Journal of the Korean Oeprations
Research Society
Vol. 7, No.1, May, 1982

A General Equilibrium Growth Model for Korea

Sehun Kim*

Abstract

A general equilbrium growth model is constructed to analyse the income distributional impacts of the 1973 world oil crisis upon the Korean economy. Our results show that all consumer groups experienced a virtually uniform percentage reduction in their income levels. This implies that a lower income groups may be more damaged in a relative sense. In any rate, there were no domestic groups who benefit from the international oil crisis. This model could be used to figure out which social group will be most vulnerable against the next possible oil crisis we may expect in the future. Our result may provide us a guideline for a compensating program to protect such groups against an external shock.

1. Introduction

There is a continual interplay between policy issues and the tools of analysis in economics. During the 1950's and early 1960's, economic development was viewed largely as a problem of economic efficiency. It might be debated whether the best way to increase the GNP might lie through an investment-oriented or through an export-oriented strategy. In either case, these issues could be viewed as an economy-wide planning problem——from the perspective of a single decision-maker. Specific numerical solutions could then be obtained through the newly developed tools of linear and nonlinear programming——and also through con trol theory. For a review of this literature, see *e.g.* Blitzer *et al.*, (1975).

Increasingly, it has become apparent that there are topics in political economy which cannot be approached from the viewpoint of a single decision-maker. Distributive economics is a key issue. The empirical evidence now indicates that in the process of development a number of groups in fact lose ground in relative and perhaps even in absolute terms. (See Chenery, *et. al.*, 1974).

An optimizing model——or a planning model——is not well suited to situations where many agents independently maximize their profits or their own utility functions. Except for the centrally planned economies, most of the countries of the world are characterized by mixed economies in which a great deal of economic activity is outside the direct control of policy makers. A decentralized economy is not based upon the single-person paradigm, but rather

* Department of Management Science, Korea Advanced Institute of Science and Technology

takes explicit account of individual classes of consumers, their resources and their individual preferences. Optimization models usually do not fully describe the characteristics of decentralized economies.

In the solution of an optimization problem, the total value of inputs will be equal to the total value of final output. Thus an overall budget constraint is automatically satisfied. However, one cannot in general expect the solution of an optimization problem to be consistent with the income and budget of each agent. Nothing guarantees that the budget constraints of the individual agents in economy are satisfied. The essence of the general equilibrium model is the reconciliation of maximizing decisions made separately and independently by various agents in an economic system.

In this paper we shall report on numerical results for an example of general equilibrium model: a dynamic multisector growth model of Korea. The world oil price in constant purchasing power has been tripled during 1973—1976 and stayed virtually constant afterwards until the second oil crisis in 1979. This model is developed to analyse the growth and distributional impacts of this external shock to the Korean economy.

2. Definition of a Competitive Equilibrium

This is a Walrasian economy containing F firms, H households, and L commodities. These are indexed by f, h, and l, respectively. Initially, each consumer group is endowed with a vector of commodities, $b_h=(b_{h1},\ldots\ldots,b_{hL})$, and a vector of shares within firms, $s_h=(s_{h1},\ldots\ldots,s_{hF})$. Preferences are represented by a concave, nonnegative utility function, $u_h(.)$. The technology available to firm f is represented by a convex production set Y_f, in which inputs are represented by positive quantities and outputs by negative quantitites. It is assumed that disposal is costless.

A competitive equilibrium is defined by an L-dimensional price vector p^*, a set of consumption vectors, $(x_1^*,\ldots\ldots,x_H^*)$, and a set of production vectors, $(y_1^*,\cdots\cdots y_F^*)$, as follows:

(1) Profit maximization:

$$\underset{y_f\in Y_f}{\text{Max}}(-p^*y_f)=-p^*y_f^*\equiv\rho_f\ (p^*),\ \text{for all } f$$

(2) Utility maximization: x_h^* is a solution of

$$\begin{aligned}&\text{Max } u_h(x_h)\\&\text{s.t. } p^*x_h\leq p^*b_h+\rho(p^*)s_h\\&x_h\geq 0,\end{aligned}$$

(3) Quantity consistency:

$$\sum_h x_h^*+\sum_f y_f^*\leq\sum_h b_h$$

3. Computational Algorithm

As a qualitative description of a general equilibrium model of economy, the Walrasian competitive equilibrium model is already in its second century of intellectual life. As a quantitative tool, however, its origins are much more recent. It was not until the fixed-point work of Scarf (1973) that rigorous numerical techniques became available for solving problems of this type. For a review of subsequent contributions along these lines, see Scarf (1977).

