Kyungpook Math. J.
Volume 21, Number 1
June, 1981.

EXPANSION OF THE GENERALIZED POLYNOMIAL SET $\{\bar{R}_n(x,y)\}$, FOR CERTAIN CLASSICAL POLYNOMIALS

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1. Introduction

The present paper aims at deriving a generating relation, involving the H-function due to Fox, [1] and a Bessel function of order ν_1 and of the first kind. The expansions of the generalized polynomials $\bar{R}_n(x, y)$ in terms of a certain classical polynomials, such as Legendre and sister-Celine under certain conditions have been arrived at. Similarly, the expansions of the Legendre and Sister-Celine polynomials in terms of the generalized polynomial set $\{\bar{R}_n(x,y)\}$ have also been deduced under certain restrictions. A number of interesting results have been worked out as particular cases of the general results obtained by the author.

A generalized polynomial set $\{\bar{R}_n(x, y)\}$ has been defined by means of the generating function (2.1). This polynomial set $\{\bar{R}_n(x,y)\}$ happens to be a generalization of as many as twenty four orthogonal and non-orthogonal polynomials such as Hermite, Laguerre, Legendre, Jacobi, Sister-celine and discrete polynomials, such as Meixner and Charlier's and several other polynomial systems. For convenience the following notations have been used for brevity.

$$(a_p)=a_1,\ a_2,\ \cdots,\ a_p;$$

$$[(a_p)]_n=\prod_{i=1}^{p}(a_i)_n=(a_1)_n(a_2)_n\cdots(a_p)_n\ ;$$

$$[(M_1(i,\ j))]_n=\prod_{j=1}^{l_2}\prod_{i=1}^{A_j}\left(\frac{i-a_j+A_jb_1}{A_j}\right)_n$$

$$[1-(M_2(i,\ j))]_n=\prod_{j=l_2+1}^{p}\prod_{i=1}^{A_j}\left(1-\frac{a_j-A_jb_1+i-1}{A_j}\right)_n\ ;$$

$$[(N_1(i,\ j))]_n=\prod_{j=l_1+1}^{\nu+1}\prod_{i=1}^{B_j}\left(\frac{i-b_j+B_jb_1}{B_j}\right)_n\ ;$$

$$[1-(N_2(i,\ j))]_n=\prod_{j=2}^{l_1}\prod_{i=1}^{B_j}\left(1-\frac{b_j-B_jb_1+i-1}{B_j}\right)_n\ ;$$

$$E=\frac{(-1)\sum_{j=1}^{l_1} B_j \prod_{j=1}^{p} A_j^{A_j}}{(-1)\sum_{j=l_2+1}^{p} A_j \prod_{j=2}^{q+1} B_j^{B_j}};$$

$$\overset{1}{\underset{k}{\Delta}}[m_4 : 1-(M_1(i, j))-n]=\prod_{j=1}^{l_2}\prod_{i=1}^{A_j}\prod_{l=1}^{m_4}\left(-\frac{(M_1(i, j))-n+l}{m_4}\right)_k$$

$$\overset{2}{\underset{k}{\Delta}}[m_4 : (M_2(i, j))-n]=\prod_{j=l_2+1}^{p}\prod_{i=1}^{A_j}\prod_{l=1}^{m_4}\left(\frac{(M_2(i, j))-n+l-1}{m_4}\right)_k ;$$

$$\overset{3}{\underset{k}{\Delta}}[m_4 : 1-(N_1(i, j))-n]=\prod_{j=l_1+1}^{q+1}\prod_{i=1}^{B_j}\prod_{l=1}^{m_4}\left(\frac{-(N_1(i, j))-n+l}{m_4}\right)_k ;$$

$$\overset{4}{\underset{k}{\Delta}}[m_4 : (N_2(i, j))-n]=\prod_{j=2}^{l_1}\prod_{i=1}^{B_j}\prod_{l=1}^{m_4}\left(\frac{(N_2(i, j))-n+l-1}{m_4}\right)_k ;$$

