論 文
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다변수 스토캐스틱 선형 계통의 추정에 관한 연구

On the Identification of the Multivariable Stochastic Linear Systems

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Abstract

The problem of parameter identification for multivariable stochastic linear systems from output measurements, which are corrupted by noises, is considered.

A modified Luenberger's input/output canonical form is used for reducing the number of unknown coefficients. A computationally and conceptionally simple systematic procedure for parameter estimation is obtained using output correlation method. The estimates are shown to be asymptotically normal, unbiased and consistent.

Numerical examples are presented to illustrate the identification method.

1. Introduction

System identification has recently received much attention in the fields of engineering, economics, statistics, and the physical sciences.

The same problem is variously described as a "modeling problem" or as a "time series analysis" problem. Even though the terminology differs from one field to another, the basic methodology is very often the same.

The problem of system identification consists of three stops: model selection, parameter estimation, and model verification. A suitable model must be selected for the final identification objective, which may be the design of a control strategy for the system, the simulation of the system, or the prediction of the system response. The unknown parameters of this model must then be identified from measurement data obtained from the actual system. Finally, to see whether the estimated model is adequate for the final objective, a model validation test must be performed on the estimated model.

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接受日字：1981年 12月 2日

A number of approaches to the identification problem have been proposed in the control literature.

Mehra[1] investigated this problem assuming a single input single output state space model with fixed and known structure. Sardis and Stein[2] developed stochastic approximation algorithms for linear stochastic system identification. The maximum likelihood estimation of the coefficients of multiple output linear dynomical systems was discussed by Kashyap[3].

One major difficulties in the extension from the single output case to multi-output case is the choice of canonical forms. A canonical model of multivariable systems was introduced by Mayne[4]. Tse and Weinert[5] considered stochastic multivariable system identification with unknown structure. Parameter identification using least squares algorithms was discussed by Hsia[6]

Baram and Sandell[7] solved consistency problem of linear system identification. Suen and Liu[8] investigated structure determination of multivariable linear systems.

The main concern of this paper is the parameter identification: more specificially, the parameter identification of multioutput stochastic linear systems from output sequences which are corrupted by noises, under assumption that structure indices are estimated by[5].

Computationally and conceptionally simple parameter identification method for multivariable stochastic linear state space models is presented.

2. Statement of Problem

Consider an n-dimensional linear time-invariant system represented by

$$X(k+1)=AX(k)+Bu(k) \tag{1a}$$

$$Z(k)=CX(k)+v(k) \tag{1b}$$

where $X(k)\in R^n, Z(k)\in R^m, k=1,2,3,\cdots$ and A,B,C are constant matrices conformable to X,Z,u,R is real field, and I is identity matrix.

We assume that

$$E\{u(i)\}=0,\quad E\{u(i)u(j)^t\}=I\delta_{ij}$$

$$E\{v(i)\}=0,\quad E\{v(i)v(j)^t\}=Q\delta_{ij}$$

$$E\{u(i)v(j)^t\}=0,$$

$$E\{x(i),v(j)^t\}=0,\ \text{for all } i \text{ and } j$$

Let $\theta=\{A,B,C,Q\}$, suppose

1) The matrix A is nonsingular and stable i.e. all eigenvalues are nonzero and lie inside unit circle.
2) The system is completely controllable and observable[9].
   $\text{rank}[B,AB,\ldots,A^{n-1}B]=n$
   $\text{rank}[C^t,A^tC^t,\ldots,(A^{n-1})^tC^t]=n.$
3) The $I/0$ sequence $\{Z(k)\},\{u(k)\}$ is identifiable [10], or roughly speaking, it contains all the information of system(1).
4) The identification scheme is started after the system has reached a steady state.

The objective is to estimate θ using measurement data $Z^N=\{z(1),z(2),\ldots,z(N)\}$, and the identifiability condition was discussed[11]~[13].

A block diagram of the identification scheme is shown in Fig. 1.

3. Canonical Forms for Identification

One of the most difficulties in the multivariable system identification is the choice of canonical forms.

In this paper, a modified Luenberger's[14],[15] $I/0$ canonical form was used[16].