To our knowledge, all of the fixed-point algorithms have been limited to small-scale numerical examples. This has been satisfactory for expository purposes, but not for policy applications. Larger numerical problems have therefore been solved by a variety of heuristics——including tâtonnement, Jacobi iterations, linear and nonlinear complementarity problems and sequences of linear programs. See e.g. Ginsburgh and Waelbroeck (1975) and Dervis, de Molo and Robinson (forthcoming, Appendix B).

Manne,Chao and Wilson(1980) developed another heuristic approach for "piecewise linear" economies——in which production and utility functions were restricted to those that could be described in terms of linear activity analysis. Chao, Kim and Manne (1982) extended this procedure to allow for the possibility of nonlinear production and utility functions. For a detail description of the algorithm, see Chao, Kim and Manne(1982) or S. Kim (1981). The following model has been solved using this algorithm.

4. A Dynamic Model for Korea

This is a recursively dynamic growth model for Korea. The overall dynamic model is partitioned into a static single period equilibrium model and a separate between period model which provides the necessary dynamic linkages and shifts the sectoral supply and demand functions. Profit maximization occurs in each year, with factor demands responding to present prices.

Here, there is a 12-year planning horizon——divided into four three-year periods. Thus the time index $t=0,1,2,3$. The year 1973 (just before the oil crisis) is chosen as the benchmarking year. Hence, the planning horizon is from 1973 to 1982. The base year data is based upon the PROLOG model orignally constructed by Inman *et al.*, (1977).

4.1 Production and factor use

All rural production processes are aggregated into one sector, and urban production processes are disaggregated into six sectors (Table 4.1). In each time period, each sector produces a single output by employing a combination of intermediate inputs from other sectors, and two substitutable primary factors (capital and labor). Intermediate inputs are determined by fixed coefficients per unit of output.

Capital and labor are viewed as the primary inputs to production, The base-year data show wide differences between the wages of agricultural and urban labor, but small variations in the rate of return to capital between sectors. We have therefore assumed perfect mobility of capital, but no mobility of labor between the urban and rural sectors. Hence there is one type of capital and two types of labor——agricultural and urban.

To allow for the continuation of existing wage differentials between urban sectors and also differentials in the rate of return to capital, we have imposed both a quantity constraint and a value constraint for urban labor and capital. These primary fators are measured in physi-

Table 4.1 Classification of Industries

Rural sector:

(1) AG; agriculture, forestry and fisheries.

Urban sectors:

(2) LE; labor-intensive, export-oriented manufactures,

(3) KE; capital-intensive, export-oriented manufactures,

(4) LD; labor-intensive, domestic-oriented manufactures,

(5) KD; capital-intensive, domestic-oriented manufactures,

(6) SV; services and social overheads,

(7) CN; construction.

cal units in the quantity constraint for each sector, and in terms of value units (determined by the base year wage and capital rental rates) in the value constraint.

Urban and rural labor each grow at an exogenously specified rate. One possible extension would be to base the labor migration rates upon relative wage differentials as in Harris and Todaro (1970). We have assumed "Harrod-neutral" productivity improvement for each type of labor. This allows us to express labor in terms of "efficiency units".

For simplicity, it is supposed that capital depreciates at a constant geometric rate.

The value-added from capital and labor is determined through a Cobb-Douglas production function for each sector.

4.2 Household demands

Private consumers are aggregated into three groups in each time period:

(1) farmers: owner-operators plus hired laborers in the rural sectors,

(2) urban laborers: wage and salary earners in the urban sectors,

(3) urban entrepreneurs: entrepreneurs in the urban sectors.

All returns to agricultural capital and labor belong to group 1. All returns to urban labor are assigned to group 2. Capital returns in the urban sectors are assigned to group 3.

Each private consumer group (household h=1, 2, 3) has a direct "addilog" utility function (Houthakker, 1960) of the following additively separable form:

$$u_h(x_{1h}, \cdots\cdots, x_{7h}) = \sum_{i=1}^{7} a_{ih} x_{ih}^{c_{ih}}$$

where x_{ih} denotes the quantity of the commodity produced by sector i, consumed by an individual in income group h. The coefficients a_{ih} and c_{ih} are derived from the base-year data for each consumer group. This benchmarking will now be described.