$$\overline{R}_n(x, y)=\overline{R}^{m;m_1;m_2;m_3;m_4;\alpha;u;(a_p, A_p);l_2}_{n;\xi;v;\alpha_1;\nu_1;(b_{q+1}, B_{q+1});r_1;l_1}(x, y) ;$$

$$\overline{R}_n(x, y)=0=\alpha_1=\nu_1 ;$$

2. Generalized polynomials set $\{\overline{T}_n(x, y)\}$

THEOREM. *If the polynomial set* $\{\overline{R}_n(x, y)\}$ *is defined by the generating function*

$$\sum_{n=0}^{\infty}\overline{R}_n(x, y)t^n=QJ_{\nu_1}\frac{(\alpha_1 x^{m_1} y^{m_2} t^{m_3})}{(1-vx^{-m}t^{m_4})^{\alpha}}$$
$$\times H^{l, l_2}_{p, q+1}\left[\frac{-y^{r_1}\mu t}{(1-vx^{-m}t^{m_4})^{\beta}}\,\middle|\,\begin{matrix}(a_1, A_1), (a_2, A_2), \cdots, (a_p, A_p)\\(b_1, 1), (b_2, B_2), \cdots, (b_{q+1}, B_{q+1})\end{matrix}\right] \tag{2.1}$$

then

$$\overline{R}_n(x, y)=\sum_{k=0}^{[n/m_4]}\sum_{s=0}^{[n/2m_3]}\frac{[(M_1(i, j))]_{n-m_4k-2m_3s}[1-(M_2(i, j))]_{n-m_4k-2m_3s}}{k!\; s!\; (n-m_4k-2m_3s)!\; (\nu_1+1)_s x^{mk-2m_1s}}$$
$$\times\frac{v^k(\mu E)^{n-m_4k-2m_3s}y^{r_1(n-m_4k-2m_3s)+2m_2s}\left(-\frac{1}{4}\right)^s(\alpha_1)^{2s}}{[(N_1(i, j))]_{n-m_4k-2m_3s}[1-(N_2(i, j))]_{n-m_4k-2m_3s}}$$
$$\times\frac{(\alpha+\beta b_1)_{n\beta-m_4\beta k-2m_3\beta s+k}}{(\alpha+\beta b_1)_{n\beta-m_4\beta k-2m_3\beta s}} \tag{2.2}$$

where

$\mu\neq 0$, $v\neq 0$, $\alpha\geq 0$, m *and* m_1 $(i=1, 2, 3, 4)$ *are positive integers,*

$$\sum_{j=1}^{p} A_j-\sum_{j=2}^{q+1} B_j\leq 1,$$

$$\sum_{j=1}^{l_2} A_j-\sum_{j=l_2+1}^{p} A_j+\sum_{j=2}^{l_1} B_j-\sum_{j=l_1+1}^{q+1} B_j=\lambda'>0,$$

$$\left|\arg \frac{\mu y^{r_1}t}{(1-vx^{-m}t^{m_4})^{\beta}}\right|<\frac{1}{2}\Pi\lambda'$$

and

$$Q=\frac{\prod_{j=l_2+1}^{p}\Gamma(a_j-A_jb_1)\prod_{j=l_1+1}^{\nu+1}\Gamma(1-b_j-B_jb_1)\,2^{\nu_1}\,\nu_1!}{\prod_{j=1}^{l_2}\Gamma(1-a_j+A_jb_1)\prod_{j=2}^{l_1}\Gamma(b_j-B_jb_1)(-\mu y^{r_1}t)^{b_1}(\alpha_1x^{m_1}y^{m_2}t^{m_3})^{\nu_1}}$$

