狀態變數에 遲延要素를 갖는 시스템의 安定化 方法에 관한 硏究

論 文
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Feedback Stabilization of Linear Systems with Delay in State

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Abstract

This paper suggests easy stabilization methods for linear time-varying systems with delay in the state. While existing methods employ the function space concept, the methods introduced in this paper transform the delay systems into the non-delay systems so that the well known methods for finite dimensional systems can be utilized. Particularly the intervalwise predictor is introduced and shown to satisfy an ordinary system. Control laws stabilizing the non-delay systems satisfied by this predictor will be shown to at least pointwise stabilize the delay systems with the additional strong possibility of true stabilization. In order to combine two steps of the predictor method, first transformation and then stabilization, an intervalwise regulator problem is suggested whose optimal control laws incorporate the intervalwise predictor as an integral part and also at least pointwise stabilize the delay systems. Since the above mentioned methods render the periodic feedback gains for time invariant systems the pointwise predictor and regulator are introduced in order to obtain the constant feedback gains, with additional stability properties. The control laws given in this paper are perhaps simplest and easiest to implement.

1. Introduction

The stabilization method used to be the first step to the design problems. For ordinary systems there exist many different methods. But this is not the case for the time-delay systems, which appear in many industrial systems, particularly in chemical processes. It is well known that the delay system is stabilizable if and only if the decomposed part of the unstable finite dimensional subsystem is completely controllable[1]. Under the spectral controllability[2], functionwise controllability[3], and F-controllability[4], the delayed systems are known to be stabilizable. Thus a general constructive method requiring the decomposition and reconstruction of the infinite dimensional state space can be employed for the stabilization[1]~[4]. This method is quite difficult to apply due to the exact computation of unstable poles and projections. For the second general method we can, as expected, utilize the steady state control of linear quadratic regulator which has been known to stabilize the system under the stabilizability condition[7]. But the computation of the associated operator Riccati-type equation is very difficult to solve since it satisfies a two-point boundary partial differential equation. Under some special conditions similar to functionwise controllability, there exist some special stabilization methods[8]~[12].

Meanwhile for the systems with delay in the control, the Smith predictor method[13] and its exten-

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接受日字 : 1981年 11月 16日

sions[13]~[16] have been extensively used for the stabilization purpose, although the Smith predictor method suffers from the mis-matching of the model. It also has been shown that the spectrums of the closed-loop systems can be arbitarily assigned[5]. Recently a new transformation, which is actually a predictor, has been introduced for the first time in[17]. This new predictor not only transforms the delay systems into the non-delay systems but also can be utilized for the general method for the stabilization. This concept has been carried out to more general distributed delay system together with additional applications[18].

In this paper an attempt will be made to utilize the predictor method for the stabilization of linear systems with delay in the state. The predictor introduced for the systems with delay in the control[17] employes the pointwise moving horizon concepts[20],[21]. It will be the intervalwise moving horizon concepts[22] that the predictors for linear systems with delay in the state will employ in the section 3 of this paper. In the section 4, the control laws derived from intervalwise regulator problems are shown to employ the intervalwise predictors as an integral part and stabilize the systems under some conditions. The advantages of the predictor or the regulator methods will be the simplicity of the feedback control laws and its transformation of the delay systems into the nondelay systems, which makes the analysis easier.

2. System Description

The systems which we will consider in this paper are represented by

$$\dot{x}(t)=A_0(t)x(t)+A_1(t)x(t-h)+B(t)u(t), \qquad (2.1)$$

where $x(t)\varepsilon R^n, u(t)\varepsilon R^m$, and $A_0(t)$, $A_1(t)$, and $B(t)$ are $n\times n$, $n\times n$, and $n\times m$ piecewise continuous bounded matrices. The solution of (2.1) is given by

$$x(t)=x(t;x_{t_0}, t_0, u(.))=\Phi(t, t_0)x(t_0)+\int_{t_0-h}^{t_0}\Phi(t, \tau+h)A_1(\tau+h)x(\tau)d\tau+\int_{t_0}^{t}\Phi(t, \tau)B(\tau)u(\tau)d\tau \qquad (2.2)$$

where $x_t(\tau)=x(t+\tau)$, $-h\leq\tau\leq 0$, and the state transition matrix $\Phi(t,\tau)$, $t_0\leq\tau\leq t$, is the $n\times n$ absolutely continuous matrix solution of the following matrix differential equation

$$\frac{\partial}{\partial t}\Phi(t, \tau)=A_0(t)\Phi(t, \tau)+A_1(t)\Phi(t-h, \tau), \quad t_0\leq\tau\leq t, \qquad (2.3)$$

with $\Phi(t, t)=I$ and $\Phi(t, s)=0$ for $t\leq s$. The system (2.1) is said to be completely pointwise (functionwise, repectively) controllable on $[t_0, t_1]$ if for every initial state x_{t_0} and every terminal point x_1 (terminal function $\Psi(t)$, $t\varepsilon[0, h]$, respectively), there exists a control law $u_{(t_0, t_1)}$ such that the corresponding trajectory of the system (2.1) satisfies the condition $x(t_1)=x_1(x_{t_1}=\Psi$, respectively). The term "completely" will be dropped in the forth-coming statements in this paper. It is well known that the pointwise complete system (2.1) is pointwise controllable on $[t_0, t_1]$ if and only if the controllability Gramian matrix