Frisch (1959) showed that if a utility function is separable, then for a given "money flexibility" (Money flexibility is the elasticiy of the marginal utility of income with respect to income) all price elasticities are determined from the income elasticities and expenditure shares. Hence the income elasticities, expenditure shares and money flexibility are sufficient data to determine a separable utility function. The assumption of additive separability simplifies the problem of parameter estimation, but in this case there can be no inferior goods. Benchmarking on the base year data, the coefficients a_{ih} and c_{ih} may be derived as follows:

$$c_{ih}=1+\frac{\omega_h}{\eta_{ih}},\ \text{and}$$

$$a_{ih}=\frac{p_i}{c_{ih}}\left(\frac{\theta_{ih}I_h}{p_i}\right)^{1-C_{ih}}$$

for all i and h, where

ω_h: money flexibility for consumer h,

η_{ih}: income elasticity of commodity i for consumer h,

θ_{ih}: expenditure share of commodity i for consumer h,

I_h: income of consumer h,

p_i: price of commodity i.

4.3 Government

Basically, there are two different sources of government revenue——indirect taxes and tariffs, and direct taxes. The latter are collected from private consumers and the the former from industries and importers.The government is viewed as a producer as well as a consumer. As a producer, the government provides "government services" to industries and importers. As a consumer, the government is simply a spending unit with its direct taxes based on a share of the incomes accruing to private consumers. We assume that in its role as a consumer, the government has a Cobb-Douglas utility function.

4.4 International trade

This model is not intended for comparative advantage calculations on individual commodities. We have therefore represented all competitive imports by a single activity. The import-mix is assumed to be the same in the future as in the base year. Exports from individual sectors are also expressed in base-year proportions.

For each period t, there is a row that measures the inputs and outputs of foreign exchange from competitive and "noncompetitive" imports and exports. The 'resource gap' between export earnings and import expenditures is to be financed through exogenously specified foreign capital inflows. The returns from these inflows are distributed to the owners of agricultural and urban capital. Thus, at each stage in formulating this type of model, it is essential to be explicit about the ownership of physical and financial assets.

5. Numerical results

Table 5.1 shows the key assumptions about growth parameters. The benchmark numerical data are provided by Roger Norton, Development Research Center, World Bank.

The "no surprise" scenario assumes that during the whole planning horizon the wolrd oil price in constant purchasing power has been stayed at the same level before the oil crisis. Figures 5.1—5.3 show the resulting gross national product, investment and consumption of individual groups. Note that the growth rates tend to be smooth.

The 1973—82 growth rate of GNP is projected at 6.3%/year——substantially lower than the actual GNP growth rate of 10.4%/year during 1970—77 (World Development Report, 19

