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—Research Paper—

Time-varying Network Model of Conveyor Systems

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Abstract

This paper presents the network models for general dynamic conveyor systems which are characterized by transporting and storing materials between work stations over time. With an appropriate choice of time-slice the conveyor system can be represented exactly as a dynamic flow network which can be solved by an efficient pure network algorithm.

I. INTRODUCTION

This paper describes analytical tools which assist design engineers to analyze general dynamic conveyor systems.

In modern manufacturing, a conveyor system is one of the most effective handling methods for transporting and storing material between work stations. Because these modest manufacturing and material handling systems are becoming more sophisticated and more complex to design and operate, more effective techniques are required to analyze the system performance. These techniques usually make use of mathematical models. Although a number of models recently have been developed, none seem to offer a satisfactory description of time-dependent flows in general conveyor systems.

"Conveyor Theory" was first developed in the late 1950's by Kwo [13] and has since been extended by a number of authors [1—26]. While most of these studies restricted to the conveyor in its simplest form has yielded feasibility conditions for conveyor operation, the following are not yet fully developed:

(1) System with time-varying flow rates
(2) Optimization procedure
(3) Generalization for complex conveyor systems.

In view of these deficiencies, linearized models discussed here seem to provide an optimization procedure for the dynamic behavior of flow in general conveyor systems. In order to develop a linearized model, we investigate the effect of the time-discretization scheme in detail, because the success of the linearization of the dynamic behavior depends on

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the discretization of the time scale. We derive a characteristic description of the dynamic behavior of conveyor flow without an extensive discussion on the formulation of a performance measure. In Chapter II, the general flow equations and capacity constraints are derived using the law of conservation. In Chapter III, by examining time-discretization, we formulate a dynanic flow network for the conveyor system. Our model can be solved by using a standard programming package such as Structured Linear Programming or the more efficient Out-of-Kilter network algorithm.

II. SYSTEM DESCRIPTION

In this chapter we will describe the general conveyor system which is characterized by transportation and storage functions. The general conveyor system studied consists of the following major parts:

- loading stations with storage spaces
- unloading stations with storage spaces
- merge stations
- accumulating conveyors

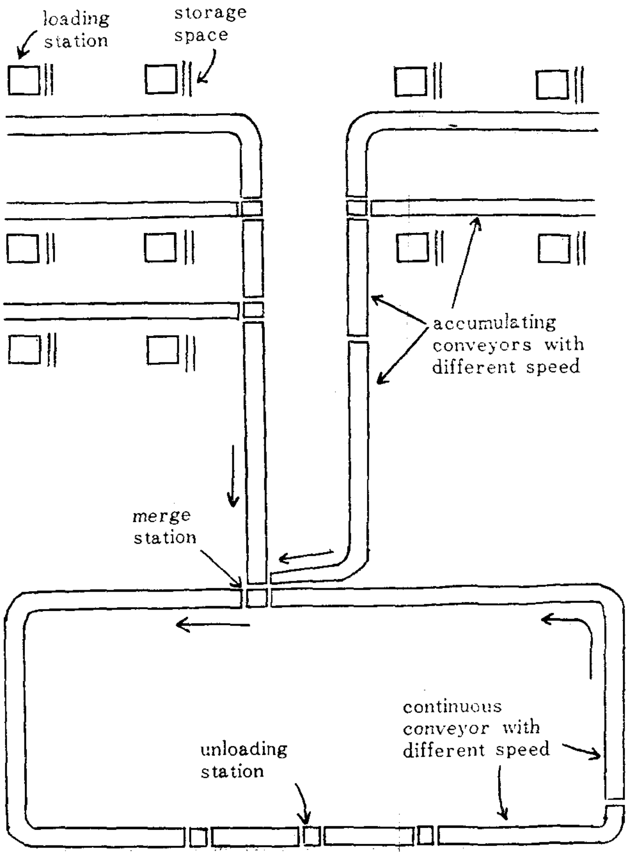

Figure-1. General conveyor system.

- continuous conveyors
- circulating conveyors.

The physical conveyor system shown in Figure-1 has various types of conveyors with different speed. A *conveyor* is defined as the one with its properties assumed to be homogeneous and bounded by merge stations. If accumulation is allowed on the conveyor, it is called the *accumulating conveyor*. When downstream flow is stopped on an accumulating conveyor, the first item in the platoon of items stops and a queue builds up with maximum item density. However the items upstream still move toward the end of the queue at the conveyor speed. A conveyor which moves continuously at a constant speed without interruption or accumulation is called a *continuous conveyor*.

Throughout this paper, the following basic assumptions are made: (1) The system conveys items which are identical in size. (2) Time is divided into discrete, equal intervals called time-slices. (3) The physical position of items in time-slice is not considered.

II.A. Network Components

To model a complex conveyor system and specify the data requirements, we represent the conveyor system as a network suitable for analysis. The system is divided into the following three elementary components.

1. Conveyor Section

A conveyor is divided into one or more *conveyor sections* by loading, unloading or merge stations. The conveyor section i is designated as the directed arc i represented by the small-square. According to the type of the conveyor, the conveyor section is called an *accumulating conveyor section* or a *continuous conveyor section*. A conveyor section i is characterized by the following requirements and capacity constraints:

(1) The *length* and *speed* of conveyor section i are $d(i)$ and $v(i)$, respectively.

(2) The *holding capacity* of the conveyor section i, $P(i)$, is stated as the maximum number of items that the conveyor section i can accomodate. For the accumulating conveyor section i, $P(i)$ is defined as follows:

$$P(i)=d(i)/L$$

where L=length of an item. For the continuous conveyor section,

$$P(i)=d(i)/(L+\text{gap allowance between items}).$$

(3) The *travel time*, $T(i)$, is defined as the time required to travel the entire conveyor section i without delay due to the accumulation on the conveyor section. Thus the length and the speed of the conveyor section i can be measured in time units;

$$T(i)=d(i)/v(i).$$

(4) The *flow capacity*, $f(i)$, is defined as the maximum number of items which can pass a given point on the conveyor section i per unit time-slice. $f(i)$ can be obtained as follows:

$$f(i)=(v(i)\times \Delta t)/(L+\text{gap allowance})$$

2. Merge Station

A conveyor section i is terminated at both ends with a merge station such as a loading station, an unloading station or a junction between different kinds of conveyors. The merge station i is labeled as the immediate upstream node of the conveyor section i.

The merge station i is characterized by the *merge capacity*, $m(i)$, which is defined as the maximum number of items which can pass the merge station i per unit time slice. $m(i)$ is affected by physical and operating conditions of the merge station i.

3. Feeder Line

A loading station with temporary storge space is conceptually a feeder line with zero travel time but holding capacity. That is, items arriving at the feeder line are released instantaneously if loading is required, otherwise items accumulate in front of the merge station at maximum density. The arc i will be used for the feeder line i. *Holding capacity*, $Q(i)$, for the feeder line i is defined as the maximum number of items that the storge space on the loading station can accomodate. An unloading station is also considered as a feeder line with a negative arrival and release.

The conveyor system can be defined as the network of merge stations connected by a set of conveyor sections and feeder lines as shown in Figure-2.

