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SEQUENTIAL CONTINUITY

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In this paper we improve a result of M. Jaegermann [1]. AC^w is the countable axiom of choice.

THEOREM. *AC^w is equivalent to SC.*

SC: Each sequentially continuous real valued function on a metric space is continuous.

PROOF. That AC^w implies SC is standard.

For the proof of the converse, be $(A_n)_{n\in w}$, $w=\{0,1,\ldots\}$, $A_n\neq\phi$, a counterexample to AC^w. $C_n\neq\phi$ is the set of all finite sequences $(a_k)_{k\in n}$, such that $a_k\in A_k$; $C_n\cap C_m=\phi$ for $n\neq m$. If a function c on $S\subseteq w$ satisfies $c(k)\in C_k$ for $k\in S$, S is finite, for otherwise $a(k)=c(n(k))_k$, $n(k)=\min\{n\in S: k\in n\}$, defines a choice function on $(A_n)_{n\in w}$. We set $A=\bigcup\{C_n: n\in w\}$, $X=A\cup\{a\}$ for $a\notin A$ and construct a metric d on X through $d(x,x)=0$, $d(x,y)=1$ for $x\neq y$ in A and $d(x,y)=\frac{1}{n+1}$ for $x\in C_n$. f is the characteristic function of A: Since $a\in A^-$, f is not continuous. f is sequentially continuous: Be (x_n) a sequence in A. We set $a(x)=\frac{1}{d(x,a)}-1$, so $x\in C_{a(x)}$. S is the set of all $a(x_n)$ such that $a(x_m)<a(x_n)$ for $m<n$, and $y_n=x_m$ for $n=a(x_m)\in S$: Since $y_n\in C_n$ for all $n\in S$, S is finite and each x_n is in $\bigcup\{C_n: n\leq \max S\}$. Thus $\lim x_n=a$ is impossible, proving the claim.

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REFERENCE

[1] M. Jaegermann, *The axiom of choice and two definitions of continuity*, Bull. Acad. Polon. 13(1965), 699—704.