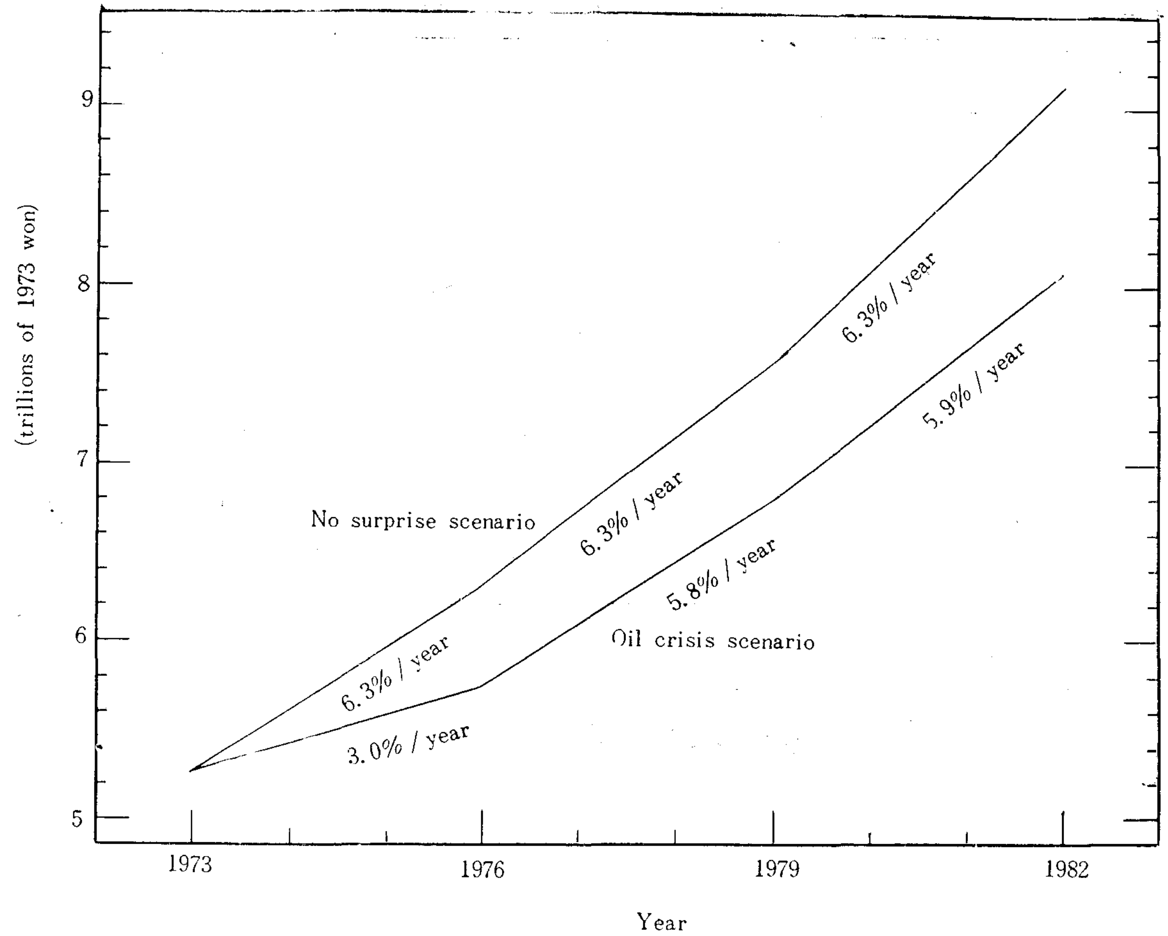

Figure 5.1 Gross National Product

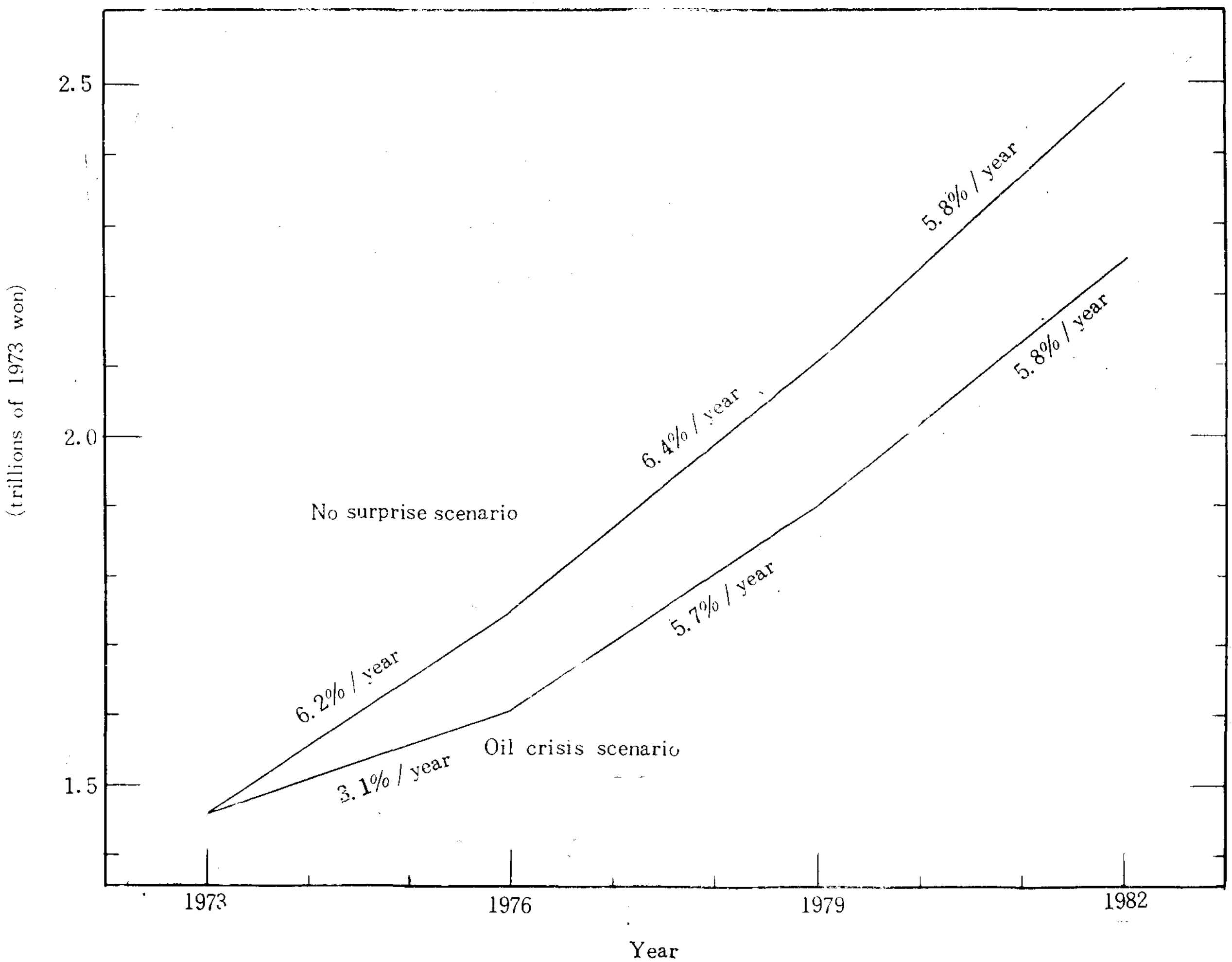

Figure 5.2 Investment

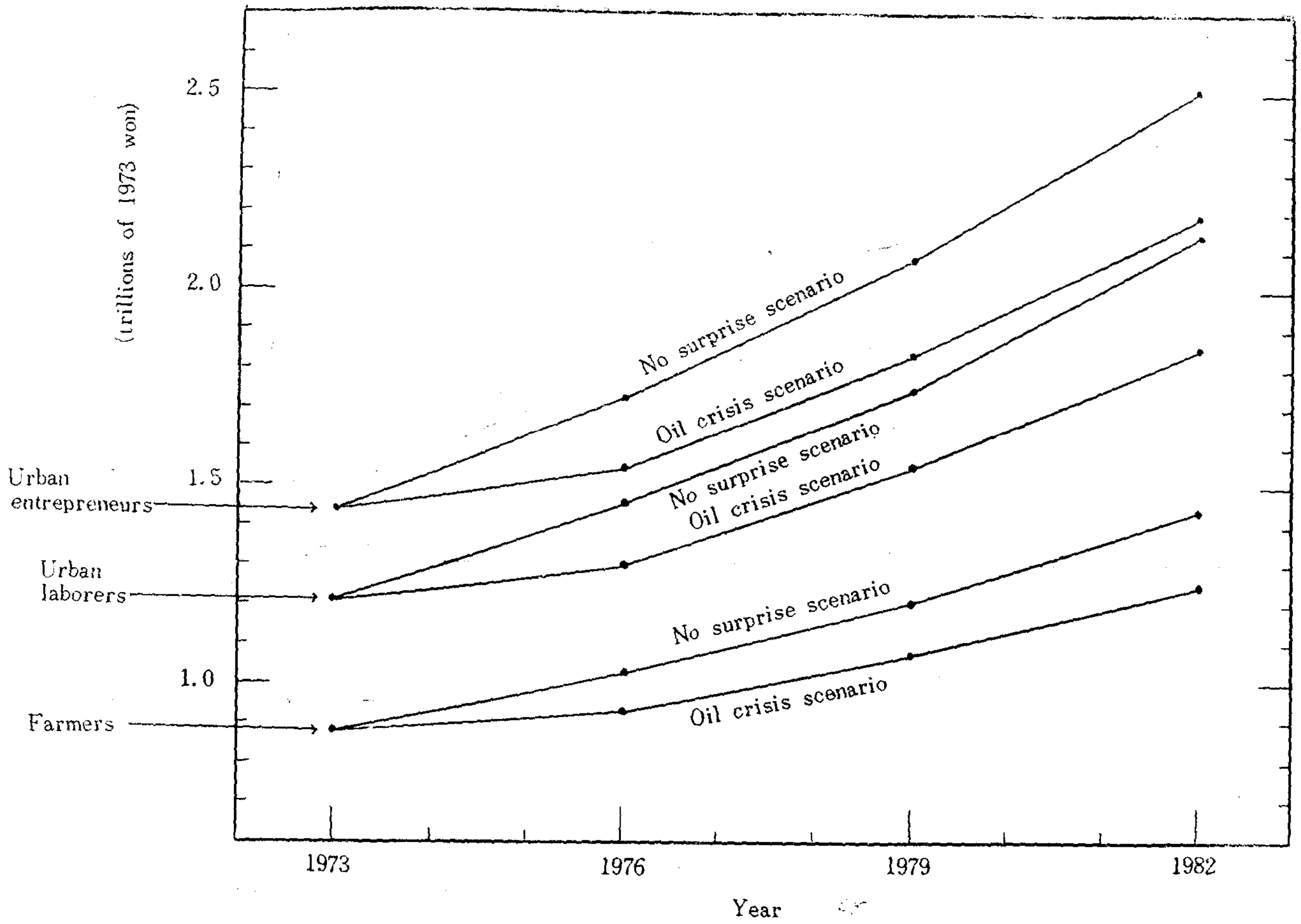

Figure 5.3 Consumption of individual groups

79). Consumption of each consumer group grows virtually uniformly over the planning horizon. Total private consumption grows at 6.2%/year which is slightly lower than the actual rate of 6.9%/year during 1970——77. This difference may be due to an underestimate of the government's non-price incentives to exporter during this period. Altogether, this is a "no surprise" scenario for a smoothly growing economy. This is the course of the economy based on our particular choices of labor growth rates, productivity growth rates and capital depreciation rates and our implicit assumptions about export markets and capital inflows. To replicate the high growth rates in GNP, it may be necessary to include the government's non-price incentives to exporters.

Table 5.1 Key Assumptions for the Korean Model

Labor Force Growth Rates :	
Rural Sectors	1%/year
Urban Sectors	3%/year
Productivity Growth Rates (Harrod-neutral):	
Rural Sectors	3%/year
Urban Sectors	4%/year
Capital Depreciation Rate	4%/year
Capital Inflow Growth Rate	5%/year