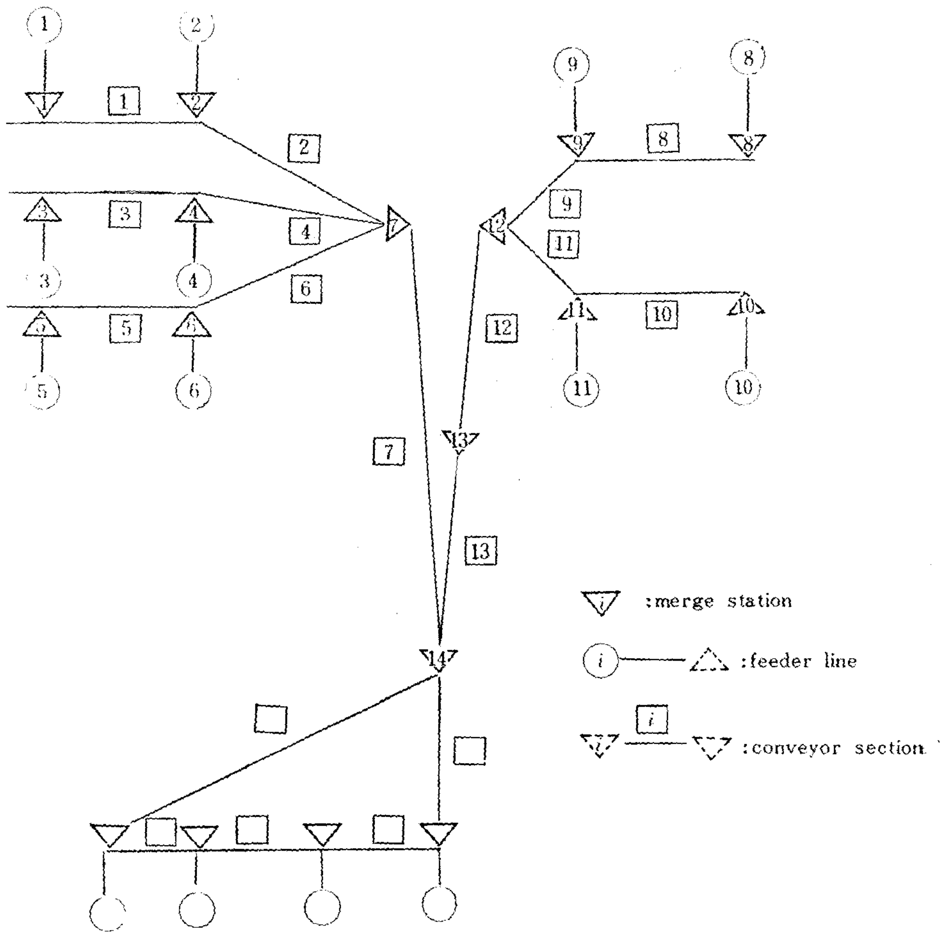

Figure-2. Equivalent network to the conveyor in Figure-1.

II.B. Time-discretization

We establish a scheme for discretization of the time scale to provide discrete, equally spaced time intervals called time-slices. In general, the success of our analysis of dynamical flow depends on an appropriate choice of the time-slice. As a general rule, we select the duration of a time-slice in order to (1) minimize the number of time intervals, and to (2) maximize the sensitivity of the flow analysis.

Now suppose that our choice of duration of a time-slice is Δt. To examine the implication of Δt on the specific conveyor section i which has travel time $T(i)$, let

$\Delta t = T(i)/k$, where k is an integer number.

Depending upon the value of k, the time-slice is classified into the following three categories:

1) If $k=1$, $\Delta t = T(i)$ is called the *unit-time-slice.*
2) If $k<1$, $\Delta t > T(i)$ is called the *fractional-time-slice.*
3) If $k>1$, $\Delta t < T(i)$ is called the *multiple-time-slice.*

If we analyze conveyor section i with the unit-time-slice the starting flow from the upstream merge station reaches the immediate downstream merge station within one time-slice. With the choice of the multiple-time-slice it takes k time-slices for first flow to reach the next merge station. With the choice of a fractional-time-slice the flow might pass over the next downstream merge station. The characteristics of each time-discretization will be discussed in detail in Chapter III.

II.C. Notation

For the chosen time slice Δt, the following notation is used, where i refers to feeder line or conveyor section and t refers to time-slice.

$a(i,t)$ = the number of items arriving per Δt at feeder line i during the time-slice t.

$r(i,t)$ = the number of items released per Δt from the feeder line i during the time-slice t.

$Q(i,t)$ = the number of items in the queue waiting to be released at the end of the time-slice t.

$b(i,t)$ = the number of items arriving per Δt at the conveyor section i during the time-slice t.

$w(i,t)$ = the number of items released per Δt from the conveyor section i during the time-slice t.

$P(i,t)$ = the number of items on the conveyor section i at the end of the time-slice t. For the accumulating conveyor section i, $P(i,t)$ is the number of units moving or accumulated in the section i at the end of time-slice t, and for the continuous conveyor section i, items are those moving.

Note that variables $a(i,t)$, $b(i,t)$, $w(i,t)$, and $r(i,t)$ are numbers of items flowing during the chosen time interval. If the arrival rate, $\alpha(i,t)$, is given per unit time, say, per hour or minute, we need to convert the given arrival rate into a per-unit-time-slice basis by

$$a(i,t) = \int_{\Delta t} \alpha(i,s)\ ds.$$

Variables $Q(i,t)$ and $P(i,t)$ are the instantaneous number observed at a specific time t, *i.e.*, the end of the time-slice t or the beginning of the time-slice $t+1$.

Throughout this paper, upper case denotes instantaneous numbers such as queue length or holding capacity and lower case is used for flow such as arrival rate, release rate, marge capacity or flow capacity. Figure-3 illustrates three basic components with notation.

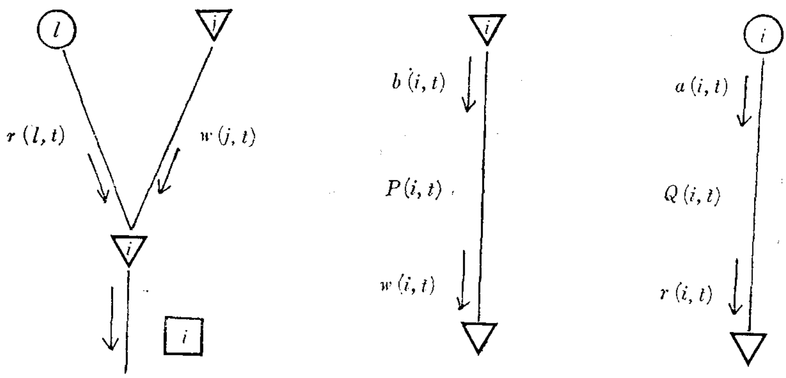

Figure-3. Basic components with notation.

II.D. Characteristics of Dynamic Flow

In this section we will describe the flow conservation of dynamic behavior of the conveyor system and the flow characteristics in terms of the capacity constraints.

1. Flow Conservation

Since items on the conveyor system are neither created nor destroyed in the tranportation and storage process, the law of conservation can be applied to the feeder line, conveyor section and merge station.

Flow Equations

Based on the principle of the flow conservation, the number of items in the conveyor section i or the feeder line i at the end of time-slice t equals the number in the system at the beginning of the time interval t plus the difference between the number arriving at and released from the system i during Δt. In referring to the diagrams shown in Figure-3, this may be written as

(2.1) $$Q(i,t)=Q(i,t-1)+a(i,t)-r(i,t)$$

for the feeder line i, and

(2.2) $$P(i,t)=P(i,t-1)+b(i,t)-w(i,t)$$

for the conveyor section i.

Merge Equation

Since at the merge station i storage is not allowed, the amount of flow that enters the merge station i during the time-slice t is equal to the amount that leaves the merge station i during time-slice t, *i.e.*

(2.3) $$b(i,t)=\sum_{j\in Ui} w(j,t)+\sum_{l\in Ui} r(l,t)$$

where Ui=set of conveyor sections $\{j\}$ and feeder line $\{l\}$ which merge at merge station i.

2. Capacity Constraints

From the definition of the basic conveyor components,

(2.4) $P(i,t)\leq P(i)$ for the conveyor section i,

(2.5) $Q(i,t)\leq Q(i)$ for the feeder line i.

Now define the *merge flow capacity*, $b(i)$, as follows:

$$b(i)=\text{Min}\ \{f(i),\ m(i)\}.$$

Then,

(2.6) $b(i,t)\leq b(i)$ for the merge station i.