$$W(t_0, t_1)=\int_{t_0}^{t_1}\Phi(t_1, \tau)B(\tau)B'(\tau)\Phi'(t, \tau)d\tau \qquad (2.4)$$

is positive definite. Since the positive definiteness of the controllability Gramian matrix (2.4) implies the pointwise controllability regardless of the pointwise completeness, the system (2.1) will be defined to be pointwise controllable in this paper for convenience if the matrix in (2.4) is positive definite. The system (2.1) will be said to be uniformly pointwise controllable with an index $\delta_c>0$ if for some $\alpha_1>0$, $\alpha_2>0$, the following condition,

$$\alpha_1 I\leq W(t, t+\delta_c)\leq\alpha_2 I \qquad (2.5)$$

holds. The system (2.1) is said to be stabilizable if the system (2.1) with a control law of a form

$$u(t)=K_0(t)x(t)+\int_{t-h}^{t}K_1(t, \tau)x(\tau)d\tau \qquad (2.6)$$

is asymptotically stable, i.e., $x(t)\to 0$ as $t\to\infty$.

3. Stabilization Via Intervalwise Moving Predictors

For any trajectory x_t of the system (2.1) the predictor with the horizon point $s>t$ will be defined by the zero-input solution at the point s, i.e.,

$$x^s(t)=x(s;\ x_t, t, u(.)=0) \qquad (3.1)$$

This is a predictor in the sense that the unknown input in the future $[t, s]$ can be considered zero as often seen in stochastic systems with random noise inputs. The predictor (3.1) is given by

$$x^s(t)=\Phi(s, t)x(t)+\int_{t-h}^{t}\Phi(s, \tau+h)A_1(\tau+h)x(\tau)d\tau \quad t\varepsilon[t_0, s] \qquad (3.2)$$

It is noted that the value s is a fixed value and $x^s(t)$ is defined on the interval $[t_0, s]$. The interesting point is that the predictor (3.1) satisfies an ordinary differential system.

Lemma 3.1. For a fixed s, the predictor $x^s(t)$ satisfies the following ordinary differential system,

$$\frac{d}{dt}x^s(t)=\Phi(s,t)B(t)u(t),\ t\leq s \tag{3.3}$$

Moreover the delay system (2.1) is pointwise controllable on $[t_0, s]$ if and only if the nondelay system (3.3) is controllable on $[t_0, s]$.

Proof: It is well known that the derivative of the state transition matrix with respect to the second argument satisfies the following adjoint equation[23],

$$\frac{\partial}{\partial t}\Phi(s,t)=-\Phi(s,t)A_0(t)-\Phi(s,t+h)A_1(t+h) \tag{3.4}$$

Taking the derivative of $x^s(t)$ in (3.2), we will get

$$\begin{aligned}\dot{x}^s(t)&=\dot{\Phi}(s,t)x(t)+\Phi(s,t)\dot{x}(t)+\Phi(s,t+h)A_1(t+h)x(t)\\&\quad-\Phi(s,t)A_1(t)x(t-h)\\&=(-\Phi(s,t)A_0(t)-\Phi(s,t+h)A_1(t+h))x(t)\\&\quad+\Phi(s,t)(A_0x(t)+A_1(t)x(t-h)+B(t)u(t))\\&\quad+\Phi(s,t+h)A_1(t+h)x(t)-\Phi(s,t)A_1(t)x(t-h)\\&=\Phi(s,t)B(t)u(t)\end{aligned} \tag{3.5}$$

The usual controllability matrix for the system (3.3) is given by

$$W(t_0,s)=\int_{t_0}^{s}\Phi(s,t)B(t)B'(t)\Phi(s,t)dt \tag{3.6}$$

since the state transition matrix of the system (3.3) is the identity matrix. The matrix (3.6) is the same to the matrix (2.4) with t_1 replaced by s. This completes the proof.

The similar transformation (3.2) has been introduced in[19] without the predictor concept. It is noted the predictor in (3.1) is defined on a finite interval $[t_0, s]$ and thus it must be extended to the infinite interval in case it is utilized for the stabilization. For this purpose the intervalwise moving predictor will be introduced here. First, the infinite time interval $[t_0, \infty)$ will be devided into equal finite time intervals, $[t_0, t_1)$, $[t_1, t_2)\dots[t_i, t_{i+1})\dots$ where $t_i=t_0+iL$ and $L>0$ is called the interval length. On each interval, say $[t_{i-1}, t_i)$, the moving horizon time s_i is defined by

$$s_i=t_i+T \tag{3.7}$$

where $T>0$ is called the intervalwise horizon distance. That is, the moving horizon $s(t)$ is defined by

$$s=s(t)=s(t;L,T)=\begin{cases}s_1=t_1+T \text{ for } t\ \varepsilon\ [t_0,t_1)\\ s_2=t_2+T \text{ for } t\ \varepsilon\ [t_1,t_2)\\ \dots\dots\dots\dots\\ s_i=t_i+T \text{ for } t\ \varepsilon\ [t_i,t_{i+1})\end{cases} \tag{3.8}$$

The intervalwise moving horizon $s(t)$ is pictured in Fig. 1a and the arguments of the horizon, t, L and T, will be suppressed whenever they are clear from contents. From this intervalwise moving horizon (3.8) the intervalwise moving predictor denoted by $x^s(t)$ or $x^{L,T}(t)$, will be defined by

$$\begin{aligned}x^s(t)=x^{L,T}(t)&=x(s;x_t,t,u(.)=0)\\&=\Phi(s(t),t)x(t)+\int_{t-h}^{t}\Phi(s(t),\tau+h)A_1\\&\quad(\tau+h)x(\tau)d\tau,\end{aligned} \tag{3.9}$$

where indices L and T denote the interval leghth and horizon distance respectively. In this paper the predictor will always mean the one in (3.9) defined over $[t_0, \infty)$, not the one in (3.2). From Lemma 2.1 the predictor (3.9) satisfies the following property.