PROOF. On using the expansion

$$H_{p,q+1}^{l_1,l_2}\left[x\left|\begin{matrix}(a_1,A_1),(a_2,A_2),\cdots,(a_p,A_p)\\(b_1,1),(b_2,B_2),\cdots,(b_{q+1},B_{q+1})\end{matrix}\right.\right]$$

$$=x^{b_1}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{l_2}\Gamma[1-a_j+A_j(b_1+n)]\prod_{j=2}^{l_1}\Gamma[b_j-B_j(b_1+n)](-x)^n}{n!\prod_{j=l_2+1}^{p}\Gamma[a_j+A_j(b_1+n)]\prod_{j=l_1+1}^{\nu+1}\Gamma[1-b_j+B_j(b_1+n)]}$$

$$J_{\nu_1}(\alpha_1x^{m_1}y^{m_2}t^{m_3})=\sum_{s=0}^{\infty}\frac{(-1)^s\left(-\frac{\alpha_1x^{m_1}y^{m_2}t^{m_3}}{2}\right)^{2s+\nu_1}}{s!\Gamma(s+\nu_1+1)}$$

and the lemma

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A(k,n)=\sum_{n=0}^{\infty}\sum_{k=0}^{[n/m_4]}A(k,n-mk) \tag{2.3}$$

We get (2.2).

3. Applications

We consider the expansion of the particularized polynomial set $\{\bar{R}_n(x,y)\}$ in terms of the Legendre polynomials $P_n(y)$, such as :

THEOREM. *If*

$$\sum_{n=0}^{\infty}\bar{R}_n^{2m_4}(x,y)t^n=\sum_{n=0}^{\infty}\sum_{k=0}^{[n/2m_4]}\frac{[(M_1(i,j))]_{n-2m_4k}}{R!\,(n-2m_4k)!\,[(N_1(i,j))]_{n-2m_4k}}$$

$$\times\frac{[1-(M_2(i,j))]_{n-2m_4k}v^k(\mu E\,y^{r_1})^{n-2m_4k}(\alpha+\beta b_1)_{n\beta-2m_4\beta k+k}t^n}{[1-(N_2(i,j))]_{n-2m_4k}x^{mk}(\alpha+\beta b_1)_{n\beta-2m_4\beta k}} \tag{3.1}$$

then

$$\overline{R}_n^{2m_4}(x, y)=\sum_{h=0}^{[n/2]}\frac{[(M_1(i, j))]_n[1-(M_2(i, j))]_n(\mu E)^n}{h!\,[(N_1(i, j))]_n[1-(N_2(i, j))]_n}$$

$$\times\frac{(2n-4h+1)P_{n-2h}(y^{r_1})}{2^n(3/2)_{n-h}}$$

$$\times F\left[\begin{array}{l}\Delta(m_4;-h),\ \Delta^3(2m_4;1-(N_1(i, j))-n),\\ \Delta^4(2m_4;(N_2(i, j))-n),\ \Delta(m_4;-\frac{1}{2}-n+h),\\ \Delta(2m_4\beta;1-\alpha-\beta b_1-n\beta);\\ \Delta^1(2m_4;1-(M_1(i, j))-n),\ \Delta^2(2m_4;(M_2(i, j))-n),\\ \Delta(2m_4\beta-1;1-\alpha-\beta b_1-n\beta);\\ \dfrac{-v(2m_4\beta)^{2m_4\beta}(2m_4)^{2m_4(\sum_2^{q+1}B_j-\sum_1^{p}A_j+1)}}{(2m_4\beta-1)^{2m_4\beta-1}(\mu E)^{2m_4}x^m}\end{array}\right] \qquad (3.2)$$

