論 文

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# Adaptive Optimal Output Feedback Control
# (適應 最適 出力 制御)

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## 要 約

이산 시간계에서 실수의 극을 갖는 다입력, 다출력 프로세스에 대하여 유용한 제어방법이 제안되었다. 이 제어방법은 적응제어와 최적 제어의 장점을 모두 가지며, 프로세스의 변수가 서서히 변한다는 가정하에서, 다이내믹스를 갖는 제어기의 설계에 응용될 수 있다. 프로세스 변수의 감식은 ARMA형태로 이루어지며 궤환 메트릭스의 최적화는 상태 변수 공간에서 이루어진다.

## Abstract

A practical and robust control scheme is suggested for MIMO discrete time processes with real simple poles. This type of control scheme, having the advantages of both the adaptiveness and optimality, may be successfully applicable to structured dynamic controllers for plants whose parameters are slowly time-varying. The identification of the process parameters is undertaken in ARMA form and the optimization of the feedback gain matrix is performed in the state space representation with respect to a standard quadratic criterion.


## I. Introduction

Strictly speaking, nearly all the real processes are nonlinear and time-varying.[11]. Nevertheless, many practical controllers are designed as if the processes under control are linear and time-invariant at its operating points, and this approach often works well. However, in case when the operating points change on purpose or due to disturbances and/or the time-varying effect of the process parameters is not negligible, the controllers must be adaptive.

For single-input single output (SISO) systems, various adaptive direct control algorithms are proposed by many researchers[12,14].

But for general multi-input multi-output (MIMO) systems, adaptive control schemes are rare and further main emphasis is given on the stability property. For example, G. C. Goodwin and his co-workers[2] have recently established an globally convergent adaptive control algorithm, but the resulting controller may not be optimal. In this paper, a method of designing an adaptive and optimal controller is suggested for a class of MIMO systems.

More specifically, for a given multivariable feedback control system with output proportional control structure, an on-line controller adjustment algorithm is given in which the adjustment is made to minimize a given cost functional. In the suggested adaptive control scheme, it is assumed that the changes of the process parameters are moderately slow so that an optimized feedback matrix at the i-th iteration stabilizes the (i+1)th identified system.

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This assumption is required to avoid the computation of an initial feedback matrix which stabilizes the overall closed loop system for each iteration[3]

In overall system configuration is shown in Fig. 1. For the adaptive optimal feedback control, it is firstly needed that a statevariable model of the controlled process be at hand. Hence it is proposed in Section II that a well-known typical identification scheme be used to obtain a matrix transfer function of the process and then a simple realization scheme be employed for the state-space model. In section III, the discrete-time optimal output feedback matrix is investigated. In section IV, an illustrative example is given to show the effectiveness of the method. In section V, some concluding remarks are provided.

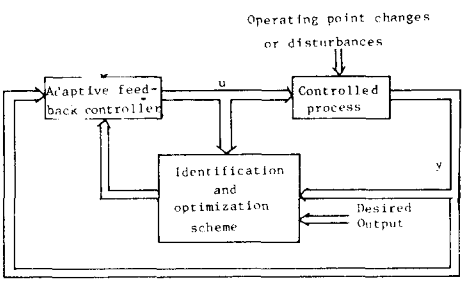


**Fig. 1.** Adaptive optimal control.

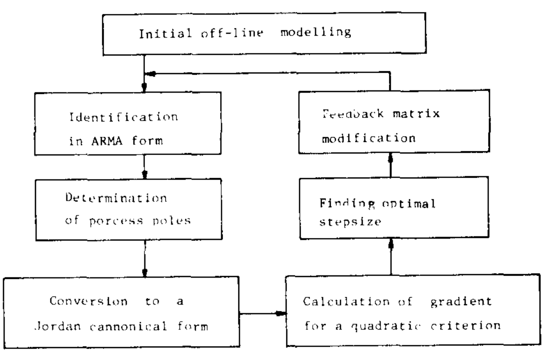


**Fig. 2.** Flow diagram of adaptive optimal control.

## II. Identification and Realization

In this section, a well-established identification algorithm is discussed and then a simple state-space realization scheme is proposed for optimal control.