Lemma 3.2 For a fixed L and T, the intervalwise moving predictor (3.9) satisfies the ordinary system

$$\frac{d}{dt}x^s(t)=\Phi(s(t),t)B(t)u(t) \tag{3.10}$$

except for countable points $t_0, t_1, t_2\dots t_i\dots$

The functionwise controllability is considered the natural counterpart to the controllability of the finite dimensional systems although it is too restirictive. Since the pointwise controllability is a weaker concept, the weaker concept on the stabilizability will be introduced for the delay systems.

Definition 3.1. The delay system (2.1) with a feedback control law (2.6) is said to be pointwise asymptotically stable if there exists a sequence $\tau_1, \tau_2, \tau_3, \tau_i, , \tau_i\to\infty$ such that $\lim_{i\to\infty} x(\tau_i)=0$.

The delay system (2.1) is said to be pointwise stabilizable if there esists a feedback control law (2.6) such that the closed-loop system is pointwise asymptotically stable. It is noted that the pointwise asymptotic stability implies that the trajectory approaches the origin but it does not say that the origin is the only one point of convergence. We define a set of function on $\Sigma[t_i-T, t_j]$ by

$$\begin{aligned}\Sigma[t_i-T,t_j]=\{&\Psi(t),\ t_i-T\leq t\leq t_j:\\&x(t_j+T;\Psi_t,t,u(.)=0)=0,\ t_i\leq t\leq t_j\}\end{aligned} \tag{3.11}$$

for a closed-loop delay system. It is believed that in most practical systems $\Sigma[t_i-T, t_i]$ will contain only the null function.

Theorem 3.1 If the ordinary system (3.10) is stabilized by a feedback control law

$$u(t)=K(t)x^{*}(t), \tag{3.12}$$

then the control

$$u(t)=K(t)x^{*}(t)=K(t)[\Phi(s(t), t)x(t)+\int_{t-h}^{t}\Phi(s(t), \tau+h)A_1(\tau+h)x(\tau)d\tau] \tag{3.13}$$

can stabilizes the delay system (2.1) in the following sense:

(i) $x(s_i)=x(t_i+T)=x(t_0+iL+T)\to 0$ as $i\to\infty$ (i.e., pointwise asymptotically stable) (3.14)

(ii) $x(t)$, $t_{i-1}-T\leq t\leq t_i$ approaches $\Sigma[t_{i-1}-T,t_i]$ as $i\to\infty$, (3.15)

Proof: For the delay system (2.1) with a feedback control law (3.13), an intervalwise predictor $x^{*}(t)=x^{L,T}(t)$ in (3.9) is introduced. Then this predictor satisfies (3.10) with a feedback control law (3.12). Since the system (3.10)~(3.12) is asymptotically stable, there exists $\alpha>0$ such that $\|\Phi_K(t, t_0)\|\leq ce^{-\alpha(t-t_0)}$ where $\Phi_K(t,t_0)$ is the state transition matrix of (3.10) and (3.12). Thus we have $|x^{*}(t)|\leq ce^{-\alpha(t-t_0)}|x^{*}(t_0)|$ and $|u(t)|\leq\|K(t)\|\cdot|x^{*}(t)|\leq Mce^{-\alpha(t-t_0)}|x^{*}(t_0)|$ where $M=\max_t\|K(t)\|<\infty$. The predictor (3.9) can be rewritten as

$$x^{*}(t)=x(s; x_t, t, u(.)=0)=x(s; x_t, t, u(.))-\int_t^s\Phi(s, \tau)B(\tau)u(\tau)d\tau$$

Thus we have

$$\begin{aligned}|x(s)|&=|x(s; x_t, t, u(.))|\\&=|x^{*}(t)+\int_t^s\Phi(s, \tau)B(\tau)u(\tau)d\tau|\\&\leq|x^{*}(t)|+\int_t^s\|\Phi(s, \tau)B(\tau)\|\cdot|u(\tau)|d\tau\\&\leq|x^{*}(t)|+N\int_t^s|u(\tau)|d\tau\\&\leq(ce^{-\alpha(t-t_0)}+NMce^{-\alpha(t-t_0)}L\cdot|x^{*}(t_0)|\\&\to 0 \text{ as } t\to\infty\end{aligned}$$

where $N=\sup_{t\leq\tau\leq s}\|\Phi(s, \tau)B(\tau)\|\leq\sup_{s-L\leq\tau\leq s}\|\Phi(s, \tau)B(\tau)\|<\infty$. The property (ii) follows from the assumption that the system (3.10) is asymptotically stable. This completes the proof.