PROOF. Using the known result given by Rainville [2, p. 181(u)] in the expression (3.1), we have

$$\sum_{n=0}^{\infty}\overline{R}_n^{2m_4}(x, y)t^n=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{h=0}^{[n/2]}\frac{[(M_1(i, j))]_n[1-(M_2(i, j))]_n v^k(\mu E)^n}{k!\,[(N_1(i, j))]_n[1-(N_2(i, j))]_n}$$

$$\times\frac{(\alpha+\beta b_1)_{n\beta+k}(2n-4h+1)P_{n-2h}(y^{r_1})t^{n+2m_4k}}{(\alpha+\beta b_1)_{n\beta}2^n x^{mk}h!(3/2)_{n-h}}$$

$$=\sum_{n=0}^{\infty}\sum_{h=0}^{\infty}\sum_{k=0}^{[h/m_4]}\frac{[(M_1(i, j))]_{n+2h-2m_4k}[1-(M_2(i, j))]_{n+2h-2m_4k}}{k!\,[(N_1(i, j))]_{n+2h-2m_4k}[1-(N_2(i, j))]_{n+2h-2m_4k}}$$

$$\times\frac{v^k(\mu E)^{n+2h-2m_4k}(\alpha+\beta b_1)_{(n+2h-2m_4k)\beta+k}}{x^{mk}2^{n+2h-2m_4k}(\alpha+\beta b_1)_{(n+2h-2m_4k)\beta}}$$

$$\times\frac{(2n+1)P_n(y^{r_1})t^{n+2h}}{(3/2)_{n+h-2m_4k}\,(h-m_4k)!}$$

On replacing n by $n-2h$ and comparing the coefficient of t^n from both sides, we finally arrive at (3.2).

Now we shall obtain the expansion of Legendre polynomials in terms of the generalized polynomial set $\{\overline{R}_n(x, y)\}$, under certain conditions in terms of the finite series.

THEOREM. *If*

$$\sum_{n=0}^{\infty} p_n(y^{r_1})t^n = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{(-)^k\left(\frac{1}{2}\right)_{n-k} 2^n y^{r_1 n} t^{n+2k}}{k!\ n!} \tag{3.3}$$

then

$$p_n(y^{r_1}) = \sum_{h=0}^{n} \frac{\left(\frac{1}{2}\right)_n 2^n [(N_1(i,j))]_n [1-(N_2(i,j))]_n (v/x)^{n-h}}{(n-h)!\,[(M_1(i,j))]_n [1-(M_2(i,j))]_n (\alpha+b_1)_h (\mu E)^n}$$
$$\times(-)^{n-h}(\alpha+b_1)_n \bar{R}^{1:-:-:-:1}_{h:1:0:0}(x,y)$$
$$\times F\left[\begin{matrix}\Delta(2;-n+h),\ \Delta^1(2;1-(M_1(i,j))-n),\ \Delta^2(2;(M_2(i,j))-n);\\ \Delta(2;1-\alpha-b_1-n),\ \frac{1}{2}-n,\ \Delta^3(2;1-(N_1(i,j))-n),\\ \Delta^4(2;(N_2(i,j))-n);\\ \dfrac{2^{2(\sum_1^p A_j-\sum_2^{q+1} B_j-1)}(\mu E x)^2}{v^2}\end{matrix}\right] \tag{3.4}$$

PROOF. From [3, *p.* 157], we have

$$y^{r_1 n} = \frac{[(N_1(i,j))]_n [1-(N_2(i,j))]_n (\alpha+b_1)_n (-)^n}{[(M_1(i,j))]_n [1-(M_2(i,j))]_n (\mu E)^n}$$
$$\times\sum_{k=0}^{n} \frac{\bar{R}^{1:-:-:-:1:\alpha:\mu:(a_p,A_p):l_2}_{k:1:v:0:0:(b_{q+1},B_{q+1}):r_1:l_1}(x,y)(-n)_k}{(\alpha+b_1)_k (v/x)_{k-n}}$$

On making the substitution $y^{r_1 n}$ in the right hand side of (3.3), we arrive at

$$\sum_{n=0}^{\infty} p_n(y^{r_1})t^n = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{(-)^k\left(\frac{1}{2}\right)_{n+h+k} 2^{n+h} [(N_1(i,j))]_{n+h}}{k!\,n!\,[(M_1(i,j))]_{n+h}[1-(M_2(i,j))]_{n+h}}$$
$$\times\frac{[1-(N_2(i,j))]_{n+h} v^n (\alpha+b_1)_{n+h}}{(\mu E)^{n+h} x^n (\alpha+b_1)_h} \bar{R}^{1:-:-:-:1}_{h:1:0:0}(x,y)t^{n+h+2k}$$

Now, on using the lemma (2.3), and comparing the coefficient of t^n from both sides we obtain (3.4).