Note that the constraints $w(j,t)\leq b(i)$ and $r(i,t)\leq b(i)$ are bounded by Eq. (2.3) and $b(i,t)\leq b(i)$.

3. Availability Constraints

The maximum amount which can be released from the feeder line i or from the conveyor section i during the given time-slice t depends on the number of items available. These are defined as the *availability* $Ar(i,t)$ and $Aw(i,t)$, respectively. Thus, $r(i,t)$ and $w(i,t)$ are bounded by availability as $r(i,t)\leq Ar(i,t)$, $w(i,t)\leq Aw(i,t)$, respectively. The availabilities $Ar(i,t)$ and $Aw(i,t)$ consists of two parts, namely,

(1) portion of queues at the beginning of the time-slice t, $Q(i,t-1)$, $P(i,t-1)$ respectively, and

(2) portion of arrival during time-slice t, $r(i,t)$, $b(i,t)$ respectively.

To derive the availability constraints, we have to determine these two parts which depend on the time-discretization.

In summary, the system can be characterized by the following behavioral relationships of material flow: 1) Flow equations, 2) Merge equations, 3) Capacity constraints, 4) Availability constraints. Since the first three are independent of time-discretization or the conveyor length and speed, Eqs. (2.1)−(2.6) can be applied to any conveyor system without further modifications. However, since there is no general expression for the availability constraints, we derive acceptable availability constraints by examining each time-discretization in Chapter III.

II.E. Measure of Performance

For the measurement of performance, we can consider two objectives, design problem of the future conveyor facilities and operation problem of the existing conveyor system, for which we assume all variables as optimizable. We do not present an extensive discussion on the formulation of the performance measurement. The following are several linear objective functions of practical interest:

- Maximize total input flow rate from all loading stations

  $$\text{Max } Z=\sum_{i,t} r(i,t)$$

- Maximize total throughput from the system

  $$\text{Max } Z=\sum_{t} b(N,t),$$

  where N is the last conveyor section of the system.
- Minimize total excess capacity

  $$\text{Min } Z=\sum_{i,t} (P(i)-P(i,t))$$

- Minimize maximum queue length at the loading station

  $$\text{Min } Z=\text{Max } \{Q(i,t)\}.$$

We can optimize any criteria subject to the constraints which we develop in the next two chapters.

III. ELEMENTARY CONVEYOR SUBSYSTEM

In this chapter we examine the basic elementary conveyor subsystem which consists of three basic components of the conveyor system; an accumulating conveyor section with a feeder line merging at a merge staton.

In Figure-4, the elementary conveyor subsystem is shown with the notation. The conveyor section i-1 is numbered as immediately upstream of merge station i, and the merge station $i+1$ as immediately downstream of conveyor section i.

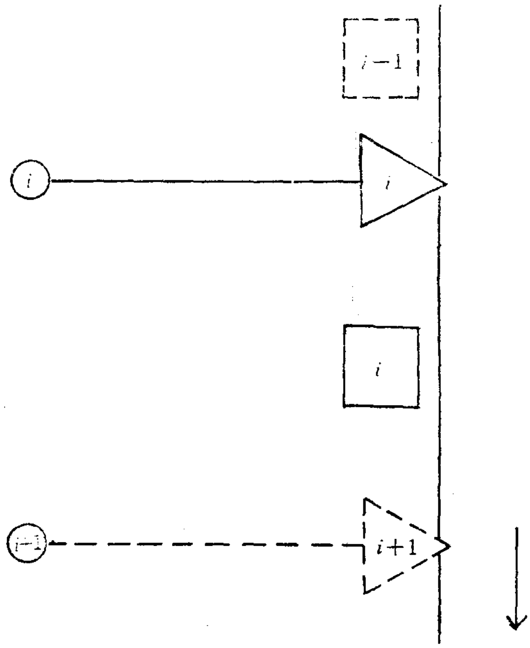

Figure-4. Elementary conveyor subsystem.

Our main purpose in this chapter is to derive availability constraints to complete our description of system behavior. We express the availability for each time-discretization scheme.

III.A. Unit-time-discretization

1. Availability of $w(i,t)$; $Aw(i,t)$

Consider the following diagram to obtain dynamic relations between $P(i,t)$, $b(i,t)$ and $w(i,t)$. If the time-slice Δt is chosen to be equal to the travel time $T(i)$, the maximum distance of travel of items on the conveyor section i is $d(i)$ during Δt, or equivalently $T(i)$ in time units.

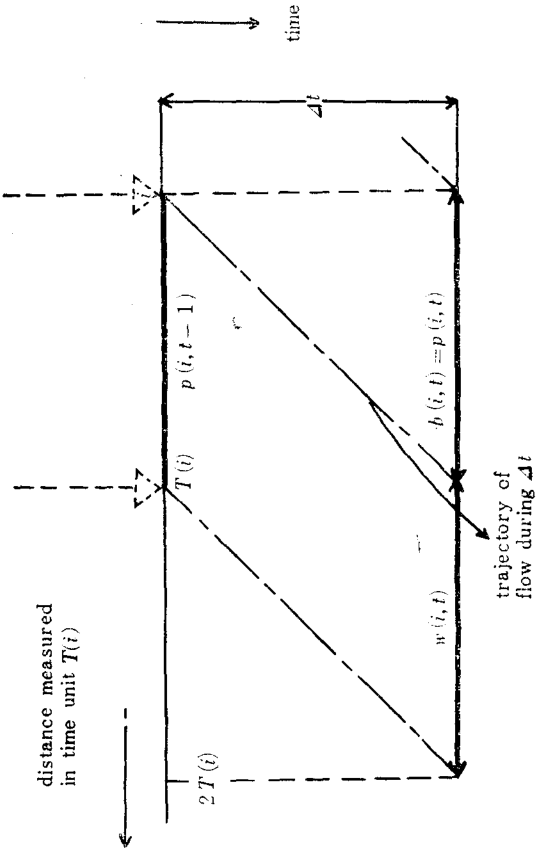

Figure-5. Relations between $b(i,t)$, $w(i,t)$ and $P(i,t)$.

Referring to Figure-5, we observe that, during time-slice $\Delta t = T(i)$, none of the items from merge station i, $b(i,t)$, exits from the merge station $i+1$. In other words, all items of $b(i,t)$ are included in $P(i,t)$, and $w(i,t)$ does not include any item of $b(i,t)$. Therefore $b(i,t)$, the second part of $Aw(i,t)$ defined in section Ⅱ. D is not available for $w(i,t)$ during Δt. However, since the maximum travel distance is $d(i)$ during Δt, the items on the conveyor section i at the beginning of time-slice t are all available to be released during Δt. Therefore the maximum number of items available is determined by the first part of availability as follows:

(3.1) $\quad Aw(i,t) = P(i,t-1)$.

By definition of availability, we have the following constraint for $w(i,t)$:

(3.2) $\quad w(i,t) \leq P(i,t-1)$.

If during the time-slice t, part or none of $P(i,t-1)$ is released, then the leftover part of $P(i,t-1)$ is concentrated at maximum density at the merge station $i+1$. Let us define this contracted leftover during time slice t as $G(i,t)$. Thus $P(i,t-1)$ is split into two parts; release part $w(i,t)$ and leftover part $G(i,t)$. We can rewrite Eq. (3. 2) as:

(3. 3) $\quad w(i,t)+G(i,t)=P(i,t-1),$

which is the availability constraint when we use a unit-time-slice.

2. Availability of $r(i,t)$; $Ar(i,t)$

Consider the diagram shown in Figure-6 where the feeder line i is scaled to one time unit.