The condition in (3.15) stronly suggests that any oscillation will not likely occur and thus the origin is the unique point of convergence, although we can not prove this formally. Many simulation tests confirmed this conjecture. It remains to check how we can stabilize the ordinary system (3.10). The finite dimensional system (3.10) is uniformly controllable with an index δ if there exist $\alpha_1>0$ and $\alpha_2>0$ such that

$$\alpha_1 I\leq\hat{W}(t, t+\delta)=\int_t^{t+\delta}\Phi(s(\tau),\tau)B(\tau)B'(\tau)\Phi'(s(\tau), \tau)d\tau\leq\alpha_2 I \tag{3.16}$$

This condition can be utilized for the stabilization problems. Thus we define the following concept.

Definition 3.2 The delay system (2.1) is said to be uniformly pretictively pointwise controllable with indices $\{\delta L, T\}$ if the non-delay system (3.10) with L and T is uniformly controllable with an index δ (i.e., the condition (3.16) holds).

It has been known[21] that the following control law

$$u(t)=-B'(t)\Phi'(s(t), t)\hat{W}^{-1}(t, t+\hat{T})x^{*}(t) \tag{3.17}$$

stabilizes the system (3.10) provided the system (3.10) is uniformly controllable with the index $\hat{T}$. Another possibility is the intervalwise moving control law[22]

$$u(t)=-B(t)\phi(s(t), t)\hat{W}^{-1}(t, \hat{s}(t))x^{*}(t) \tag{3.18}$$

where the new horizon $\hat{s}(t)$ is defined similarly as in (3.8) with L and T replaced by the new interval length and horizon distance $\hat{L}$ and $\hat{T}$. Although in[22] the stability property of the control law (3.18) is given for the time invariant systems it can be easily extended to time-varying systems. It is noted that the difference between the pointwise controllability in (2.4) and the predictive pointwise controllability in (3.16) is the first argument of the matrix $\Phi(\ ,\ )$ in the integrand. We summarize the above results.

Theorem 3.2. If the delay system (2.1) is uniformly predictively pointwise controllable with some indices $\{\delta, L, T\}$, then there exists a stabilizing control law, particularly the control law (3.17) or (3.18), in the sense of Theorem 3.1.

For ordinary systems the pointwise controllability, predictive pointwise controllability, spectral controllability and functionwise controllability are all equivalent, but it is believed that the predictive pointwise controllability conditionis much weaker than the spectral and functionwise controllability conditions and slightly stronger than the pointwise controllability condition in delay systems as can be seen in Example 3.1 for time-invariant systems. It is not clear so far whether the delay system (2.1) is stabilizable under the pointwise controllability.

For time-invariant systems predictors and control laws have simpler forms. For the time-invariant system (2.1) with constant matrices, A_0, A_1, and B, the solution is given by

$$x(t)=x(t;\ x_{t_0},\ t_0,\ u(.))=\Phi(t-t_0)x(t_0)$$
$$+\int_{t_0-h}^{t_0}\Phi(t-\tau-h)A_1x(\tau)d\tau$$
$$+\int_{t_0}^{t}\Phi(t-\tau)Bu(\tau)d\tau \qquad (3.19)$$

where the state transition matrix is the solution to the matrix differential equation

$$\frac{d}{dt}\Phi(t)=A_0\Phi(t)+A_1\Phi(t-h) \qquad (3.20)$$

with $\Phi(0)=I$ and $\Phi(t)=0$ for $t<0$. The controllability matrix (2.4) is defined by

$$W(t)=\int_0^t\Phi(t-\tau)BB'\Phi'(t-\tau)d\tau \qquad (3.21)$$

The predictor (3.9) is defined by

$$x^s(t)=x^{L,T}(t)=\Phi(s(t)-t)x(t)$$
$$+\int_{t-h}^{t}\Phi(s(t)-\tau-h)A_1x(\tau)d\tau \qquad (3.22)$$

The non-delay system (3.10) obtained from time invariant delay systems is unfortunately a time-varying system as can be seen as

$$\frac{d}{dt}x^s(t)=\Phi(s(t)-t)Bu(t) \qquad (3.23)$$

since the argument $s(t)-t$ is not constant. Unlike the ordinary time invariant systems the controllability matrix in (3.16) can not be made of one argument and is given by

$$\hat{W}(t,\ t+\delta)=\int_t^{t+\delta}\Phi(s(\tau)-\tau)BB'\Phi'(s(\tau)-\tau)d\tau \qquad (3.24)$$

The control law law (3.17) will be given by

$$u(t)=-B'\Phi'(s(t)-t)\hat{W}^{-1}(t,\ t+\hat{T})x^s(t), \qquad (3.25)$$

and the one (3.18) by

$$u(t)=-B'\Phi'(s(t)-t)\hat{W}^{-1}(t,\ \hat{s}(t))x^s(t) \qquad (3.26)$$

It is seen that the control laws (3.25) and (3.26) have periodic feedback gains with the period L since $s(t+L)=s(t)+L$ and $W(t+L,\ t+L+T)=W(t,\ t+T)$. An attempt will be made to obtain constant feedback gain control laws in Section 5.