Given a finite dimensional linear system of order n having m outputs z and r inputs u, and the system matrices $A,B,C,$ then it is always possibie to construct an equivalent system having as A matrix, A^*, given by

$$A^*=TAT^{-1}=[A_{ij}],\ i,j=1,2,\ldots,m$$

$$\underset{(n_i\times n_i)}{A_{ii}}=\begin{pmatrix}0 & 1 & 0 & \ldots & 0\\ 0 & 0 & 1 & \ldots & 0\\ \cdot & \cdot & \cdot & \ldots & \cdot\\ a_{ii1} & a_{ii2} & a_{ii3} & \ldots & a_{iin_i}\end{pmatrix} \tag{2}$$

$$\underset{(n_i\times n_j)}{A_{ij}}=\begin{pmatrix} & 0 & \\ \cdots & \cdots & \cdots\\ a_{ij1} & \ldots & a_{ijn_j}\end{pmatrix},\ i>j \tag{3}$$

where T : observability matrix

n : structure index associated with ith input

and Gupta and Fairman[16] prove that

$$a_{ijk}=0,\ k>n_i+1$$
$$=0,\ k=n_i+1 \text{ and } i<j \tag{4}$$

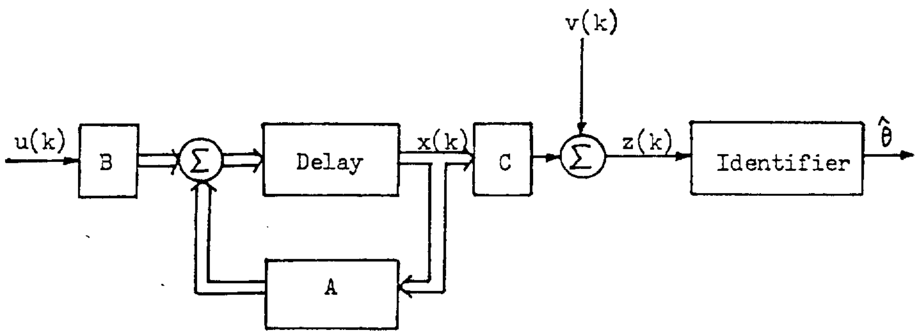

Fig. 1. A block diagram of the identification scheme

Consider the rows of the observability matrix in the following order:

$$T=\begin{pmatrix} C_1 \\ C_1A \\ \cdot \\ C_1A^{n_1-1} \\ \cdot \\ C_mA^{n_m-1} \end{pmatrix} \quad (5)$$

where C_i is the ith row of C. Since n_i is the structure index, $C_iA^{n_i}$ is linearly dependent on the vectors in all proceding rows of (5).

And

$$\sum_{j=1}^{m} n_j = n$$

From the definition of n_i, there exist a unique set a_{ijk}, such that for $i=1,2,\ldots,m$

$$C_iA^{n_i}=\sum_{j=1}^{i}\sum_{k=0}^{n_j-1} a_{ijk+1}C_jA^k, \text{ if } n_i>0$$
$$C_i=\sum_{j=1}^{i-1}\sum_{k=0}^{n_j-1} a_{ijk+1}C_jA^k, \text{ if } n_i=0 \quad (6)$$

Then the rows of canonical form of C is constructed as follows

$C_i^*=[0,\ldots,0,1,0,\ldots,0]$, $n_i>0$

where C_i^* is ith row of C^* which is canonical form of C, and the 1 in C_i^* is in column $1+n_1+n_2+\ldots,+n_{i-1}$, and

$C_i^*=[a_{i11},\ldots,a_{i1n_1},a_{i21},\ldots,a_{i2n_2},\ldots,$
$a_{i,i-1,n_i-1},0,\ldots,0]$, $n_i=0$

Let B^* be canonical form of B, which has no special form, then the elements of B^* are uniquely determined by Z^∞.

