=\frac{\lambda_i\lambda_j}{s_1 s_2}\qquad \text{where}\qquad \lambda_0=\mu$$

$$A(\infty)=1-\frac{(\lambda_1+\lambda_2)^2}{s_1 s_2}$$

REFERENCES

1. Elsayed, E.A. and Zebib, A., A Reparable Multistate Device, IEEE Transactions on Reliability, Vol. R-28 No. 1, 81-82, 1979.

2. Proctor, C. L., A Reparable 3-State Device,' IEEE Transactions on Reliability, Vol. R-25, No. 3, 210-211, 1976.

3. Rau, J. G., Optimization and Probility in System Engineering, Van Nostrand Reinhold Company, 1970.

4. Shooman, M.L., Probabilistic Reliability : An Engineeting Approach, McGraw-Hill, New York, 1968.