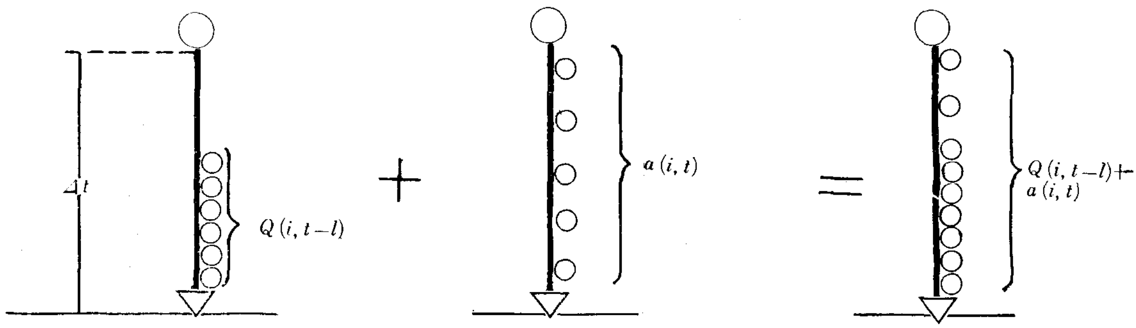

Figure-6. Illlustration of availability $Ar(i,t)$.

Since the feeder line is assumed to have zero travel time, $Q(i,t-1)$ is concentrated in front of the merge station. In referring to Figure-6, during Δt, items on the length Δt measured in time unit can be released, that is,

(3. 4) $\quad Ar(i,t)=Q(i,t-1)+a(i,t).$

Then by definition of the availability,

$$r(i,t) \leq Q(i,t-1)+a(i,t).$$

But this constraint is automatically satisfied by the flow equation (2. 1).

3. Network Model

For the elementary conveyor subsystem, Eq. (2. 3) reduced to

(3. 5) $\quad b(i,t)=r(i,t)+w(i-1,t).$

Substituting Eq. (3. 3) into Eq. (2. 2), we obtain

$$P(i,t)=P(i,t-1)+b(i,t)-w(i,t)$$

(3. 6) $\quad =G(i,t)+b(i,t)$

Thus the notwork model for the elementary conveyor subsystem is formulated as follows with the unit-time-discretization;

$$Q(i,t)=Q(i,t-1)+a(i,t)-r(i,t)$$
$$P(i,t)=G(i,t)+b(i,t)$$
$$P(i,t-1)=G(i,t)+w(i,t)$$
$$b(i,t)=r(i,t)+w(i-1,t)$$
$$Q(i,t)\leq Q(i)$$
$$P(i,t)\leq P(i)$$

$G(i,t) \leq P(i)$
$b(i,t) \leq b(i)$
$r(i,t) \leq b(i)$
$w(i-1,t) \leq b(i)$

Note that since $w(i,t)$ is not bounded on the conveyor section i but bounded on the conveyor section $i+1$ as $w(i,t) \leq b(i+1)$, the model of the elementary conveyor subsystem does not contain the capacity constraint of $w(i,t)$. Figure-7 is the flow-accumulation network for the elementary conveyor subsystem during the time-slice t.

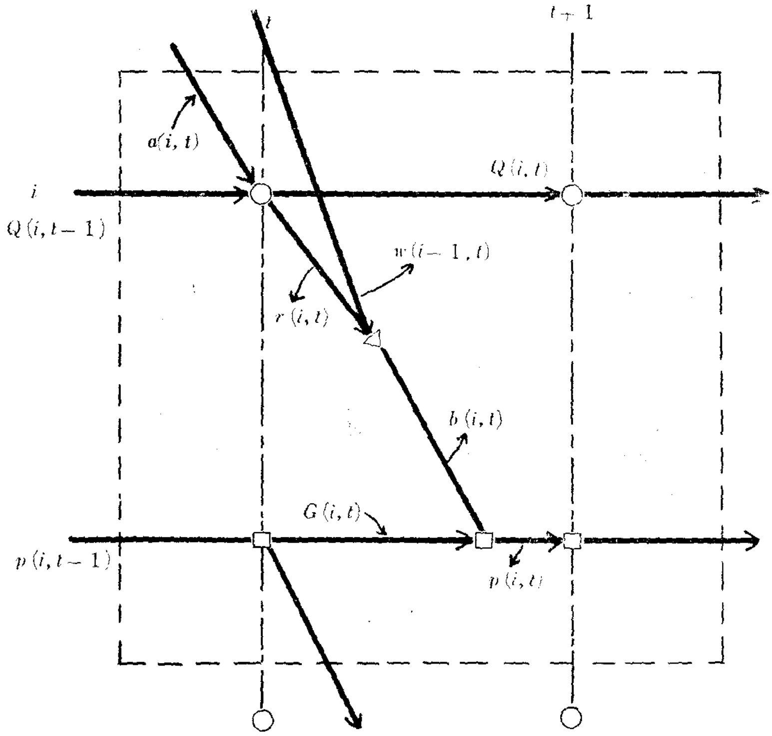

Figure-7. Dynamic flow-accumulation network for elementary accumulating conveyor subsystem.

Note that in the dynamic flow-accumulation network,

(1) arc $G(i,t)$ hold the key to modeling the accumulating conveyor. This has certain capacities to preserve the dynamical relations between $P(i,t-1)$ and $w(i,t)$,

(2) there are two kinds of directed arc;

- accumlation variables, $Q(i,t)$, $P(i,t)$ and $G(i,t)$ are shown as horizontal arcs and,
- flow variables $r(i,t)$, $w(i,t)$, and $b(i,t)$ as vertical arcs, and

(3) all arcs are capacitated by the capacity constraints.

4. Continuous Conveyor Subsystem

A continuous conveyor subsystem is a special case of the elementary accumulating conveyor subsystem. The essential difference between the accumulating and the continuous conveyor section is that when we choose $\Delta t = T(i)$ the leftover on the continuous conveyor section is

$$G(i, t) = 0, \text{ or } Aw(i, t) = w(i, t).$$

Thus Eqs. (3.3) and (3.6) become

(3.7) $\quad w(i, t) = P(i, t-1)$ and

(3.8) $\quad P(i, t) = b(i, t)$

Eq. (3.8) implies that with unit-time-slice the merge flow capacity $b(i)$ of merge station i is equal to the holding capacity $P(i)$ of continuous conveyor section i, that is, $P(i) = b(i)$. Combining Eq. (3.7) and (3.8), we have

(3.9) $\quad b(i, t) = w(i, t+1)$.

Substituting this equation into Eq. (3.5), we obtain the following relation between $w(i, t)$ and $w(i-1, t)$:

(3.10) $\quad w(i, t+1) = r(i, t) + w(i-1, t)$.

Then the network model for the continuous conveyor segment with unit-time-slice is

$$Q(i, t) = Q(i, t-1) + a(i, t) - r(i, t)$$
$$w(i, t+1) = r(i, t) + w(i-1, t)$$
$$w(i, t+1) \leq b(i)$$
$$w(i, t) \leq b(i)$$
$$r(i, t) \leq b(i)$$
$$Q(i, t) \leq Q(i).$$

Note that we need only $w(i, t+1)$ and $w(i-1, t)$ to describe the continuous conveyor section i, and both are bounded by $b(i)$. In Figure-8 the corresponding network is constructed.

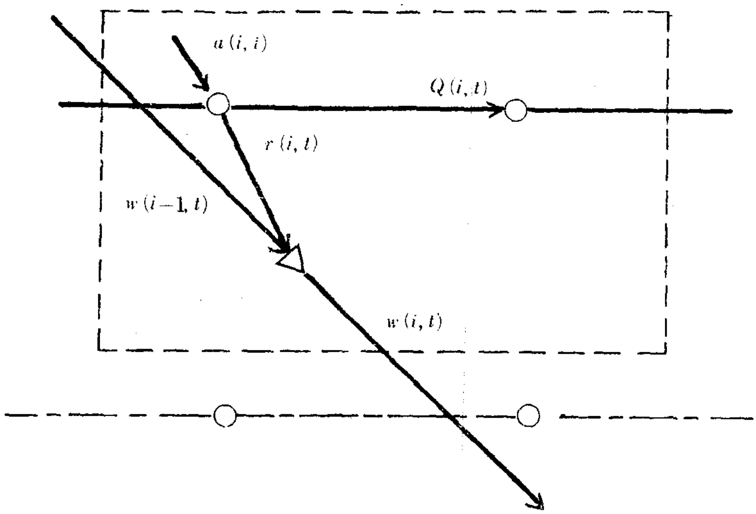

Figure-8. Network for continuous conveyor segment.

