Laminar Convective Heat Transfer in Vertical Square Duct with Variational Symmetric Heat Flux

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An analysis of convection, in a fully developed laminar steady flow through the vertical square duct under the condition of variational symmetric heat flux. is considered. Finite element solution algorithm by Galerkin's method with triangular elements and linear interpolation polynominals for the temperature and velocity profiles are derived for the vertical square duct.

The comparison of temperature distribution due to variational symmetric heat flux in the duct were made with available the other datas when the condition of peripheral heat flux were uniform and zero. Numerical values for the dimensionless temperatures and Nusselt numbers at selected Rayleigh numbers and pressure gradient parameters were obtained at a few nodal points for the vertical square ducts and effects of corner in the duct were investigated.

Introduction

The problems of incompressible laminar steady flow convection in vertical duct, under the condition of constant axial and variational symmetric peripheral wall heat flux. has important application in heat exchanger and heat transmission where design consideration may make duct with successful production. Although numerious studies on this problems have been conducted both theoretically and experimentally, the analytical solutions have been confined to ducts under condition uniform peripheral wall temperature(Han, 1959) and heat flux(Ayglawala, 1969). For more complicated boundary conditions, where analytical solutions are not possible, recent development in numerical techniques suggest that it can be best handled by means of the finite element methods(FEM) for the zero heat flux(Giudice, 1978) and uniform heat flux (Nayak, 1975).

Recently, the problem of laminar convective flow for the vertical circular duct under the condition of variational peripheral heat flux (Choi, 1981) was studied by means of the finite element method. In this paper, finite element solution algorithm with triangular elements and piecewise linear interpolation polynominals for the temperature and velocity profiles were derived for the vertical square duct with variational symmetric heat flux. The governing equations of the problem was formulated by Galerkin's method (Segerlined, 1976; Huebner, 1975).

By the specifying of the vertical square duct due to the boundary condition and other parameter of Rayleigh numbers and pressure gradients, numerical values for the temperature distributions, velocity and Nusselt numbers in the duct with varying boundary conditions were obtained. Comparison of these numerical results with the other FEM soultions, in which the

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peripheral heat flux are uniform (Nayak, 1975) and zero (Giudice, 1978), show that they have a good agreement. The behavior of the heat transfer in vertical square duct with variational symmetric heat flux due to variation of the Rayleigh numbers, pressure gradient parameters in nodal points and corner effects were suggested.

Governing Equations and Boundary Conditions

As in (Nayak, 1975), the mathematical formulation of the problem is based on the following assumptions;

1. The fluid assumed to be viscous and heat conducting and in a steady motion.
2. Fully developed velocity and temperature profiles are assumed.
3. Fluid properties are assumed to be constant, except the density is linear to and varying with temperature.
4. Frictional heating due to viscous is neglected.
5. No internal heat generation.
6. Heat input in the axial direction, i. e. Z-direction is constant.
7. Heat flux in peripheral wall is assumed to be variation.

For a fully developed incompressible laminar flow, the momentum equation (Ansari, 1970) in a Z-direction gives

$$\mu\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial y^{*2}}\right)=\rho^* g \frac{dp^*}{dz^*}. \qquad (1)$$

With the condition $u^*=v^*=0$ for the x and y direction, the assumptions (5) and (6) the energy equation becomes

$$k_f\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right)=\rho^* c_p w^* \frac{\partial T^*}{\partial z^*}. \qquad (2)$$

For a fully developed temperature profile and constant axial heat flux, the wall temperature (Shimazaki, 1951) is given by

$$T^*(x^*, y^*, z^*)=T_w^*(z^*)+t^*(x^*, y^*). \qquad (3)$$

Substituting equation (3) into equation (1) and (2) and asumming the density varies linearly with temperature, the density is of the form

$$\rho^*=\rho_w^*[1-\beta(T^*-T_w^*)]. \qquad (4)$$

We have

$$\mu\left(\frac{\partial^2 w^*}{\partial x^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)=\frac{\partial p^*}{\partial z^*}+\rho_w^* g(1-\beta t^*), \qquad (5)$$

$$k_f\left(\frac{\partial^2 t^*}{\partial x^{*2}}+\frac{\partial^2 t^*}{\partial y^{*2}}\right)=\rho^* c_p w^*. \qquad (6)$$