Consider a linear time-invariant process whose n-dimensional state vector x(k), m-dimensional control vector u(k), and r-dimensional output vector y(k) are related by

$$x(k+1) = A\,x(k) + B\,u(k),\; x_o = x(o) \qquad (1)$$

$$y(k) = C\,x(k). \qquad (2)$$

It is well known that the ARMA form is often used in identification because it is convenient to manipulate directly the output sequence as well as the input sequence without the state information. So, in terms of unit delay operator $q^{-1}$, the system (1) and (2) is assumed to be represented as

$$\begin{bmatrix} A_1(q^{-1}) \ldots & 0 \\ & \ddots & \\ 0 & \ldots & A_r(q^{-1}) \end{bmatrix} y(k) = q^{-1} \begin{bmatrix} B_{11}(q^{-1}) \ldots & B_{1m}(q^{-1}) \\ \vdots & \vdots \\ B_{r1}(q^{-1}) \ldots & B_{rm}(q^{-1}) \end{bmatrix} \cdot u(k) \qquad (3)$$

where

$$A_i(q^{-1}) = 1 - \alpha_{i1} q^{-1} - \alpha_{i2} q^{-2} - \cdots - \alpha_{in_i} q^{-n_i},\quad n_i \le n \qquad (4)$$

$$B_{ij}(q^{-1}) = \beta_{ijo} + \beta_{ij1} q^{-1} + \cdots + \beta_{ijm_{ij}} q^{-m_{ij}},\quad m_{ij} < n_i \qquad (5)$$

for i=1, 2, ..., r and j=1, 2, ..., m.

For a specific output $y_i$, (3) can be thought of as a multiple input single-output (MISO) system, for each i=1, 2, ..., r. If we let

$$Q_i' = [\alpha_{i1}, \ldots, \alpha_{in_i}, \beta_{i1o}, \ldots, \beta_{i1m_{i1}}, \beta_{i2o}, \ldots, \beta_{imm_{im}}] \qquad (6)$$

and

$$\psi_i' = [y_i(k), \ldots, y_i(k-n_i+1), u_1(k), \ldots, u_1(k-m_{i1}), u_2(k), \ldots, u_m(k-m_{im})], \quad (7)$$

where ( )′ means the transpose, an orthogonal projection of Q(k-1) on the hypersurface defined by the equation $y(k)-\psi'(k-1)Q=0$ yields the following identification algorithm[2].

$$Q_i(k) = Q_i(k-1) + \psi_i(k-1)\,[\Delta + \psi_i'(k-1)\,\psi_i(k-1)]^{-1} \cdot [y_i(k) - \psi_i'(k-1)\,Q_i(k-1)],$$
$$\text{for } i=1, 2, \ldots, r \quad (8)$$

where $\Delta$ in $[\Delta + \psi_i'(k-1)\,\psi_i(k-1)]$ is incorporated to avoid any singular case.

Recall that, from the system equation (3), one can easily obtain the following matrix transfer function:

$$G(q^{-1}) = q^{-1}\begin{bmatrix} B_{11}(q^{-1})/A_1(q^{-1}) & \cdots & B_{1m}(q^{-1})/A_1(q^{-1}) \\ \vdots & & \vdots \\ B_{r1}(q^{-1})/A_r(q^{-1}) & \cdots & B_{rm}(q^{-1})/A_r(q^{-1}) \end{bmatrix} \quad (9)$$

Each entry of $G(q^{-1})$ is to be determined by using the algorithm in Eq. (8).

Next, in order to design an optimal feedback controller, Eq. (9) is represented by a state space model. This realization can be done using the Smith-McMillan form. While this form has the advantage of yielding a minimal realization directly as a combination of partial realizations related to the elementary matrices of rank 1, it has a drawback due to the amount of computations required[1].

In case the process has only simple real poles, one can get the Jordan form easily, using the Gilbert's method. For multiple poles, the realization may be obtained by decomposing the transfer matrix into a sum of rank one matrices. In this approach, non-minimal state equations are usually constructed first and then reduction to minimal representation is followed. For complex poles, the transformation technique given in[13] may be used.

Hence, for simplicity, we confine our interests to the processes whose poles are simple and real.