Special cases of (3.4).

(i) On taking $m=m_4=1=\beta=\mu=x=r_1=l_1;\ p=0=q=\alpha_1=b_1=\nu_1=l_2$

$v=-1\,;\ \alpha=\frac{1}{2}\,;\ y=Y^2$, we obtain

$$P_n(y)=\sum_{h=0}^{n}\frac{\left(\frac{1}{2}\right)_n\left(\frac{1}{2}\right)_n(-)^h 2^n H_{2h}(Y)}{(n-h)!\left(\frac{1}{2}\right)_h 2^{2h} h!}\,F\left[\begin{matrix}\Delta(2\,;\,-n+h)\,; & \\ \Delta\left(2\,;\,\frac{1}{2}-n\right),\ \frac{1}{2}-n\,; & -\frac{1}{4}\end{matrix}\right]$$

(ii) On setting $\alpha=1+\lambda\,;\ m=m_4=1=\beta=v=x\,;\ p=0=q=b_1=\alpha_1=\nu_1$ $l_2=0\,;\ r_1=1=l_1$, we arrive at

$$P_n(y)=\sum_{h=0}^{n}\frac{\left(\frac{1}{2}\right)_n 2^n(1+\lambda)_n(-)^h L_h^{(\lambda)}(y)}{(n-h)!\ (1+\lambda)_h}\ F\left[\begin{matrix}\Delta(2\,;\,-n+h)\,; & \\ \Delta(2\,;\,-\lambda-n),\ \frac{1}{2}-n\,; & \frac{1}{4}\end{matrix}\right]$$

Next, we shall consider the expansion of the polynomials set $\{\overline{R}_n(x, y)\}$ in terms of the sister-celine polynomials $C_n(y)$ and vice-versa.

THEOREM. *If*

$$\sum_{n=0}^{\infty}\overline{R}_n(x, y)t^n=\sum_{n=0}^{\infty}\sum_{k=0}^{[n/m_4]}\frac{[(M_1(i, j))]_{n-m_4k}[1-(M_2(i, j))]_{n-m_4k}}{k!(n-m_4k)!\,[(N_1(i, j))]_{n-m_4k}}$$

$$\times\frac{(\alpha+\beta b_1)_{n\beta-m_4\beta k+k}t^n}{[1-(N_2(i, j))]_{n-m_4k}x^{mk}(\alpha+\beta b_1)_{n\beta-m_4\beta k}} \tag{3.5}$$

then

$$\overline{R}_n(x, y)=\sum_{h=0}^{n}\frac{[(M_1(i, j))]_n[1-(M_2(i, j))]_n(\mu E)^n\left(\frac{1}{2}\right)_n n!\,[(f_a)]_n(-)^h}{[(N_1(i, j))]_n[1-(N_2(i, j))]_n[(d_e)]_n(n-h)!(n+h+1)!}$$

$$\times(2h+1)C_h(y^{r_1})$$

$$\times F\left[\begin{matrix}\Delta(m_4\,;\,-n+h),\ \Delta(m_4\,;\,-n-h-1),\ \Delta(m_4\,;\,1-(d_e)-n), \\ \Delta(m_4\beta\,;\,1-\alpha-\beta b_1-n\beta),\ \overset{3}{\Delta}(m_4\,;\,1-(N_1(i, j))-n), \\ \overset{4}{\Delta}(m_4\,;\,(N_2(i, j))-n)\,; \\ \overset{1}{\Delta}(m_4\,;\,1-(M_1(i, j))-n),\ \overset{2}{\Delta}(m_4\,;\,(M_2(i, j))-n), \\ \Delta(m_4\beta-1\,;\,1-\alpha-\beta b_1-n\beta),\ \Delta(m_4\,;\,\frac{1}{2}-n), \\ \Delta(m_4\,;\,-n),\ \Delta(m_4\,;\,1-(f_a)-n)\,; \\ \dfrac{-v(m_4\beta)^{m_4\beta}(-m_4)^{m_4(\sum_{2}^{q+1}B_j-\sum_{1}^{p}A_j-a+e)}}{(m_4\beta-1)^{m_4\beta-1}(\mu E)^{m_4}x^{m}}\end{matrix}\right] \tag{3.6}$$