The world oil price in constant purchasing power has been tripled during 1973—1976 and stayed constant afterwards until the second oil crisis in 1979. Our "oil crisis" scenario analyses the impact of the oil crisis upon the Korean economy. The adjustment of the production and sectoral consumption patterns (not the savings patterns) to an external shock is simulated using this model. Since oil import is aggregated in the noncompetitive imports in our model and oil import constitutes 30% of the total noncompetitive imports at base year, we approximately assumed that the price of noncompetitive imports has been doubled.

Figures 5.1—5.3 shows the results comparing with the "no surprise" scenario. The reduction in GNP growth rate due to the oil crisis is 3.3%/year, 0.5%/year and 0.5%/year, respectively, during 1973—1976, 1676—1979, and 1979—1982. Most of the reduction occurs during the first period. The GNP growth rate is only 3.0%/year during 1973—1976. After adjusting the economy to the high world oil prices, the Korea was able to achieve 5.8%/year of growth rate. Similar patterns are observed for investment and consumption.

Table 5.2 shows the impact of the oil crisis upon the income distribution of the economy. For example, farmers experienced 9.7% reduction in their income in 1976 due to the oil crisis. Our results show that all consumer groups experienced a virtually uniform perc entage

Table 5.2 Reduction (%) in individual consumption due to oil crisis

	1973	1976	1979	1981
1. Farmers	0.0	9.7	10.7	12.0
2. Urban laborers	0.0	11.0	13.0	13.6
3. Urban entrepreneurs	0.0	10.4	11.5	12.0

reduction in their income levels. In terms of income levels, urban laborers were slightly more worse off than farmers.This is due to the greater direct effects of the high oil prices upon the urban sectors than the agricultural sectors. The indirect effects make only a small difference on the income distribution.

However, percentage income level reduction does not measure the real amount of "damages" they experienced. Since we assumes only the existence of an ordinal (not cardinal) utility functions for each consumer groups, it is not possible to compare the "happinesses" of two different income groups. As a result, we report only the changes of income levels. In a very crude sense, we may be able to say that a lower income group may be mere damaged when all groups experienced a uniform percentage reduction in their income levels. In any rate, there were no domestic groups who benefited from the international oil crisis. The oil crisis was a pain for everyboby in our country.