Example (Conveyor subsystem with the feeder conveyor)

Now consider the conveyor subsystem shown in Figure-9 which has the feeder accumulating conveyors instead of the feeder lines in the elementary conveyor subsystem. We can apply to the feeder conveyor the same constraints as for the conveyor section, introducing leftover variable $H(i, t)$ on the feeder conveyor as $G(i,t)$ on the conveyor section. The network model can be written as follows;

$$Q(i,t)=H(i,t)+a(i,t)$$
$$Q(i,t-1)=H(i,t)+r(i,t)$$
$$Q(i,t)\leq Q(i)$$
$$H(i,t)\leq Q(i)$$
$$P(i,t)=G(i,t)+b(i,t)$$
$$P(i,t-1)=G(i,t)+w(i,t)$$
$$b(i,t)=r(i,t)+w(i-1,t)$$
$$P(i,t)\leq P(i)$$
$$G(i,t)\leq P(i)$$
$$b(i,t)\leq b(i)$$
$$r(i,t)\leq b(i)$$
$$w(i-1,t)\leq b(i)$$

The corresponding network is shown in Figure-9.

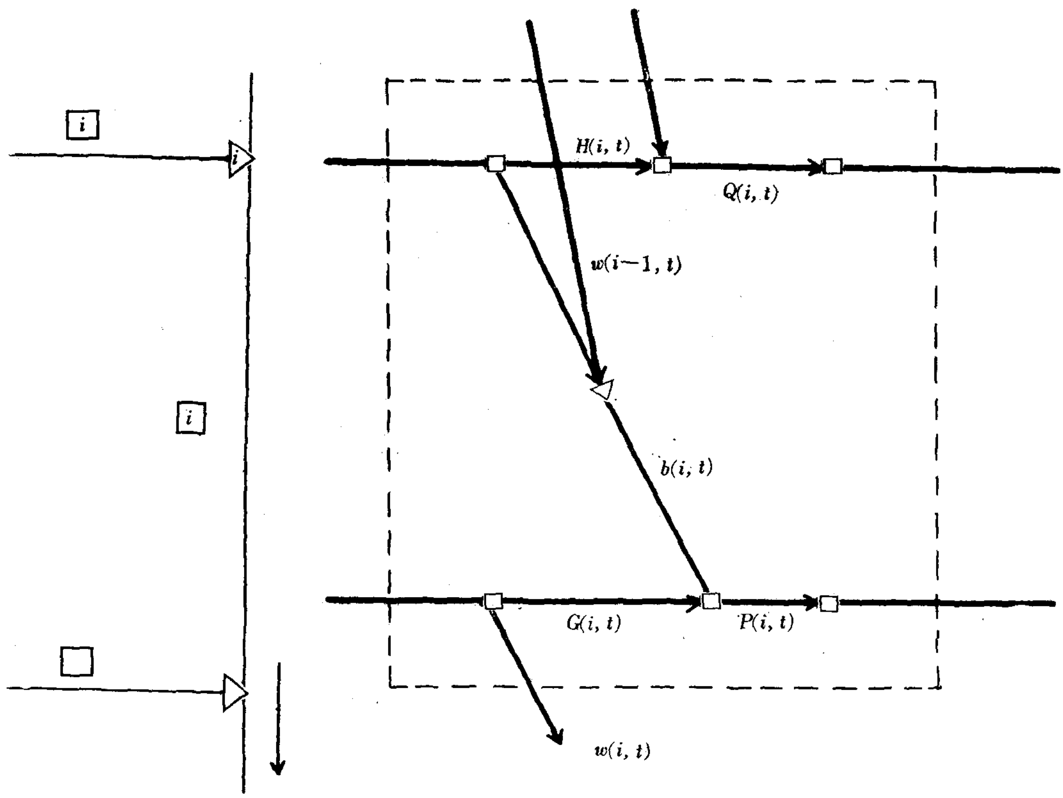

Figure-9. Network for the conveyor segment with the feeder conveyor.

III.B. Fractional-time-eiscretization

In the last section, we showed that for the conveyor section i if the travel time $T(i)$ is chosen as the time-slice Δt the constraints of the network model represent exactly the dynamical behavior of the elementary conveyor segment and some simple systems. However, since the travel time is generally less than a few minutes we might have enormous number of time-slices to analyze a working cycle time or congestion time periods.

In this section, we will examine the time-discretization scheme where a time-slice is chosen longer than the travel time between merge stations in light of possible reduction of the number of time-slices. We derive availabilities $Aw(i, t)$ and $Ar(i, t)$ choosing $\Delta t = 2T(i)$ for the simple illustration.

1. Availability of $w(i, t)$; $Aw(i, t)$

Consider the relstionship between $P(i, t-1)$, $w(i, t)$, and $b(i, t)$ shown in Figure-10.

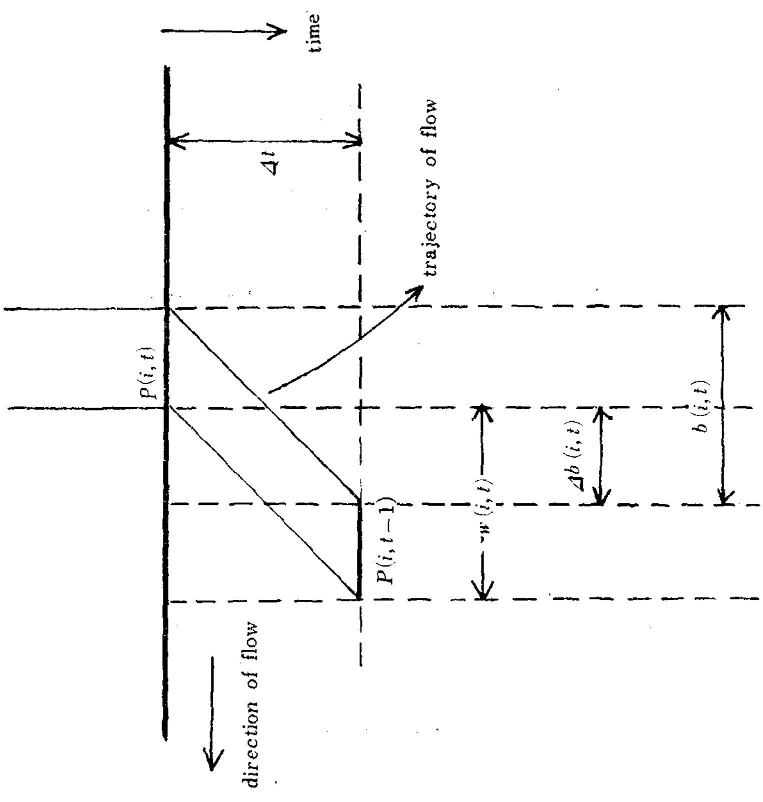

Figure-10. $Aw(i, t)$ for fractional-time-discretization.

Availability of $w(i, t)$ can be expressed by

$$Aw(i, t) = P(i, t) + \Delta b(i, t)$$

where Δ indicates a portion of $b(i, t)$ during $\Delta/2$. Due to the discontinuity of Δt, *i.e.* $\Delta t/2$, no simple way is found to represent $\Delta b(i, t)$.