Example 3.1 Consider the delay system (2.1) with constant matrices A_0, A_1, and B. If $\{A_0,\ B\}$ is controllable, the the system (2.1) is pointwise controllable regardless of A_1. But for special cases of A_1, the system (2.1) can not be functionwise, nor F-controllable[3,4]. It can be shown that if $\{A_0,\ B\}$ is controllable, then the system (2.1) is predictively pointwise controllable with indices $\delta=h/2$, $L=h$, and $T=h/4$, regardless of A_1. This example looks to show that the predictive pointwise controllability is weaker than the functionwise controllability and F-controllability.

The methods introduced in this section is two-stage methods in the sense that first the predictors are obtained and then the feedback control law are sought for the ordinary predictor systems. In the next section a moving horizon regulator problem is introduced which combines two-stage procedures into one and requires weaker conditions.

4. Stabilization Via Intervalwise Moving Regulator

A performance index for the regulator has been considered a design parameter in order to get a satisfactory feedback control law. Thus we take a performance index as a closed-loop type index since it automatically renders a closed-loop solution. We will find the optimal control of the system (2.1) which minimizes

$$J(u)=\int_t^{s(t)}u'(\tau)u(\tau)d\tau \qquad (4.1)$$

subject to

$$x(s(t))=0 \qquad (4.2)$$

where $s(t)$ is defined in (3.8). The constraint (4.2) can be writted as

$$0=x(s)=\Phi(s,\ t)x(t)+\int_{t-h}^{t}\Phi(s,\ \tau+h)A_1(\tau+h)x(\tau)d\tau$$
$$+\int_t^s\Phi(s,\ \tau)B(\tau)u(\tau)d\tau \qquad (4.3)$$

If the system (2.1) is pointwise controllable on $[t,\ s]$ then the control which minimizes (4.1) is given by

$$u(t)=-B'(t)\Phi'(s(t),\ t)W^{-1}(t,\ s(t))[\Phi(s(t),\ t)x(t)$$
$$+\int_{t-h}^{t}\Phi(s(t),\ \tau+h)A_1(\tau+h)x(\tau)d\tau]$$
$$=-B'(t)\Phi'(s(t),\ t)W^{-1}(t,\ s(t))x^{L,T}(t) \qquad (4.4)$$

where $x^{L,T}(t)=x^s(t)$ is defined in (3.9). It is the equation (4.4) where the predictor is introduced first. It is noted that the control law (3.18) and (4.4) are quite different since the former uses the predictive controllability matrix (3.16) and the latter the pointwise controllability matrix (2.4). In order to avoid the singularity in the feedback gain we take $T>0$. The important properties of the control law (4.4) are stated in Theorem 4.1.

Theorem 4.1 If the system (2.1) is uniformly pointwise controllable with an index δ, then the control law (4.4) is defined with $T>\delta, L>0$. Moreover if the system (2.1) is uniformly predictively pointwise controllable with indices $\{\delta, L, T\}$ then the control law (4.4) stabilizes the system (2.1) in the sense of (3.14) and (3.15).

Proof: Likewise as in (3.10) the predictor $x^{L,T}(t)$ in (4.4) can be written as

$$\begin{aligned}\dot{x}^{L,T}(t)&=\Phi(s(t),t)B(t)u(t)\\&=-\Phi(s(t),t)B(t)B'(t)\Phi'(s(t),t)W^{-1}(t,s(t))x^{L,T}(t)\\&\triangleq F(t)x^{L,T}(t)\end{aligned}\tag{4.5}$$

Consider an adjoint system of (4.5)

$$\dot{z}(t)=-F'(t)z(t)\tag{4.6}$$

with an associated scalar valued function

$$V(z(t),t)=z'(t)W(t,s(t))z(t)\tag{4.7}$$

where $W(t,s(t))$ is a piecewise continuous function and has a property[21]

$$W(t,\tau_1)\leq W(t,\tau_2),\quad \tau_1\leq\tau_2\tag{4.8}$$

Since the system (2.1) is uniformly pointwise controllable with an index δ, we have

$$\alpha_1|z|^2\leq V(z,t)\leq\alpha_2|z|^2\tag{4.9}$$

for some $\alpha_1>0$, $\alpha_2>0$ and $T>\delta$. Taking the derivative of (4.7) yields

$$\begin{aligned}\dot{V}(z(t),t)&=\dot{z}'(t)W(t,s(t))z(t)+z'(t)\dot{W}(t,s(t))z(t)\\&\quad+z'(t)W(t,s(t))\dot{z}(t)\\&=z'(t)\Phi(s(t),t)B(t)B'(t)\Phi'(s(t),t)z(t)\geq 0\end{aligned}\tag{4.10}$$

except discontinuous points $\{t_0+iL\}$ $i=1,2,\ldots$. By integrating of (4.10) we can get, for $t_\delta\geq t_\alpha+\delta$ and $\Omega=\{t_0+iL,\ i=1,2,..\}$,

$$\begin{aligned}&V(z(t_\delta),t_\delta)-V(z(t_\alpha),t_\alpha)\\&=\int_{t_\alpha}^{t_\delta}\dot{V}(z(t),t)dt+\sum_{t\in(t_\alpha,t_\delta)\cap\Omega}z'(t)(W(t_+,s(t_+))\\&\quad-W(t_-,s(t_-))z(t)\\&\geq\int_{t_\alpha}^{t_\delta}z'(t)\Phi(s(t),t)B(t)B'(t)\Phi'(s(t),t)z(t)dt\end{aligned}\tag{4.11}$$