This model could be used to figure out which social groups will be most damaged from the next possible oil crisis we may expect in the future. Our result may provide us a guideline for a compensating program to protect such groups against an external shock.

Appendix: Algebraic Formulation of Korean Model

In formulating this model, the following conventions are employed:

(1) Exogenously specified parameters are denoted either by barred variables or by Greek letters,

(2) All endogenous variables are indicated by capital letters with one or more subscripts. The subscripts distinguish:

(1) 3 income groups (subscript h),

(2) 7 sectors (or outputs) (subscripts i or j).

A. 1 Material balance equations

A. 1. 1 Consumable and investible goods

These constraints ensure a balance between supplies and demands for all sectors. Supplies include domestic production and competitive imports. Demands include interindustry, private and government consumption, investments and exports.

$$\sum_j \overline{a_{ij}} X_j - \sum_{h=1}^{3} C_{ih} - (GCD_i + \overline{g_i} GCI) - \overline{v_i} IN - \overline{e_i} EX + \overline{m_i} MC \geqq 0,$$

where

$\overline{a_{ij}}$: output coefficients (+) or inputs (−)in the (I–A) matrix of interindustry transactions per unit of gross output,

X_j: gross output in sector j,

C_{ih}: quantity of commodity produced by sector i, consumed by income group h,

GCD_i: government consumption, commodity i, based on direct tax revenues,

$\overline{g_i}$: proportion of government consumption spent on commodity i,

GCI: government consumption based on indirect tax revenues,

$\overline{v_i}$: proportion of investment originating in sector i,

IN: total investment,

$\bar{e}_i$: proportion of total exports originating in sector i,

EX: total exports,

$\bar{m}_i$: proportion of competitive imports originating in sector i,

MC: total competitive imports.

A. 1. 2 Factor use

$$L_1 \leqq \overline{LA},$$

$$\sum_{j=2}^{7} L_j \leqq \overline{LU},$$

$$\sum_{j=2}^{7} \frac{\bar{w}_i}{\bar{w}_u} L_j \leqq \overline{LU},$$

$$\sum_{j=1}^{7} K_j \leqq \bar{K},$$

$$\sum_{j=1}^{7} \frac{\bar{r}_j}{\bar{r}} K_j \leqq \bar{K}.$$

where

L_j: labor inputs employed in sector j,

K_j: capital inputs employed in sector j,

$\bar{K}$: capital stock available,

$\overline{LA}$: agricultural labor available,

$\overline{LU}$: urban labors available,

$\bar{w}_j$: actual wage in sector j in base year data,

$\bar{w}_u$: average urban wage in base year data,

$\bar{r}_j$: actual rental rates in sector j in base year data,

$\bar{r}$: average rental rates in base year data.

A. 1. 3 Value added

$$Y_j = \bar{A}_j K_j^{\alpha_j} L_j^{1-\alpha_j}, \text{ for all } j,$$

where

Y_j: value-added by capital and labor inputs, sector j,

α_j: capital value share, sector j,

$\bar{A}_j$: productivity coefficient, sector j.

A. 1. 4 Noncompetitive imports

$$\sum_{j=1}^{7} \bar{n}_j X_j \leqq MN,$$

where

$\bar{n}_j$: noncompetitive imports input-output ratio, sector j,

MN: noncompetitive imports.

A. 1. 5 Government services

$$\sum_{j=1}^{7} \tau_j X_j + (\bar{t}_n MN + \bar{t}_c MC) = GS,$$

where

τ_j: indirect tax rate, sector, j,

$\bar{t}_n$: tariff rate, noncompetitive imports,

$\bar{t}_c$: tariff rate, competitive imports,

GS: government services.

A. 1. 6 Balance of payments

$$(1-\overline{t}_c)MC+(1-\overline{t}_n)MN-EX\leqq\overline{CI}$$

where

$\overline{CI}$: capital inflows (exogenous).

B Income Equations for consumer groups.