The boundary conditions are the form for a vertical square duct with variational symmetric heat flux

$$w^*=0 \quad k_f\frac{\partial t^*}{\partial n^*}=f^*(x^{c*}, y^{c*}). \qquad (7)$$

We shall now introduce the following dimensionless variables(Igbal, 1970):

$$x=x^*/d, \qquad y=y^*/d,$$
$$t=t^*/\rho^* c_p d^2 w_m^*/k_f, \qquad (8)$$

where

$$w_m^*=\frac{1}{A}\iint w^* dx^* dy^*$$

is the mean velocity in the Z-direction, and $d=4A/P$ in the equivalent diameter with A denoting the cross sectional area and P is the perimeter of the duct.

Equation (5) and (6) in dimensionless form are given by

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+R_a t+L=0, \qquad (9)$$

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}-w=0. \qquad (10)$$

Where L and R_a are respectively the pressure gradient parameter and the Rayleigh number given by $L\equiv -d^2[dp^*/dz^*+\rho_w^* g/\mu w_m^*$ and $R_a\equiv \rho^{*2} g c_p c\beta d^4/\mu k_f$. The boundary conditions in term of dimensionless variables are

$$w=0 \quad \frac{\partial t}{\partial n}=f(x^c, y^c). \qquad (11)$$

Application of Finite Element Method

With the triangular element, we assume piecewise linear interpolation polynominals for velocity and temperature distributions in Fig. 1. Thus we have

$$w^e(x,\ y)=\sum_{i=1}^{n}N_i(x,y)w_i=[N^e]\{w^e\} \qquad (12)$$

$$t^e(x,\ y)=\sum_{i=1}^{n}N_i(x,y)t_i=[N^e]\{t^e\} \qquad (13)$$

where $\{w^e\}$, $\{t^e\}$ and $[N^e]$ are column matrix given by

$$\{w^e\}=\begin{Bmatrix} w_i \\ w_j \\ w_k \end{Bmatrix}, \quad (14) \qquad \{t^e\}=\begin{Bmatrix} t_i \\ t_j \\ t_k \end{Bmatrix}, \quad (15)$$

$$[N^e]=\frac{1}{2A^e}\begin{Bmatrix} a_i & b_i & c_i \\ a_j & b_j & c_j \\ a_k & b_k & c_k \end{Bmatrix}\begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix}, \qquad (16)$$

where $A_e\equiv\frac{1}{2}|x_{ij}y_{jk}-x_{jk}y_{ij}|$ is the area of the triangular element and $x_{ij}\equiv x_j-x_i$, $y_{jk}\equiv y_k-y_j$, $a_i=x_jy_k-x_ky_j$, $b_i=y_j-y_k$, $c_i=x_k-x_i$ with the indices $(i,\ j,\ k)$ permute cyclily in the elements. For a formulation of the finite element method. it will be convenient to use the Galerkin's method(Segerlind, 1976; Huebner, 1975).

With the aid of the scheme, equation (10) can be written as

$$\iint_{D(e)}[N^e]^T\Big[\frac{\partial}{\partial x}\Big(\frac{\partial t^e}{\partial x}\Big)+\frac{\partial}{\partial y}\Big(\frac{\partial t^e}{\partial y}\Big)-w^e\Big]dxdy=0 \qquad (17)$$

where

$$\frac{\partial}{\partial x}\Big([N^e]^T\frac{\partial t^e}{\partial x}\Big)=[N^e]^T\frac{\partial^2 t^e}{\partial x^2}+\frac{\partial[N^e]^T}{\partial x}\frac{\partial t^e}{\partial x} \qquad (18)$$

$$[N^e]^T\frac{\partial^2 t^e}{\partial x^2}=\frac{\partial}{\partial x}\Big([N^e]^T\frac{\partial t^e}{\partial x}\Big)-\frac{\partial}{\partial x}[N^e]^T\frac{\partial t^e}{\partial x} \qquad (19)$$