Let all the poles of $G(q^{-1})$ in Eq. (9) be $\lambda_1, \lambda_2, \ldots, \lambda_{n_o}$ where $\lambda_i > \lambda_j$ if $i > j$. Also, let $P_i$ be the rank of $M_i$, where

$$M_i = \lim_{q \to \lambda_i} (q-\lambda_i) \cdot G(q^{-1}) \quad (10)$$

Then $\sum_{i=1}^{n_o} P_i = n$ = the number of states.

**Lemma II-1**

Suppose $G(q^{-1})$ in Eq. (9) has real poles with simple order. Then the Jordan representation (A, B, C) of $G(q^{-1})$ is given as follows:

$$A = \text{diag}\{\overbrace{\lambda_1', \ldots, \lambda_1'}^{P_1}, \overbrace{\lambda_2', \ldots, \lambda_2'}^{P_2}, \ldots, \lambda_{n_o}\} \quad (11)$$

$$B = \begin{bmatrix} B_1 \\ \hline B_2 \\ \hline \vdots \\ \hline B_{n_o} \end{bmatrix}$$

$$C = \left[\begin{array}{c|c|c|c} C_1 & C_2 & \cdots & C_{n_o} \end{array}\right]$$

where $(P_i \times m)$ matrix $B_i$ and $(r \times P_i)$ matrix $C_i$ satisfy

$$C_i B_i = M_i$$

for $i=1, 2, \ldots, n_o$.

Proof:

Since each element of $G(q^{-1})$ is proper, $G(q^{-1})$ can be decomposed as,

$$G(q^{-1}) = \sum_{i=1}^{n_o} \frac{1}{q-\lambda_i} M_i \tag{12}$$

where $M_i$ is given by (10). Since $\lambda_i$'s are in simple order, one can take A as in (11).

From the relation

$$G(q^{-1}) = C(qI - A)^{-1} B,$$

one can get

$$G(q^{-1}) = \left\{ \sum_{k=1}^{n} \frac{1}{q - a_{kk}} c_{ik} b_{kj} \right\} \begin{matrix} i=1, r \\ j=1, m \end{matrix} \tag{13}$$

From equations (10) to (13), one can easily obtain

$$C_i B_i = M_i \text{ for all } i=1, 2, ..., n_o . \tag{14}$$

Solving (14) for $C_i$ and $B_i$ is not complicated. However, in real online control, there may be errors in identification and in numerical calculation. Moreover, the process itself may be higher order than the identified model. In such cases least squared-error fitting is reasonable. Here we suggest a method in which pseudo inverses are used iteratively.

**Lemma II-2**

Let $B_i$, $C_i$, $M_i$ be p x m, r x p, r x m matrices respectively, where $p \leq pi$ = rank of $M_i$ Then

(1) for a fixed $B_i$,

$C_i = M_i B_i' [B_i B_i']^{-1}$ minimizes

$\| M_i - C_i B_i \|$.

(2) For a fixed $C_i$

$B_i = [C_i' C_i]^{-1} C_i' M_i$ minimizes

$\| M_i - C_i B_i \|$ .

The proof of lemma II-2 is lengthy but straightforward.[16] So it is not detailed. Note that, by using the above results of (1) and (2) in lemma II-2 alternatively, one can get $B_i$ and $C_i$ in least squared-error sense.

In practice, the problem of finding poles of the process during on-line control must be solved. When we know the dynamics of the process completely, this may be easy. But in many cases we do not know enough about the process to predict the variations of process poles. As a solution, the following algorithm is suggested. Here the number of states in Jordan canonical form is not fixed, but it is determined by the two given error bounds $\epsilon$ and $\delta$.