4. Estimation of System Matrices

Let P denote the covariance matrix of the states in (1) and define

$$R(k)\equiv E\{z(i)z^t(i-k)\} \quad (7)$$

An expression for $R(k)$ can be easily obtained from (1)

$$R(0)=CPC^t+Q \quad (8)$$
$$R(k)=CA^{k-1}S, \text{ for } k>0 \quad (9)$$

where

$$P=E\{X(i)X^t(i)\}=APA^t+BB^t \quad (10)$$
$$S=APC^t \quad (11)$$

Let $r_{ij}(k)$ denote the i,jth element of $R(k)$ and S_j denote the jth column of S, then (6) and (7) imply

$$r_{ij}(n_i+t)=C_iA^{n_i}A^{t-1}S_j$$
$$=\begin{cases}\sum_{l=1}^{i}\sum_{k=0}^{n_l-1} a_{ilk+1}C_lA^kA^{t-1}S_j, n_i>0 \\ \sum_{l=1}^{i-1}\sum_{k=0}^{n_l-1} a_{ilk+1}C_lA^kA^{t-1}S_j, n_i=0\end{cases} \quad (12)$$

where $t=1,2,\ldots$ Using (9) again yields

$$r_{ij}(n_i+t)=\begin{cases}\sum_{l=1}^{i}\sum_{k=0}^{n_l-1} a_{ijk+1}r_{ij}(k+t), \ n_i>0 \\ \sum_{l=1}^{i-1}\sum_{k=0}^{n_l-1} a_{ijk+1}r_{ij}(k+t), n_i=0\end{cases} \quad (13)$$

Now for the case $i=1$, (13) becomes

$$r_{ij}(n_i+t)=\sum_{k=0}^{n_l-1} a_{ijk+1}r_{ij}(k+t), t=1,2,\ldots \quad (14)$$

For $t=1,2,\ldots,n_1$, (14) is equivalent to the matrix equation

$$r_1=\Phi_1(n_1)a_1 \quad (15)$$

where

$r_1^t=[r_{ij}(n_1+1),\ldots, r_{ij}(2n_1)]$

$a_1^t=[a_{111},\ldots, a_{11n_1}]$

$$\Phi_1(k)=\begin{pmatrix} r_{1j}(1) & r_{1j}(2) & \ldots & r_{1j}(k) \\ r_{1j}(2) & r_{1j}(3) & \ldots & r_{1j}(k+1) \\ \cdot & \cdot & \ldots & \cdot \\ r_{1j}(k) & r_{1j}(k+1) & \ldots & r_{1j}(2k-1)\end{pmatrix}$$

Gantmacher[17] showed that $\Phi_i(k)$ is always nonsingular for $k\leq n_i$. Thus n can be estimated by testing the singularity of $\Phi_i(k)$ for $i=1,2,\ldots,n_i+1$[5].

We now turn to the estimation problem. Let $\hat{R}(k)$ denote an estimate of $R(k)$. Then using the ergodic property of $z(i)$, a reasonable estimate for $R(k)$ is

$$\hat{R}(k)=\frac{1}{N}\sum_{i=k}^{N} z(i)z^t(i-k) \quad (16)$$

It was shown that $\hat{R}(k)$ is an asymptotically unbiased, normal, and consistent estimate of $R(k)$. Thus $\hat{a}_1$ is estimated easily under assumption that $\hat{n}_1=n_1$

$$\hat{a}_1=\hat{\Phi}_1^{-1}(n_1)\hat{r}_1 \quad (17)$$

where $\hat{a}_1$ is strongly consistent estimate of a_1.

For $i=2,3,\ldots,m$, a_i is computed an analogous manner. For example, if $i=2$, write (13) for $t=1,2,\ldots,n_1+n_2$ as

$$r_2=\Phi_2(n_2)a_2 \quad (13)$$

where

$r_2^t=[r_{2j}(n_2+1), \ldots, r_{2j}(2n_2+n_1]$

$a_2^t=[a_{211}, \ldots, a_{21n_1}, a_{221}, \ldots, a_{22n_2}]$

$$\Phi_2(k)=\left(\begin{array}{ccc} & \Phi_1(n_1) & \\ \hline r_{1j}(n_1+1) & \ldots & r_{1j}(2n_1) \\ r_{1j}(n_1+2) & \ldots & r_{1j}(2n_1+1) \\ r_{1j}(n_1+k) & \ldots & r_{1j}(2n_1+k-1)\end{array}\right.$$

$$\begin{pmatrix} r_{2j}(1) & \cdots & r_{2j}(k) \\ r_{2j}(2) & \cdots & r_{2j}(k+1) \\ \cdot & \cdots & \cdot \\ r_{2j}(n_1+k) & \cdots & r_{2j}(n_1+2k-1) \end{pmatrix}$$

$\hat{a}_2$ is estimated from

$$\hat{a}_2=\hat{\Phi}_2^{-1}(n_2)\hat{r}_2 \tag{19}$$

$\hat{a}_2$ is strongly consistent estimate of a_2.