Substituting equation (18) into equation (19), it can be shown that

$$\iint_{D(e)}[N^e]^T\frac{\partial^2 t^e}{\partial x^2}dxdy=\iint_{D(e)}\frac{\partial}{\partial x}\Big([N^e]^T\frac{\partial t^e}{\partial x}\Big)dxdy-\iint_{D(e)}\frac{\partial[N^e]^T}{\partial x}\frac{\partial t^e}{\partial x}dxdy \qquad (20)$$

Now, if we use the Gauss theorem (Segerlind, 1976), equation (20) can be written as

$$\iint_{D(e)}\frac{\partial}{\partial x}\Big([N^e]^T\frac{\partial t^e}{\partial x}dxdy=\oint_{s(e)}[N^e]^T\frac{\partial t^e}{\partial x}lx\,d_s^e \qquad (21)$$

Similarly, we also have

$$\iint_{D(e)}[N^e]^T\frac{\partial^2 t^e}{\partial y^2}dxdy=\oint_{s(e)}[N^e]^T\frac{\partial t^e}{\partial y}lx\,d_s^e-\iint_{D(e)}\frac{\partial[N^e]^T}{\partial y}\frac{\partial t^e}{\partial y}dxdy \qquad (22)$$

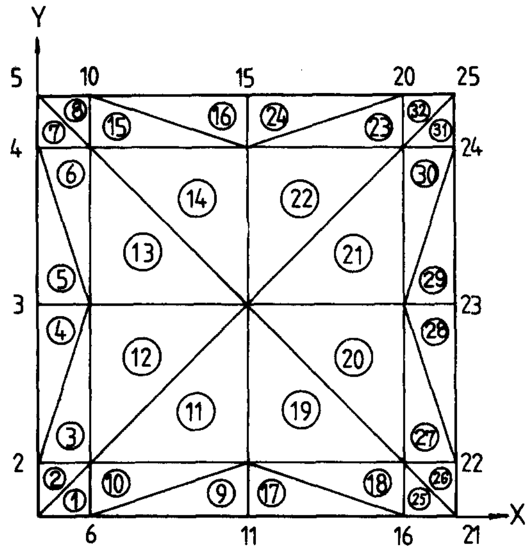

Fig. 1 Subdivision of a square duct into triangular finite elements of unequal size.

Rewriting equation (17) and simplifying gives

$$\iint_{D(e)}\Big(\frac{\partial[N^e]^T}{\partial x}\frac{\partial t^e}{\partial x}+\frac{\partial[N^e]^T}{\partial y}\frac{\partial t^e}{\partial y}\Big)dxdy+\iint_{D(e)}[N^e]^T\{w^e\}\ dxdy-\oint_{s(e)}[N^e]^T\frac{\partial t^e}{\partial n}d_s^e=0. \qquad (23)$$

Then

$$\frac{\partial t^e}{\partial x}=\frac{\partial}{\partial x}[N^e\{t^e\},\quad \frac{\partial t^e}{\partial y}=\frac{\partial}{\partial y}[N^e]\{t^e\} \qquad (24)$$

and

$$\oint_{s(e)}[N^e]^T\frac{\partial t^e}{\partial n}d_s^e=q\oint_{s(e)}[N^e]^Td_s^e. \qquad (25)$$