The algorithm can be described as follows:

1. Find all roots of the equation $q^{n_i} A_i (q^{-1}) = 0$ for $i=1, 2, ..., r$. There will be $n_t = \sum_{i=1}^{r} n_i$ roots.
   Set $\omega_i = 1$ for $i=1, 2, ..., n_t$.
2. Check if $|\lambda_i - \lambda_j| < \epsilon$, for every $i \neq j$
   If above condition is satisfied, set $\lambda_i$ as $(\omega_i \lambda_i + \omega_j \lambda_j)/(\omega_i + \omega_j)$ and remove $\lambda_j$ and set $\omega_i$ as $\omega_i + \omega_j$.
3. For each $\lambda_i$, calculate $M_i$, using (10).
4. For each $M_i$, determine the minimal rank $P_i$ of $C_i$ and $B_i$ to satisfy the following criterion.

$$\frac{\| M_i - C_i B_i \|}{1 - |\lambda_i|} < \delta \tag{15}$$

5. Calculate the system matrices (A, B, C)

In step 2, one should set $\epsilon$ small enough so that Eq. (15) can be satisfied for the given $\delta$.

With this algorithm the maximum number of eigenvalues is $n_t$, and the number of states is the total sum of $P_i$'s.

## III. Optimization of Output Feedback Matrix

Levine and Athans[5] and many others studied the continuous-time constrained state feedback problem. For discrete time systems, explicit form of necessary condition is given in[15] which an optimal output feedback gain matrix must satisfy. In this section, a discrete-time problem of determining the output feedback gain matrix and some aspects in solving the discrete-type Lyapunov equations are discussed. A simple method in calculating the optimal stepsize is also presented.

Given the system represented by (1) and (2), let the performance measure be given by,

$$J=\frac{1}{2}\sum_{k=0}^{\infty}\left\{x'(k)\,Q\,x(k)+u'(k)\,R\,u(k)\right\} \tag{16}$$

where Q is positive semidefinite and R is positive definite. Using output feedback control,

$$u(k) = -F\,y(k) \tag{17}$$

(16) can be rewritten as

$$J=\frac{1}{2}\mathrm{tr}\left\{\sum_{k=0}^{\infty} A_F'^{k}\,\tilde{Q}\,A_F^{k}\,X_o\right\} \tag{18}$$

where $A_F = A - BFC$, $X_o = x(o)\,x'(o)$, and

$$\tilde{Q} = Q + C'F'\,RFC.$$

**Lemma III-1.**

When there exists an F for which $A_F$ is stable, the necessary condition for F to be optimal with respect to (18) is that

$$(RFC - B'MA_F)LC' = 0 \tag{19}$$

where L and M are respectively the unique positive semidefinite solutions of

$$A_F\,L\,A_F' - L = -X_o \tag{20}$$

$$A_F'\,M\,A_F - M = -\tilde{Q} \tag{21}$$

The proof of the lemma III-1 is given in[15]

With (19) one can optimize F by using the gradient or the conjugate gradient algorithms[6] However, solving the discrete type Lyapunov equation, in this way, is the most time-consuming job. The method given in[8] requires transformation to Jordan canonical form, and one in[9] needs transformation to Schur form, hence they are efficient if one want to obtain various solutions for different values of $\tilde{Q}$. But in our case, $A_F$ will change for each iteration with constant $\tilde{Q}$. So the following iterative method is suggested:

$$M_{i+1} = A_F'\,M_i\,A_F + \tilde{Q} \tag{22}$$

$$L_{i+1} = A_F\,L_i\,A_F' + X_o \tag{23}$$

It is easy to see that (22) and (23) converge when $A_F$ is an asymptotically stable matrix. This iterative algorithm, having merits of being simple to program, is capable of handling easily complex or multiple poles. Furthermore, assuming that the changes of $A_F$, and hence M, are small between one iteration of identification and optimization, the calculation time will not be large. Only one drawback of (22) and (23) is that the convergence rate is highly dependent on the moduli of eigenvalues of $A_F$.