2. Availability of $r(i, t)$; $Ar(i, t)$

The amount of items which can be released during $\Delta t = T(i)/2$ is plotted by thick lines in Figure-11, where feeder lines are scaled in time units.

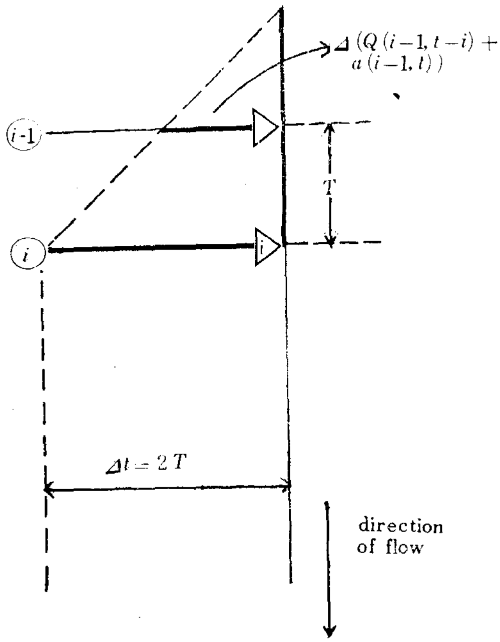

Figure-11. $Ar(i, t)$ for fractional-time-discretization.

$\Delta(Q(i-1, t-1)+a(i-1, t)$ is availability of $r(i-1, t)$ during $\Delta t/2$, which cannot be expressed explicitly because of the discontinuity of Δt.

In general, both difficulties to express the availability of $w(i, t)$ and $r(i, t)$ during $\Delta t/k$ is due to the k times discontinuities of Δt. This indicates that for exact analysis of dynamic conveyor flow, the time-slice should be chosen as $\Delta t = T(i)$, which is unit-time-slice.

III.C. Multiple-time-discretization

Multiple-time-discretization is defined as a choice of time-slice which is less than the travel time between two adjacent merge stations in the elementary conveyor subsystem.

1. Availability of $w(i, t)$; $Aw(i, t)$

Consider the diagram shown in Figure-12.
In referring to the diagram availability $Aw(i, t)$ is

$$Aw(i, t) = \Delta P(i, t),$$

where $\Delta P(i, t)$ is a portion of $P(i, t-1)$ during time-slice Δt. Two possible representations of $\Delta P(i, t-1)$ will be considered below.

Segmentation of Convevor Section

Introduce $(k-1)$'s dummy junctions, and divide the conveyor section i into k homogeneous *conveyor segments* such that the travel time of each conveyor segment is Δt. This procedure, which produces a new conveyor system model, is illustrated in Figure-13.

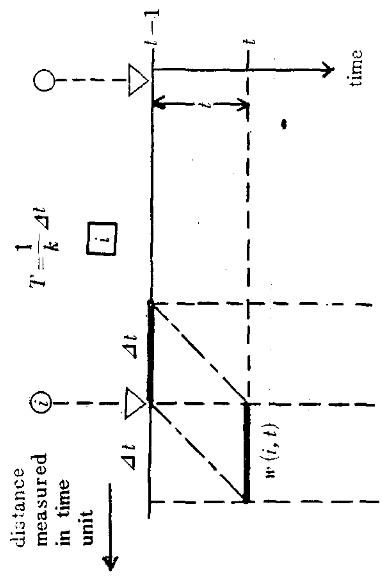

Figure-12. Availability for multiple-time-discretization.

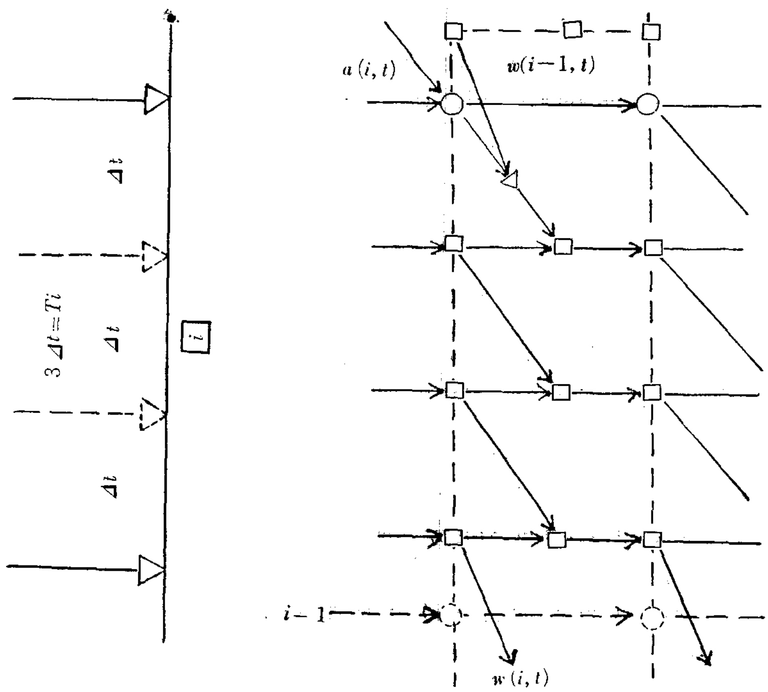

Figure-13. Introduce dummy merge stations with zero release.

Since the time-slice Δt is the unit-time-slice for a conveyor segment, we can apply the results of the unit-time-slice analysis to each of the conveyor segments, as illustrated in the example in Section Ⅱ. A.

Direct Assessment of Availability

Unlike the definition of availability in unit- and fractional-time-discretization, we now define $Aw(i, t)$ directly as

(3.11) $$Aw(i,t) = Aw(i,t-1) - w(i,t-1) + b(i,t-k) \text{ subject to}$$

(3.12) $$Aw(i,t) \leq P(i,t-1).$$

Equations (3.11) and (3.12) require that the number of available items for release $w(i,t)$ from the conveyor section i during time-slice t be defined as the difference between availability and actual release **during** previous time-slice $t-1$ plus arrival at the conveyor section i k time-slices ealier. This definition of availability is based on the fact that the items which arrived k time-slices earlier could be blocked at the merge station at time-slice t. By the definition of availability,