Similary, substitution of the same method into equation(9) with the boundary condition, $w=0$, it can be written as

$$\iint_{D(e)}\Big(\frac{\partial[N^e]^T}{\partial x}\frac{\partial[N^e]}{\partial x}\{w^e\}+\frac{\partial[N^e]^T}{\partial y}\frac{\partial[N^e]}{\partial y}\{w^e\}dxdy-R_a\iint_{D(e)}[N^e]^T[N^e]dxdy-L\iint_{D(e)}[N^e]dxdy=0. \qquad (27)$$

It will beconvenient to rewrite the equation (23) and (27) in the matrix form as

$$[F^e]\{w^e\}-R_a[M^e]\{t^e\}=L\{B^e\} \qquad (28)$$

$$[M^e]\{w^e\}+[F^e]\{t^e\}=q\{Q^e\} \qquad (29)$$

where

$$F_{ij}=\frac{b_ib_j+c_ic_j}{4A^e},\quad M_{ij}=\frac{A^e}{12},\quad B_i=\frac{A^e}{3}$$

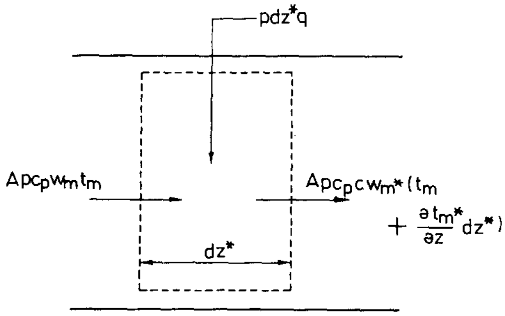

Fig. 2 Energy balance on a differential transient volume.

$$Q_i=\frac{1}{2}d_s^e \tag{30}$$

To express average temperature in term of nodal point velocities and temperatures, we note that the values can be written as

$$t_m=\frac{\sum_{e=1}^{n}\iint_{A^e} w^e t^e\,dxdy}{\sum_{e=1}^{n}\iint_{A^e} dxdy} \tag{31}$$

For the condition of peripheral constant heat flux in Fig. 2, the boundary conditions gives

$$k_f\frac{\partial t^e}{\partial n^*}=q \tag{32}$$

$$\partial t/\partial n=q/\rho^* c_p c w_m^* d \tag{33}$$

hence

$$q=A\rho^*\, c_p w_m^*\, c/p. \tag{34}$$

Substition of equation (34) into equation (33) yields

$$\frac{\partial t}{\partial n}=A/pd=0.25 \tag{35}$$

and for the variational symmetric heat flux in the vertical square duct, the values are $q=0.5(1-x)$.

We note that the Nusselt number for the duct can be written as

$$N_u=(x^e,\ y^e)=\frac{0.5f(x^e,y^e)}{\triangle t^e(x^e,y^e)} \tag{36}$$

where

$$\triangle t^e(x^e,\ y^e)=t^e(x^e,\ y^e)-t_m. \tag{37}$$

Results and Discussion

The finite element solution algorithm derived in the previous section is applicable to vertical square duct. By using the location of the interior and boundary the values of i, j, and k for the each element as well as R_a, L, and variational symmertic heat flux of temperature, velocity and Nusselt number for the duct with quarter of the cross section. To obtain the values of the duct, calculation were carried out variational heat flux, $0.5(x,y)$. The subdivision of a square duct into triangular finite elements can be done in many different ways.

But we divided into triangular elements, 25, and the duct are shown in Fig. 1(with elements of unequal size). The FEM solutions were calculated for a few selected set of parameters R_a and L taken from the datas(Choi, 1981). The result for the temperature distributions under the condition of constant heat flux(Nayak, 1975) and zero(Giudice, 1978) are compared with the analysis as the shown in Table 1 and Fig. 3. The FEM solutions have agreement in the nodal points 21 and 11.

But the values increased proportional to the heat flux at other points. In order to investigate heat transfer for the laminar covective in the vertical square duct with variational symmetric heat flux, the subdivision of the duct are shown in Fig. 1.