$$\begin{aligned}&=z'(t_\alpha)\int_{t_\alpha}^{t_\alpha}\Phi_z'(t,t_\alpha)\Phi(s(t),t)B(t)B'(t)\Phi'(s(t),t)\\&\quad\Phi_z(t,t_\alpha)dt\ z(t_\alpha)\end{aligned}\tag{4.12}$$

where $\Phi_z(t,\tau)$ is the state transition matrix of (4.6). The inequality (4.11) follows from (4.8). It is well known that $\Phi_z'(t,\tau)=\Phi_F(\tau,t)$ where $\Phi_F(t,\tau)$ is the state transition matrix of the system (4.5). Since the pair $\{0, \Phi(s(t),t)B(t)\}$ is assumed to be uniformly controllable, thus the system $\{-\Phi(s(t),t)B(t)K(t), \Phi(s(t),t)B(t)\}$ is also uniformly conformly controllable for the bounded $K(t)$. Therefore there exists a lower bound in (4.12) such that

$$V(z(t)_\delta,t_\delta)-V(z(t_\alpha),t_\alpha)\geq\alpha_3|z(t_\delta)|^2\tag{4.13}$$

for $t_\delta\geq t_\alpha+\delta$ and $\alpha_3>0$. It can be shown that $z(t)$ increases exponentially (see[22] for a similar proof). Therefore the adjoint system (4.5) of the system (4.6) is uniformly asymptotically stable. From Theorem 3.1 follows the results mentioned in the theorem. This completes the proof.

It is noted again that the control law (4.4) is defined under the pointwise controllability condition, although the stability property can be proved under the predictively pointwise controllability. There are strong indications that the control (4.4) might stabilize the system (2.1) under the pointwise controllability though we can not prove this claim. The control laws (3.17) and (3.18) are defined under the predictively pointwise controllability condition instead.

For time-invariant systems the control law (4.4) can be written by

$$\begin{aligned}u(t)&=-B'\Phi'(s(t)-t)W^{-1}(s(t)-t)(\Phi(s(t)-t)x(t)\\&\quad+\int_{t-h}^{t}\Phi(s(t)-\tau-h)A_1x(\tau)d\tau\end{aligned}\tag{4.14}$$

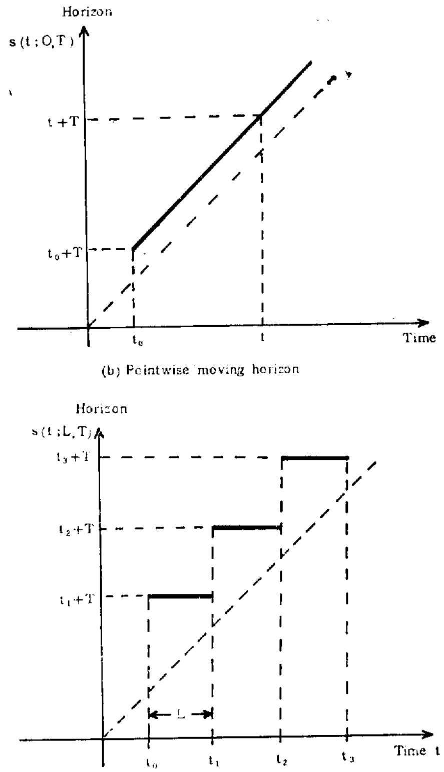
where

$$W(t)=\int_0^t\Phi(\tau-\tau)BB'\Phi'(t-\tau)d\tau.\tag{4.15}$$

The gain in the control law (4.14) is periodic and limited memory storage is necessary for implementation. In the next section we will attemnt to obtain constant feedback control laws for time-invariant systems.

5. Pointwise Moving Predictors and Regulators

While the interval moving horizons introduced in Section 3 and 4 (Fig. 1.a) have some applications for ordinary systems[22], the pointwise moving horizons (Fig. 1.b) have been more successfully applied[17,18,20,21]. The pointwise moving horizons



(b) Pointwise moving horizon

(a) Intervalwise moving horizon

Fig. 1. Moving horizon

are defined from the intervalwise moving horizons by taking the interval length $L=0$. That is, for the present time t, the horizon is defined by $s(t)=t+T$. Thus the pointwise moving predictor is defined by

$$x^*(t)=x^{0,L}(t)=x^T(t)=x(t+T;\ x_t,\ t,\ u(.)=0)$$
$$=\Phi(t+T,\ t)x(t)$$
$$+\int_{t-h}^{t}\Phi(t+T,\ \tau+h)A_1(\tau+h)x(\tau)d\tau \quad (5.1)$$