(3.13) $$w(i,t) \leq Aw(i,t),$$

Introducing leftover variable $G(i,t)$, Eq. (3.13) can be written as

(3.14) $$w(i,t) + G(i,t) = Aw(i,t).$$

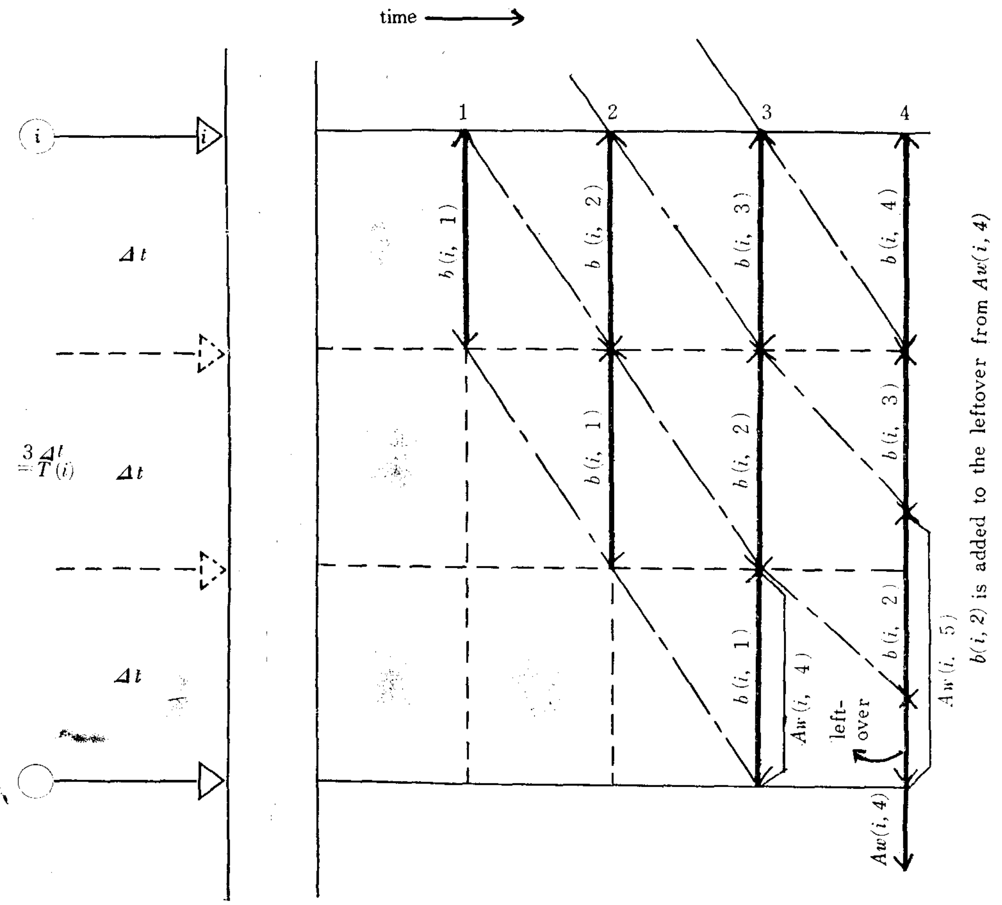

Figure-14. Illustration of availability for multiple-time-discretization.

We substitute Eq. (3.14) into Eq. (3.11) to obtain

(3.15) $$Aw(i,t)=G(i,t-1)+b(i,t-k).$$

Eq. (3.15) states that availability at time t is equal to the concentrated sum of the leftover from the previous time-slice $t-1$ and arrival during time-slice $t-k$ which contracted to the leftover from time-slice $t-1$. A schematic diagram of the relation for the case of $k=3$ is shown in Figure-14.

2. Availability of $r(i,t)$; $Ar(i,t)$

Applying the same argument of availability as in unit-time-discretization, we have

$$r(i,t)\leq Q(i,t-1)+a(i,t)$$

which is redundant by the flow equation on the feeder line.

3. Network Formuation

If we choose the time-slice Δt shorter than the travel time $T(i)$ on the elementary conveyor subsystem, we can formulate the network model as follows:

First notationally, we substitute $Aw(i,t)$ as

(3.16) $$Aw(i,t)=X(i,t-1).$$

Using $X(i,t)$, Eqs. (3.11) and (3.14) can be written as

(3.17) $$X(i,t)=X(i,t-1)-w(i,t)+b(i,t-k+1)$$

(3.18) $$X(i,t-1)=w(i,t)+G(i,t).$$

Combining these equations, we have

(3.19) $$X(i,t)=G(i,t)-b(i,t-k+1).$$

Inequality of Eq. (3.12) can be equated as

(3.20) $$P(i,t)=X(i,t)+\sum_{s=t-k+2}^{t} b(i,s)$$

Thus the constraints for the network model are;

[A]:
$$\begin{aligned}
&Q(i,t)=Q(i,t-1)+a(i,t)-r(i,t)\\
&X(i,t)=G(i,t)+b(i,t-k+1)\\
&X(i,t-1)=w(i,t)+G(i,t)\\
&b(i,t)=r(i,t)+w(i-1,t)\\
&w(i-1,t)\leq b(i)\\
&r(i,t)\leq b(i)\\
&b(i,t)\leq b(i)\\
&X(i,t)\leq P(i)\\
&G(i,t)\leq P(i)\\
&Q(i,t)\leq Q(i)
\end{aligned}$$

[B]:
$$X(i,t)+\sum_{s=t-k+2}^{t} b(i,s)\leq P(i).$$

The constraints are divided into two parts;

part [A] is same as the network model for unit-time-discretization except for the time shift of $k-1$ as shown in Figure-15, and part [B] which cannot be represented in the network, are additional constraints needed because the travel time is longer than the chosen time-slice. The left hand side of this inequality is the number of items on the conveyor section i at the end of the time-slice t:

$$X(i,t)+\sum_{s=t-k+2}^{t} b(i,s)=P(i,t).$$

This model is called a structured linear programming and cannot be solved by a pure network algorithm such as the Out-of-Kilter algorithm.

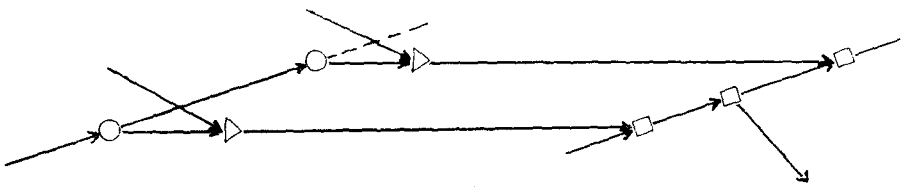

Fig-15. Flow-accumulation network for accumulating conveyor subsystem with multiple-time-slice.

We have developed two alternatives to describe the conveyor subsystem with a multiple-time-slice:

(1) Introducing dummy merge stations, segmentation of the conveyor section enables us to represent the system as a pure network at the price of additional dummy arcs and nodes, which can be solved by the pure-network algorithm.

(2) Direct availability assessment yields a network with additional linear constraints which cannot be represented in the pure network. This model can be solved by the structured linear programming algorithm which is less efficient than pure network algorithm.

4. Continuous Conveyor Subsystem

For the continuous conveyor section i, availability is

$$Aw(i,t)=b(i,t-k)=X(i,t-1),$$

and thus we have

(3.21) $$X(i,t)=b(i,t-k+1)$$
$$X(i,t-1)=w(i,t)$$
$$b(i,t)=w(i,t+k).$$

Then Eq. (3.5) becomes

$$w(i,t+k)=w(i-1,t)+r(i,t).$$

For the continuous conveyor section i, if we restrict the arrival $b(i,t)$ at the conveyor section, then the total number of items on the conveyor section is automatically restricted.

Thus the holding capacity restriction $P(i,t) \leq P(i)$ is satisfied by the merge flow constraints of $b(i,t) \leq b(i)$, and the constraint (B) is not necessary for the continuous conveyor section i. However by substituting Eq. (3.21) into Eq. (3.20), the number of items on the conveyor section i can be written as follows:

$$P(i,t) = X(i,t) + \sum_{s=t-k+2}^{t} b(i,s)$$

$$= b(i, t-k+1) + \sum_{s=t-k+2}^{t} b(i,s)$$

$$= \sum_{s=t-k+1}^{t} b(i,s)$$

The model for the continuous conveyor subsystem is the pure network with the following constraints:

$$Q(i,t) = Q(i,t-1) + a(i,t) - r(i,t)$$
$$w(i,t+k) = w(i-1,t), + r(i,t)$$
$$w(i-1,t) \leq b(i)$$
$$w(i,t) \leq b(i)$$
$$r(i,t) \leq b(i)$$
$$Q(i,t) \leq Q(i)$$

The following network in Figure-16 is for $2\Delta t = T$.

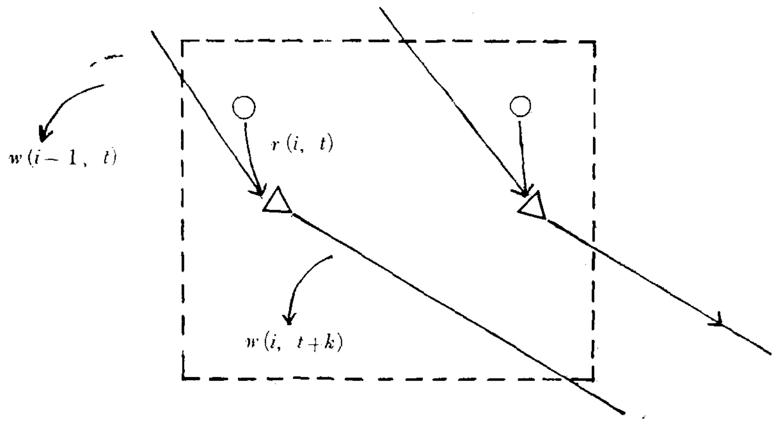

Fig-16. Continuous conveyor subsystem.