Numerical results for the dimensionless temperature and Nusselt number, N_u at selected Rayleigh numbers and pressure gradients were

Table 1 Comparison of the values(Nayak, 1975) **and FEM solutions for the temperatures in the square duct**(25 nodes) **for the nodal points**($R_a=100\pi^4$, $L=441.8$, $q=0.5(1-x)$)

Nodal point (Fig. 1)	FEM(25 nodes)	Nayak, 1975
1	0.1237	−0.0625
7	0.9874	−0.0420
12	3.1266	0.0830
13	1.1362	0.1182
17	−0.0781	−0.0420
21	−0.0625	−0.0625

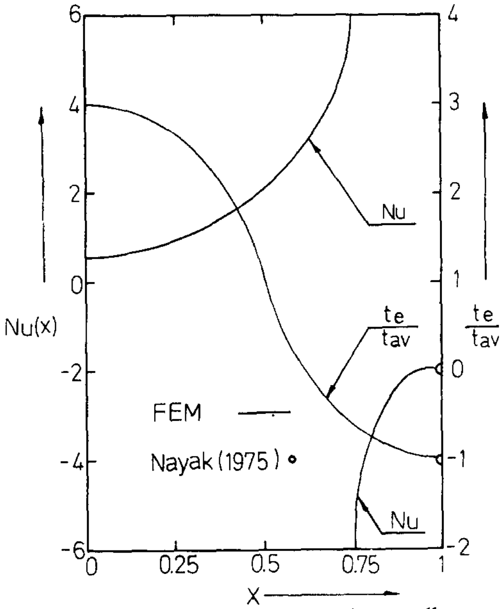

Fig. 3 Effect of circumferential heat flux variation in a square duct. (R_a=1, L=32.06)

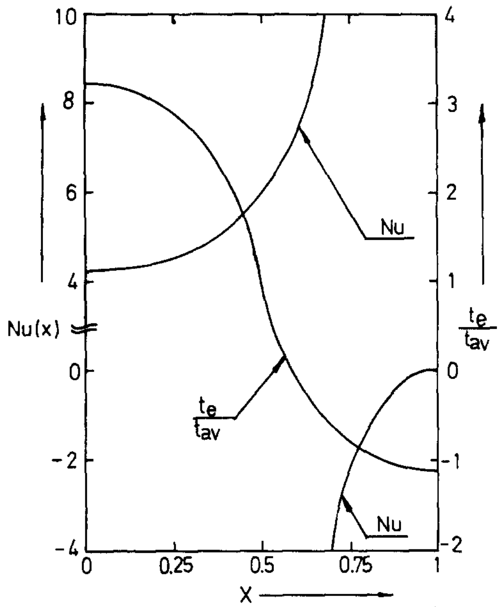

Fig. 5 Effect of circumferential heat flux variation in a squareduct. (R_a=1,000, L=85.45)

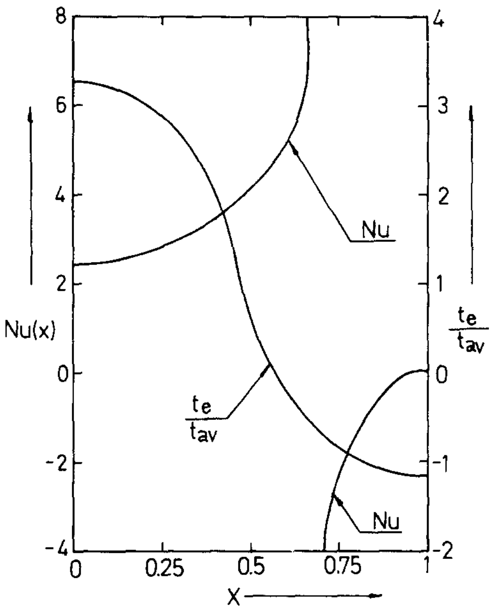

Fig. 4 Effect of circumferential heat flux variation in a square duct. (R_a=100, L=37.69)