This pointwise moving horizon is comparable to the new predictor introduced for the linear systems with delay in control only, whose solution is represented by $x(t;x(t_0),\ t_0,\ u_{t_0},\ u(.))$ and predictor is given by[17]

$$x^*(t)=x(t+T;\ x(t),t\ ,\ u_t,\ u(.)=0) \quad (5.2)$$

While the pointwise predictor (5.2) for the systems with delayed controls satisfies an ordinary system for any $T\geq 0$, the problem of the pointwise predictor (5.1) for the systems with delay in the state is that we can not obtain ordinary systems like (3.10). However the pointwise moving horizon control laws can be obtained directly from intervalwise moving horizon control laws, (3.13) and (4.4), by taking $L=0$ provided the feedback gain exists. Thus a pointwise moving horizon control law can be obtained from (3.13) as

$$u(t)=[\lim_{L\to 0} K(t)][\Phi(t+T,\ t)x(t)$$
$$+\int_{t-h}^{t}\Phi(t+T,\tau+h)A_1(\tau+h)x(\tau)d\tau] \quad (5.3)$$

provided the limit on $K(t)$ exists when it is dependent on L. Unfortunately the control laws (3.17) and (3.18) will not be defined satisfactorily in this case since the controllability matrix $W(t,\ t+\delta)$ is given by

$$W(t,\ t+\delta)=\int_{t}^{t+\delta}\Phi(\tau+T,\ \tau)B(\tau)B'(\tau)\Phi'(\tau+T,\ \tau)d\tau \quad (5.4)$$

and morelikely becomes singular unless the matrix $B(t)$ is nonsingular as can be seen for time-invariant cases

$$W(t,\ t+\delta)=\Phi(T)\int_{t}^{t+\delta}B(\tau)B'(\tau)d\tau\Phi'(T). \quad (5.5)$$

However it is fortunate that the control law (4.4), introduced from the intervalwise moving regulator, can perfectly be defined for the case of $L=0$. That is, it becomes

$$u(t)=-B'(t)\Phi'(t+T,\ t)W^{-1}(t,\ t+T)x^T(t) \quad (5.6)$$

where the predictor $x^T(t)$ is given in (5.1) and $W(t,t+T)$ in (2.4) is nonsingular under pointwise controllability. The main advantage of the pointwise moving horizon is that it renders constant gain predictors and control laws for time-invariant systems. The pointwise predictor for time-invariant systems is given by

$$x^*(t)=x^T(t)=\Phi(T)x(t)$$
$$+\int_{-h}^{0}\Phi(T-h-\tau)A_1x(t+\tau)d\tau \quad (5.7)$$

and the pointwise moving horizon control is given by

$$u(t)=-B'\Phi'(T)W^{-1}(T)x^T(t). \quad (5.8)$$

The problem for the these control laws (5.6)~(5.8) is its stability. In order to check the stability, we observe that the parameters in the intervalwise moving horizon predictors, (3.9) and (3.22), and controllers, (4.4) and (4.14), are continuous with respect to L at the neighborhood of zero and thus

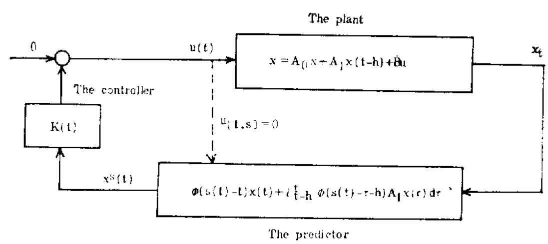


Fig. 2. Structure of predictor and regulator methods

the solution of the system (2.1) with these predictors and controllers is continuously dependent on L from the well known theorem on the continuous dependence of the solution on parameters[23]. From this we can claim the following result.

Proposition 5.1. If the system (2.1) is asymptotically stable with the intervalwise moving horizon control law (4.4) for any $L>0$, then the pointwise moving horizon control law (5.6) with the system (2.1) is at least stable.

Since the system (2.1) with the control law (4.4) is shown to stabilize the system mostlikely in Theorem 4.1, the control laws (5.6) and (5.8) are at least stable in most cases. But there are strong indications that these controls are actually asymptotically stable in the strict sense of Lyapunov. Some of indications are that as the interval L approaches as $1/n$, the distance of neighboring horizons, $(T_0+(i+1)/n+T)-(t_0+i/n+T)=1/n$, becomes smaller and thus the gains in (4.4) and (5.8) more negative in some sense. As the interval L approaches as $1/n$, the solution tends to approach to the origin from (3.14) and (3.15) and the predictive controllability with $\delta=T$ becomes the pointwise controllability. Since the time-invariant systems are more interesting the following conjecture is made for these systems from the above properties.

Conjecture If the time-invariant system (2.1) is pointwise controllable with an index $T>h$, then the control law (5.8)~(5.7) with $T>h$ stabilizes the time-invariant system (2.1).

We have checked the above conjecture for the second order systems with computer simulations and found no exceptions. If the above conjecture ever becomes true, the control law (5.8)~(5.7) is simplest among existing stabilizing control laws since existing control laws require decomposition of infinite dimensional state space, computation of the exact unstable poles from infinite number of poles, or computation of the operatortype Riccati equation. Even if the conjecture is not ture, it is clear that this control law will stabilize most systems. Thus we recommend the control law (5.7)~(5.8) as the first candidate for the test of the stabilization method of time invariant systems (2.1).

6. Conclusion

The feedback control of delay systems has been an important problem in the process industry. For those linear systems with delay in the control, the Smith predictor method[13,14] and the pointwise predictor method[17,18] can be utilized for stabilization under weak conditions, both of which transforms systems with delay into systems without delay in the frequency and time domains respectively. There have been no such corresponding results for linear systems with delay in the state.