In the iterative identification and optimization, each iteration should be completed as soon as possible. Hence in finding an optimal

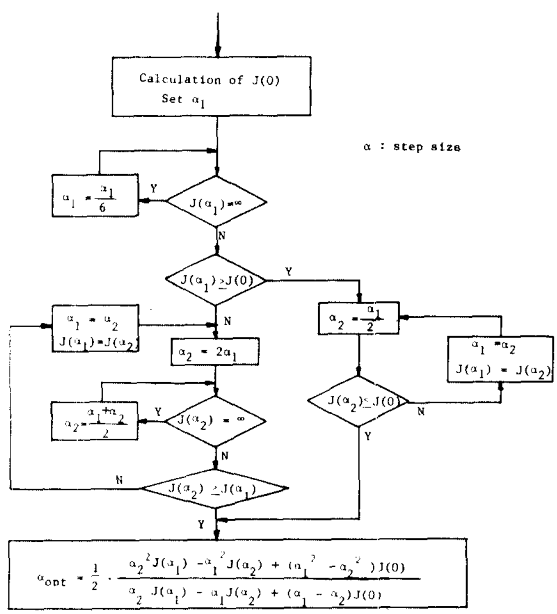


**Fig. 3.** Flow diagram of stepsize adjustment algorithm.

stepsize, a rough and fast algorithm is used, which is given in Fig. 3, which is similar to that in[10] but is far more simple. In the figure, $\alpha$ means a step size, and J is set to be function of $\alpha$. Then J(o) means the value for an initial feedback matrix.

## IV. Example

The algorithms suggested in Sections II and III are summarized in the diagram shown in Fig. 2. To show the effectiveness of the scheme, an example is provided in the following:

### Example

An output regulation problem is considered for a process whose system matrices are given by

$$A = \text{diag.}\ [.96\ .92\ .84]$$

$$B = \begin{bmatrix} .3 & .0 \\ .0 & .4 \\ .1 & .2 \end{bmatrix} \qquad C = \begin{bmatrix} 1. & 2. & 0. \\ 0. & 1. & 1. \end{bmatrix}$$

Let $I_n$ mean an identity matrix of dimension n. For the criterion J in (18), let

$$Q = C' I_2 C$$
$$R = 10 \cdot I_2$$

and

$$X_o = I_n$$

where n becomes in this example 3 or 4 depending on the number of identified states at each iteration. The optimal constant output feedback matrix is then calculated as

$$F = \begin{bmatrix} .118 & -.019 \\ .168 & .114 \end{bmatrix}$$

Suppose, during $0 < k \leq 5$, the system matrix A is smoothly changed to the following:

$$A = \text{diag.}\ [1.02\quad .976\quad .892]$$

For the initial state $x(0) = [1.77\ -.78\ 1.19]$, the results of Sections II and III are applied and the responses of the optimal constant output feedback and the adaptive optimal output feedback are compared in Fig. 4. As is obvious from the simulation result, the adaptive control regulates the process while the fixed output feedback control does not. It is found from simulations that the algorithm presented in this paper works well for characteristic value variations as well as for system gain variations.

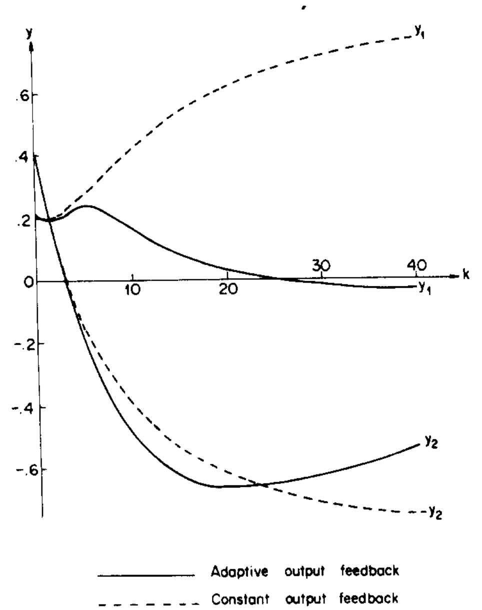


**Fig. 4.** Comparison of the responses for the example.

## V. Conclusions

A control scheme which has the properties of adaptivity and optimality is suggested for the processes with real and simple poles. Extension to general processes is briefly discussed. Some practical aspects are considered for on-line control with an approximate realization method. By only specifying error bounds, one can get an approximate realization of Jordan canonical form for a given matrix transfer function. This concept can also be applied to model reduction problems. Finally using gradient of a standard criterion function, a relatively simple feedback matrix modification algorithm is described.

It is remarked that, by introducing an augmented state vector, adaptive proportional plus integral (PI) controllers and adaptive dynamic controllers can also be designed using the method presented in this paper.

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