B. 1 Farmers $(h=1)$

Total consumption $=\sum_{j=1}^{7}P_jX_{j1}=(1-\overline{s}_1)(1-\mu_1)(W_1\overline{LA}+R\overline{KA})$

where

P_j: price of output in sector j,
W_1: agricultural wage rate,
R: rate of return to capital,

$\overline{KA}$: capital in agricultural sector,
μ_1: income tax rate for farmers,

$\overline{s}_1$: savings rate for farmers.

B. 2 Urban laborers $(h=2)$

Total consumption $=\sum_{j=1}^{7}P_jX_{j2}=(1-\overline{s}_2)(1-\mu_2)W_u\overline{LU}$

where

W_u: urban wage rate,
μ_2: income tax rate to urban laborers,

$\overline{s}_2$: savings rate for urban laborers,

B. 3 Entrepreneurs $(h=3)$

Total consumption $=\sum_{j=1}^{7}P_jX_{j3}=(1-\overline{s}_3)(1-\mu_3)R\overline{KU}$

where

$\overline{KU}$: capital in urban sectors,
μ_3: income tax rate to entrepreneurs,

$\overline{s}_3$: savings rate for entrepreneurs.

B. 4 Investment

$IN=\overline{s}_1(1-\mu_1)W_1\ \overline{LA}+R\overline{KA}+\overline{s}_2(1-\mu_2)\ W_u\ \overline{LU}+\overline{s}_3(1-\mu_3)R\ \ \overline{KU}=$ Total savings

B. 5 Government revenue from income tax

$$REV=\mu_1\ [W_1\ \overline{LA}+R\ \overline{KA}]+\mu_2\ W_u\ \overline{LU}+\mu_3\ R\ \overline{KU}.$$

C. Dynamic Linkage Model (three years per period)

C. 1 Labor (efficiency unit)

$$\overline{LA_t}=(1+\overline{g}_a)^3(1+\gamma_a)^3\ \overline{LA_{t-1}}$$

$$\overline{LU_t}=(1+\overline{g}_u)^3(1+\gamma_u)^3\ \overline{LU_{t-1}}$$

where

$\overline{g_a}$: rural labor annual growth rate,

γ_a: rural labor annual productivity growth rate,

$\bar{g}_u$: urban labor annual growth rate,

γ_u: urban labor annual productivity growth rate.

C. 2 Capital

$\overline{K_t} = (1-\delta)^3\overline{K_{t-1}} + 3\, IN_{t-1}$

where

δ: capital depreciation rate.

C. 3 Capital inflows

$\overline{CI_t} = (1+\zeta)^3\overline{CI_{t-1}}$

where

ζ: capital inflow growth rate.

REFERENCES

Blitzer, C.R., P.S, Clark, and L. Taylor (1975), *Economy-Wide Models and Development Planning*, Oxford University Press, London.

Chao, H., S. Kim, A. Manne (1982), "Computation of Equilibria for Nonlinear Economies: Two Experimental Models", *Journal of Policy Modeling*, 4(1):23-43.

Chenery, H. B., M.S. Ahluwalia, C.K.G. Bell, J.H. Duloy and R. Jolly (1974), *Redistribution with Growth*, Oxford University Press, London.

Dervis, K., J. de Melo and S. Robinson, forthcoming, *Planning Models and Development Policy*, unpublished manuscript.

Frisch, R. (1959), "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors", *Econometrica*, 27.

Ginsburgh, V. and J. Waelbroeck (1975), *A General Equilibrium Model of World Trade, Part I, Full Format Computation of Economic Equilibria*, Cowels Foundation Discussion Paper No. 412, Yale University.

Harris, J.R. and M.P. Todaro (1970), "Migration, Unemployment and Development: A Two-sector Analysis", *American Economic Review*, March.

Houthakker, H.S. (1960), "Additive Preferences", *Econometrica*, 28.

Inman, R., M.J. Kim, Y.H. Kim and R. Norton, (1977), "A PROLOG Model for Korea", unpublished manuscript.

Kim, S. (1981), *General Equilibrium Models: Formulation and Computation*, Ph. D. Dissertation, Operations Research Department, Stanford University.

Manne, A.S., H. Chao and R. Wilson (1980), "Computation of Equilibria by a Sequence of Linear Programs", *Econometrica*, November.

Scarf, H. (1973), *The Computation of Economic Equilibria*, Yale University Press, New Haven.

Scarf, H. (1977), "The Computation of Equilibrium Prices: An Exposition", *Coweles Foundation Discussion Paper No. 473*, Yale University.