The procedure continues in a similarmanner until $\sum_{i=1}^{m} n_i=n$, then canonical form A and C is obtained.

5. Estimation of Noise Covariances

Using the above procedure, we can estimate A and C. In this section we shall describe an algorithm for estimation B and Q while assuming that A, C and $R(k)$ are known exactly.

A number of approaches have been suggested in the literature. We describe here an approach similar to H_0 and Kalman[18].

Begin by constructing $nm\times nm$ Hankel matrix H:

$$H=\begin{pmatrix} R(1) & R(2) & \cdots & R(n) \\ R(2) & R(3) & \cdots & R(n+1) \\ \cdot & \cdot & \cdots & \cdot \\ R(n) & R(n+1) & \cdots & R(2n-1) \end{pmatrix} \tag{20}$$

where

$$R(k)=CA^kPC^t,\ k>0$$

H can be writen as

$$H=DW \tag{21}$$

where

$$D=\begin{pmatrix} CA \\ \cdot \\ CA^n \end{pmatrix},\quad W=[PC^t, APC^t, \ldots, A^{n-1}PC^t]$$

Both D and W are of rank n by virtue of the observability and the controllability conditions.

Therefore, one can find $nm\times nm$ nonsingular matrices E and V such that;

$$ED=\begin{pmatrix} I_n \\ \cdots\cdots \\ 0 \end{pmatrix} \tag{22}$$

and

$$WV=[I_n : 0] \tag{23}$$

Then E and V reduce H to a diagonal form.

$$EHV=ED\ WV=\begin{pmatrix} I_n & \vdots & 0 \\ \cdots\cdots & \vdots & \cdots\cdots \\ 0 & \vdots & 0 \end{pmatrix} \tag{24}$$

The determination of E and V is as follows; for example, write

$$D=\begin{pmatrix} D_1 \\ \cdots\cdots \\ D_2 \end{pmatrix}$$

such that D_1 is $n\times n$ nonsingular. Then choose

$$E=\begin{pmatrix} D_1^{-1} & \vdots & 0 \\ \cdots\cdots\cdots & & \cdots \\ -E_1D_2D_1^{-1} & \vdots & E_1 \end{pmatrix} \tag{25}$$

where E_1 is any nonsingular matrix.

H_0 and Kalman show that

$$C=[I_m : 0]\ HV\begin{pmatrix} I_n \\ \cdots\cdots \\ 0 \end{pmatrix} \tag{26}$$

$$PC^t=[I_n : 0]\ EH\begin{pmatrix} I_m \\ \cdots\cdots \\ 0 \end{pmatrix} \tag{27}$$

Moreover

$$P=APA^t+BB^t \tag{28}$$

We can obtain B and P from eqs(27), (28) using iterative method[19, 20]. (see appendix)

Then Q can be obtained from (8)

$$Q=R(0)-CPC^t \tag{29}$$

6. Numerical Examples

In this section two examples are given to illustrate the identification scheme shown in sections 4 and 5. All computing was done in Fortran Ⅳ on IBM 360 digital computer.

Example 1 : This example is a single-output four dimensional system taken from Mehra[1], which has the following parameters;

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.656 & 0.784 & -0.18 & 1.0 \end{pmatrix}$$

$$B^t=[\ 0\quad 1.0\quad 0\quad 1.0\ ]$$

$$C=[\ 1\quad 0\quad 0\quad 0\ ]$$

$$Q=0.25$$

The eigen values of A are $-0.4\pm j0.8$ and $0.9\pm j0.1$. They are all located inside the unit circle i.e. stable.

Output data were generated from (1).

First, the dimension was estimated by testing determinant of $\Phi_1(k)$[5], then we can obtain $n_1=4$.

A dimension of 4 indicates the following structure for the matrices in (1) :

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{111} & a_{112} & a_{113} & a_{114} \end{pmatrix}$$

$$B^t=[\ b_1\quad b_2\quad b_3\quad b_4\ ]$$

$C=[1 \quad 0 \quad 0 \quad 0]$

$Q=q$

Next, we can estimate $R(k)$, $k=0, \ldots, 8$ from (16), then a_1 can be obtained from (17).