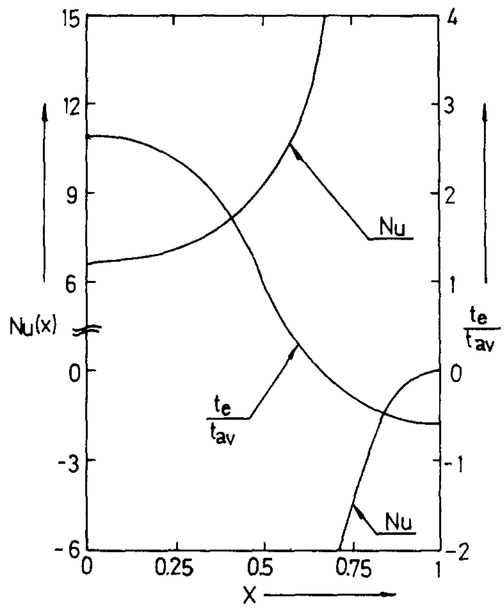

Fig. 6 Effect of circumferential heat lfux variation in a square duct. (R_a=10,000, L=441.8)

compared with zero heat flux (Giudice, 1978). As it can be seen, the values have a good agreement at R_a, 1 and L, 32.06 in Fig.3.

The temperatures and Nusselt numbers distribution over the square duct at selected R_a numbers and pressure gradient parameters are shown in Fig.4,5,6. In all cases the calculated dimensionless temperatures and Nusselt numbers were found to proportion at the selected Rayleigh number, R_a and pressure gradient, L. This tendency may be to some variable convection, some partially development laminar pressor, temperatures and heat flux in the duct. The discrepancy in the corner for the duct is of such a nature that it probably could be compensated for by the boundary layer theory due to rounding.

The temperatures and Nusselt numbers were found to lag slightly in the corner. Such a correlation, however, does not appear, if we can be able to make rounding in the limited range of the square duct.

Conclusions

From the finite element mehtod solutions in laminar convective heat transfer in vertical square duct with variational symmetric heat flux, the following results were obtained.

1. The agreement was found to be satisfactory between this solutions and other available datas when the heat flux were uniform or zero.

2. The temperature distributions vary with proportional to the heat flux and the maximum Nusselt numbers were found at near 0.7.

3. The Nusselt numbers slightly lag on decrease in duct depend on the velocity and temperature by the corner effects.

4. The dimensionless temperatures and the Nusselt numbers vary with Rayleigh numbers and pressure gradient parameters.

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Laminar Convective Heat Transfer in Vertical Square Duct with Variational Symmetric Heat Flux

非均一對稱性 熱Flux인 垂直四角 닥트內의 層流組合對流 熱傳達 效果

金 時 榮

本論文은 非均一 對稱性 熱Flux인 垂直四角 Duct內의 層流組合對流 熱傳達 效果를 解析하기 위하여 그 流動의 特性 支配 方程式 및 非均一 熱Flux의 境界條件을 無次元化 시켜 이를 Galerkin's 方法에 依해 有限要素式으로 定式化하고 이에 대하여 R_a 수 및 壓力구배 변수에 대해서 Duct 內의 溫度分布, 速度分布 및 Nusselt 수의 값을 計算하였고 溫度分布를 熱 Flux가 一定 및 없는경우와 比較하였으며 또 닥트內의 熱傳達 特性을 R_a 수, 壓力구배변수 및 Corner에 따른 변화경향을 調査하였다.

그 結果

1. 本解析의 境界壁 溫度分布 計算値와 有効資料들과의 比較에서 熱 Flux가 一定 또는 없는 경우는 그 값이 一致하였다.

2. 닥트내의 溫度分布와 Nusselt수의 값은 R_a수 및 壓力구배 변수에 比例하여 增減하였다.

3. Nusselt수는 Corner에서 流速지연에 依한 溫度分布의 特性때문에 그 값이 감소하였으며 最大値는 0.7 附近이었다.