IV. GENERAL CONVEYOR SYSTEM

We are now ready to apply the basic principles developed for elementary conveyor segments to the general conveyor system which has unequal travel time between junctions.

IV.A. Time-discretization

The parameters, conveyor speed $v(i)$ and length $d(i)$ associated with each conveyor section i are transformed into travel time $T(i)$. To avoid the fractional-time-slice and minimize the

number of time-slices, it is essential to determine the time slice such that
Δt=the Greatest Common Measure $\{T(i)$; for all conveyor section $i\}$.

IV.B. Formulation

Each component in the general system can be charaterized by the following flow equations and capacity constraints;

(Feeder Line)

$$Q(i,t)=Q(i,t-1)+a(i,t)-r(i,t)$$
$$Q(i,t)\leq Q(i)$$
$$r(i,t)\leq b(i,t)$$

(Accumulating Conveyor Section)

$$X(i,t)=G(i,t)+b(i,t-k+1)$$
$$X(i,t-1)=G(i,t)+w(i,t)$$
$$P(i,t)=X(i,t)+\sum_{s=t-k+2}^{t} b(i,s) \quad \text{for } k\geq 2.$$
$$b(i,t)=\sum_{j\epsilon Ui} w(j,t)+\sum_{l\epsilon Ui} r(l,t)$$
$$w(j,t)\leq b(i)$$
$$b(i,t)\leq b(i)$$
$$G(i,t)\leq P(i)$$
$$P(i,t)\leq P(i)$$
$$X(i,t)\leq P(i)$$

(Continuous Conveyor Section)

$$w(i,t+k)=w(j,t)+r(i,t)$$
$$w(i,t+k)\leq b(i)$$
$$w(i,t)\leq b(i)$$

where j is for the upstream conveyor section j and l is feeder line l which merges at merge station i, and Ui is set of conveyor section j and feeder line l.

It can be easily verified that the constraints for feeder line and continuous conveyor section are special cases of the description of the accumulating conveyor section with following relations.

(Feeder line)

Since zero travel time is assumed on the feeder line, $k=0$. Availability $Ar(i,t)$ and leftover on the feeder line can be written as

$$X(i,t-1)=Q(i,t-1)+a(i,t),\quad G(i,t)=Q(i,t).$$

With these three equations and replacing $b(i,t)$, $P(i,t)$, and $w(i,t)$ with $a(i,t)$, $Q(i,t)$ and $r(i,t)$ respectively, the constraints for the accumulating conveyor section are reduced to the one for the feeder line.

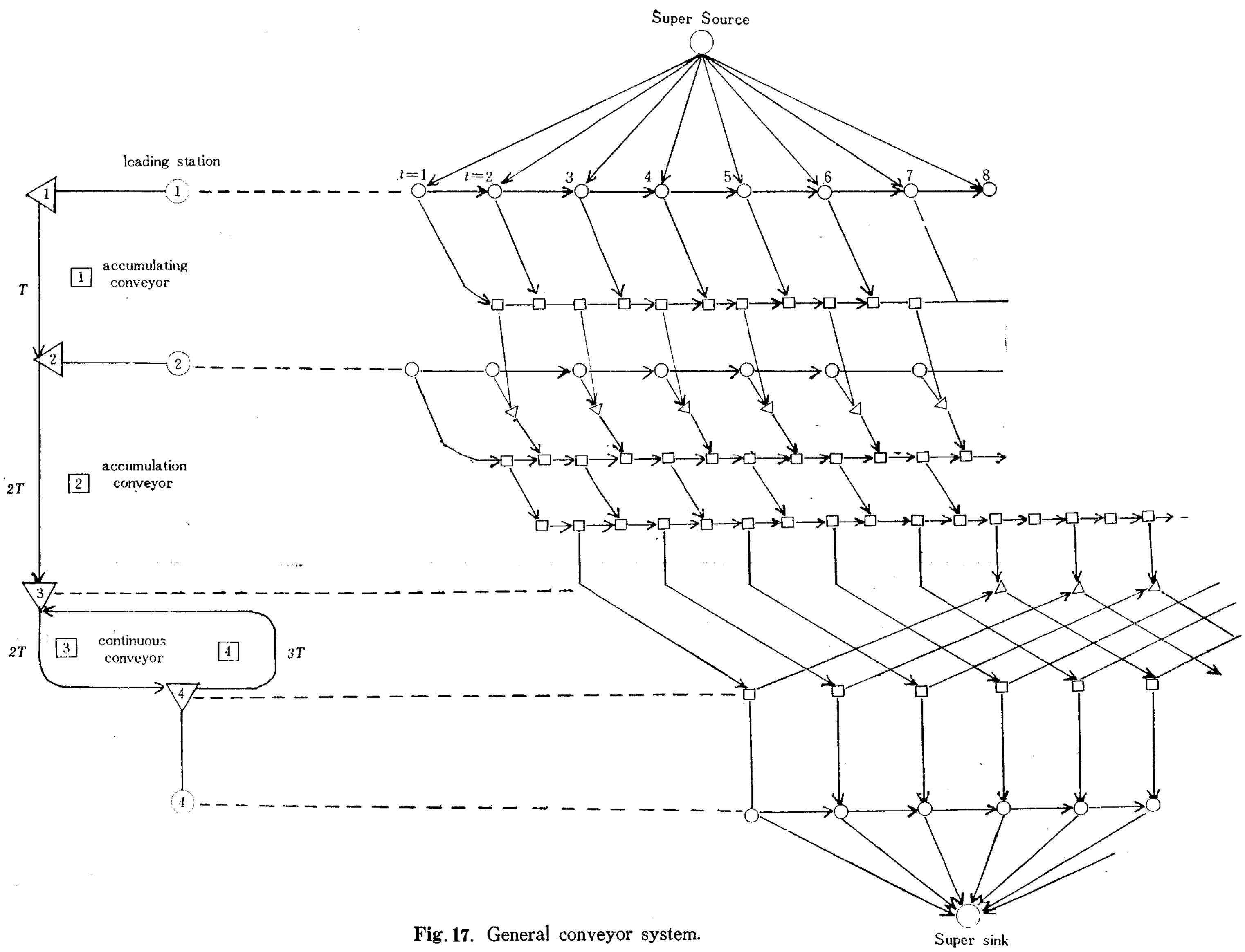

Fig. 17. General conveyor system.

(Continuous conveyor section)

Substituting $G(i, t)=0$ in the constraints of the accumulating conveyor section, the flow equations for the continuous conveyor section can be derived.

Example (Conveyor system)

In Figure-17, a general conveyor system with its flow-accumulation network is plotted.

V. CONCLUSION

Our principal aim in this work has been to derive the characteristic description of the dynamic behavior of conveyor systems.

(1) The dynamic behavior of the flow in the conveyor system can be characterized by the following linear relationships: (a) Flow equations, (b) Merge equations, (c) Capacity constraints, (d) Availability constraints.

(2) For the continuous conveyor, availability constraints are automatically satisfied by the flow equations.

(3) Depending on the time-discretization, availability constraints for the accumulating conveyor can be derived explicitly.

(4) With an appropriate choice of time-slice the general conveyor system can be represented as a pure network which is characterized by a dynamic flow network.

(5) The dynamic flow network is an exact representation of a time-dependent system with transportation and storage functions.

The network model developed can be solved by an efficient pure network algorithm such as Out-of-Kilter. Our network model may be applicable to other systems which have a storage and transportation function,

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