Table 1. Parameter estimates for example 1

Parameter	Estimates		True Values
	$N=1000$	$N=3000$	
a_{111}	−0.7475	−0.6791	−0.6560
a_{112}	0.9357	0.7960	0.7840
a_{113}	−0.1586	−0.1328	−0.1800
a_{114}	0.9090	0.9608	1.0000
b_1	0.0	0.0	0.0
b_2	0.9230	0.9252	1.0
b_3	0.0	0.0	0.0
b_4	0.9125	0.9127	1.0
q	0.2788	0.2296	0.25

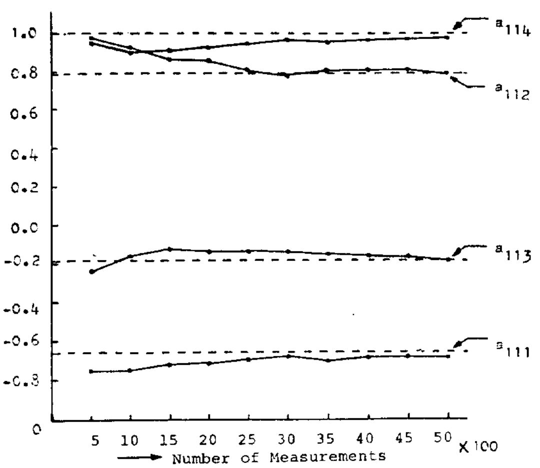

Fig. 2. Estimated values of a_1 versus N

Using these values, B was obtained from (27) and (28), and was obtained from (29).

These values obtained from 1000 measurements and 3000 measurements are shown in Table 1. And the estimated values of a_1 versus N are plotted in Fig. 2.

Example 2 : Consider a four dimensional three output system. The actual values of A,B,C and Q are

$$A=\begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -0.5 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ -1.0 & -4.0 & -0.25 & 0.0 \end{pmatrix}$$

$$B^t=[0 \quad 1.0 \quad 0 \quad 1.0]$$

$$C=\begin{pmatrix} 1.0 & 0 & 0 & 0 \\ -1.0 & -3.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \end{pmatrix}$$

$$Q=\begin{pmatrix} 0.25 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.25 \end{pmatrix}$$

The structure indices which can be obtained by[5] are; $n_1=2, n_2=0, n_3=2$.

Thus A,B,C,Q have the following structure:

$$A=\begin{pmatrix} 0 & 1.0 & 0 & 0 \\ a_{111} & a_{112} & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ a_{311} & a_{312} & a_{331} & a_{332} \end{pmatrix}$$

$$B^t=[b_1 \quad b_2 \quad b_3 \quad b_4]$$

$$C=\begin{pmatrix} 1.0 & 0 & 0 & 0 \\ a_{211} & a_{212} & 0 & 0 \\ 0 & 0 & 1.0 & 0 \end{pmatrix}$$

$$Q=\begin{pmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{pmatrix}$$

Table 2. Parameter estimates for example 2

Parameter	Estimates		True Values
	$N=1000$	$N=3000$	
a_{111}	−0.4847	−0.5058	−0.50
a_{112}	1.0401	1.0392	1.00
a_{211}	−0.9443	−1.0430	−1.00
a_{212}	−3.1656	−2.9867	−3.00
a_{311}	−1.6386	−1.3409	−1.00
a_{312}	−3.3989	−4.0306	−4.00
a_{331}	−0.3238	−0.2660	−0.25
a_{332}	0.1815	−0.0342	0.0
b_1	0.0	0.0	0.0
b_2	1.2327	0.9127	1.0
b_3	0.0	0.0	0.0
b_4	1.2435	0.9273	1.0
q_1	0.3327	0.2835	0.25
q_2	0.6237	0.5528	0.5
q_3	0.3207	0.2786	0.25

The estimates obtained from 1000 measurements and 3000 measurements are shown in Table 2. And the estimates versus N are plotted in Figs. 3-4.