This paper suggests, for the purpose of stabilization, the intervalwise predictors, (3.9) and (3.19), that transform delay systems into non-delay systems (3.10) and (3.23) for wich many stabilization methods exist. Under the predictive pointwise controllability, which is a slightly stronger condition than the pointwise controllability, feedback control laws, (3.17), (3.18), (3.25), and (3.26), are shown to at least pointwise stabilize the linear time-delay systems with additional properties. In order to combine two stages of the predictor method into one, a regulator problem, which incorporates the predictor in it, has been suggested in this paper along with stabilizing control laws, (4.4) and (4.14). While the control laws (3.25) and (3.26) utilizing the predictor method are defined under the predictive pointwise controllability, the advantage of the control laws, (4.4) and (4.14), is that they are defined under the pointwise controllability. Especially for the time invariant systems, feedback gains for the intervalwise predictor and regulator methods are pereodic with the period L. In order to obtain the constant feedback gains, the pointwise regulator problem is suggested which incorporates the pointwise predictor (5.7) and the constant feedback control law (5.8). It is strongly believed that the simplest control law (5.8) stabilizes the most practical systems under the pointwise controllability.

The approach suggested in this paperis quite different from the existing results in that the latter are mathematically oriented employing the state decomposition and infinite-time regulator in the infinite dimensional space but this paper is practically oriented employing the finite dimensional space. Control laws introduced in this paper are simple and easy to implemt. They will find many applications in the process systems with delay in state.

References

〔1〕 Y.S. Osipov; "Stabilization of controlled system with delays," Differential'nye Uraveniya, vol. 1, no. 5, pp. 605～618, 1965.

〔2〕 L. Pandolfi; "On feedback stabilization of functional differential equations," Boll. Unione Math. Italiana, vol. 12, no. 4, pp. 625～635, 1975.

〔3〕 A. Manitius and R. Triggiani, "Sufficient conditions for function space controllability and feedback stabilizability of linear retarded systems," Proceedings of 1976 IEEE Conference on Decision and Control, 1976.

〔4〕 A. Manitius; "Controllability, observability and stabilizability of retarded systems," Proceedings of 1976 IEEE Conference of Decision and Control, 1976.

〔5〕 A. Manitius and A.W. Olbrot; "Finite spectrum assignment problem for systems with delays," IEEE Trans. Automat. Contr., vol. AC-24, pp. 541～553, August 1979.

〔6〕 A.W. Olbrot; "Stabilizability, detectability, and spectrum assignment for linear autonomous system with general time delays," IEEE Trans. Automat. Contr., vol. AC-23, pp. 887～890, Oct. 1978.

〔7〕 R.H. Kwong; "A stability theory for the linear-quadratic-Gaussian problem for systems with delays in the state, control and observations," SIAM J. Control and Optimization, vol. 18, No. 1, Jan. 1980.

〔8〕 G.J. Nazaroff and G. Hewer; "Stabilization of linear autonomous differential delay system," IEEE Trans. Automat. Contr. vol. AC-18, pp. 673～674, Dec. 1973.

〔9〕 G.J. Nazaroff; "Stability and stabilization of linear differential delay systems," IEEE Trans. Automat. Contr., vol. AC-18, pp. 317～318, June 1973.

〔10〕 W.H. Kwon and A.E. Pearson; "A note on feedback stabilization of a differential-difference system," IEEE Trans. Automat. Contr., vol. AC-22, pp. 468～470, June 1977.

〔11〕 M. Ikeda and T. Ashida; "Stabilization of linear systems with time-varying delay," IEEE Trans. Automat. Contr., vol. AC-24, pp. 369～370, April 1979.

〔12〕 A. Feliachi and A. Thowsen; "Memoryless stabilization of linear delay-defferential systems," IEEE Trans. Automat. Contr., vol. AC-24, pp. 586～587, April 1981.

〔13〕 O.J. Smith; Feedback Control Systems, McGraw-Hill, 1958.

〔14〕 G. Alevisakis and D.E. Seborg; "An extension of the Smith predictor method to multivariable linear systems containing time delays," Int. J. Control, vol. 3, No. 17, 1973.

〔15〕 D.H. Mee; "An extension of predictor control for systems with control time-delays," Int. J. Control, vol. 18, No. 6, 1973.

〔16〕 J.F. Donoghue; "A comparison of the Smith predictor and optimal design approaches for systems with delay in the control," IEEE Trans. on Indus. Elect. Control Inst., vol. IECI-24, Feb. 1977.

〔17〕 W.H. Kwon and A.E. Pearson; "Feedback stabilization of linear systems with delayed control," IEEE Trans. Automat. Contr., vol. AC-25, pp. 266～269, April 1980.

〔18〕 Zvi Artstein; "Linear systems with delayed controls: A reduction," To appear

〔19〕 G. Slater and W. Wells; "On the reduction of optimal time-delay systems to ordinary ones," IEEE Trans. Automat. Control., vol. 17, Feb. 1972.

〔20〕 D.L. Kleinman; "An easy way to stabilize a linear constant system," IEEE Trans. Automat. Contr, vol. AC-15, p. 692, Dec. 1970.

〔21〕 W.H. Kwon and A.E. Pearson; "A modified quadratic cost problem and feedback stabilization of a linear system" IEEE Trans. Automat. Contr., vol. AC-22, pp. 838～842, Oct. 1977.

〔22〕 W.H. Kwon and A.E. Pearson; "Linear systems with two-point boundary Lyapunov and Riccati equations." To be published inIEEE Trans. Automat. Contr., 1981.

〔23〕 J.K. Hale; Functional Differential Equations, Springer-Verlag, N.Y., 1971.