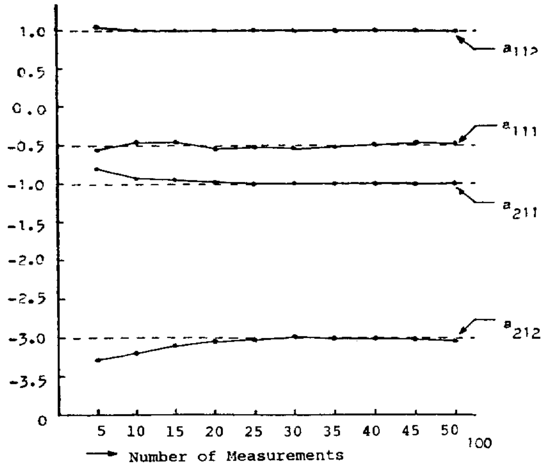

Fig. 3. Estimated Values of a_1 and a_2 versus N

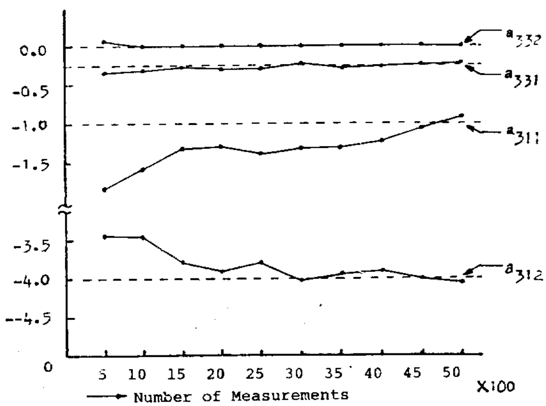

Fig. 4. Estimated values of a_3 versus N

7. Conclusion

In this paper, we considered the problem of parameter identification for multivariable stochastic linear systems.

A simple identification method for multivariable linear systems was presented and shown to converge to the true parameter values, as the number of measurements increase.

Our method need not the large computational burden, comparing with maximum likelihood approach, but we can obtain the relatively accurate parameter values.

This method can be applied to the continuous systems.

Appendix

Let $P(n\times n)$ be a symmetric positive semidefinite matrix satisfying the set of relationship

$$PC^t=G \tag{A1}$$

$$P=APA^t+BB^t \tag{A2}$$

where G,C are $n\times m, m\times n$ matrices respectively and A is $n\times n$ stable matrix.

To solve this problem, we define an equivalent optimization problem;

$$\min J=tr[(PC^t-G)(PC^t-G)^t] \tag{A3}$$

with respect to B subject to constraints (A2). It is easily shown that

$$J=\sum_{i=1}^{n}\sum_{j=1}^{m}\{[PC^t]_{ij}-[G]_{ij}\}^2$$

The solution to this optimization problem can be writen easily in terms of an $n\times n$ symmetric matrix Λ of Lagrange multipliers for (A2). Adjoiting the constraints (A2) to the performance index J, we get a modified performance index

$$\begin{aligned}J&=tr[(PC^t-G)(PC^t-G)^t]+tr[\Lambda(P-APA^t-BB^t)]\\&=tr[PC^tCP-PC^tG^t-GCP+GG^t+\Lambda P-\Lambda APA^t\\&\quad-\Lambda BB^t]\end{aligned} \tag{A4}$$

The necessary condition of optimality are easily derived using results on gradient matrices.

$$\frac{\partial J}{\partial B}=-2\Lambda B=0$$

$$\begin{aligned}\frac{\partial J}{\partial P}&=PC^tC+C^tCP-GC-C^tG^t+\Lambda\\&\quad-A^t\Lambda A=0\end{aligned} \tag{A5}$$

or

$$\Lambda=A^t\Lambda A+GH+H^tG^t-PC^tC-C^tCP. \tag{A6}$$

A numerical procedure for solving these equations is as follows

1) Pick an initial estimates B_0 and solve(A2) for P_0. One way of obtaining P_0 is to solve recursively for the steady state of the following:

$$P_0(n+1)=AP_0(n)A^t+B_0B_0^t, P_0(0)=0 \tag{A7}$$

2) Solve (A6) for Λ_0 using P_0. This can be done by solving till steady state

$$\begin{aligned}\Lambda_0(n+1)&=A^t\Lambda_0(n)A+(GC+C^tG^t-P_0C^tC\\&\quad-C^tCP_0),\ \Lambda_0(0)=0\end{aligned} \tag{A8}$$

3) Let

$B_1 = B_0 + 2\alpha A_0 B_0$

where α is a suitable step size

4) Repeat 1)~3) till convergence is achived.

This procedure is a gradient procedure and will converge to a local extremum of J.

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