PROOF. On making use of the result [2, p. 292(12)] in the equation (3.5) and after a little simplification we get (3.6).

Special cases of (3.6)

Similar to (3.4), we get the following results for Legendre Jacobi, Gegenbanrer, Bateman's and Ricc's polynomials

(i) $$P_n(Y)=\sum_{h=0}^{n}\frac{\left(\frac{1}{2}\right)_n\left(\frac{1}{2}\right)_n 2^n n!\,[(f_a)]_n(-)^h(2h+1)}{(n-h)!\;[(d_e)]_n(n+h+1)!}\times F\begin{bmatrix}-n+h,\ -n-h-1,\ 1-(d_e)-n\,; & \\ & -2\\ -2n,\ \frac{1}{2}-n,\ 1-(f_a)-n\,; & \end{bmatrix}C_h(Y-1)$$

(ii) $$P_n^{(a',b')}(Y)=\sum_{h=0}^{n}\frac{\left(\frac{1}{2}\right)_n 2^n n!\,[(f_a)]_n(-)^h(2h+1)(1+a')_n}{(n-h)!\,[(d_e)]_n(n+h+1)!(1+a'+b')_n}\times F\begin{bmatrix}-n+h,\ -n-h-1,\ 1-(d_e)-n,\ -a'-n\,; & \\ & -2\\ -a'-b'-2h,\ 1-(f_a)-n,\ -n,\ \frac{1}{2}-n\,; & \end{bmatrix}C_h(Y-1)$$

(iii) $$C_n^{\eta}(Y)=\sum_{h=0}^{n}\frac{(\eta)_n\left(\frac{1}{2}\right)_n 2^n n!\,[(f_a)]_n(-)^h(2h+1)}{[(d_e)]_n(n-h)!(n+h+1)!}$$
$$\times F\begin{bmatrix}-n+h,\ -n-h-1,\ 1-(d_e)-n,\ \frac{1}{2}-n-\eta,\ -n-\eta\,; & \\ & -2\\ 1-2n-2\eta,\ 1-n-\eta,\ \frac{1}{2}-n,\ -n,\ 1-(f_a)-n\,; & \end{bmatrix}C_h(Y-1)$$

(iv) $$Z_n(y)=\sum_{h=0}^{n}\frac{\left(\frac{1}{2}\right)_n 2^n\left(\frac{1}{2}\right)_n[(f_a)]_n(-)^h(2h+1)n!}{(n-h)!\,[(d_e)]_n(n+h+1)!}\times F\begin{bmatrix}-n+h,\ -n-h-1,\ -n,\ 1-(d_e)-n\,; & \\ & -2\\ \frac{1}{2}-n,\ -2n,\ 1-(f_a)-n\,; & \end{bmatrix}C_h(y)$$

(v) $$H_n(\xi,P,V)=\sum_{h=0}^{n}\frac{(\xi)_n\left(\frac{1}{2}\right)_n\left(\frac{1}{2}\right)_n(-2)^{2n}n!\,[(f_a)]_n(-)^h(2h+1)}{(P)_n[(d_e)]_n(n-h)!(n+h+1)!}\times F\begin{bmatrix}-n+h,\ -n-h-1,\ 1-(d_e)-n,\ 1-P-n\,; & \\ & 1\\ 1-\xi-n,\ -2n,\ 1-(f_a)-n\,; & \end{bmatrix}C_h(V)$$

THEOREM. *If*

$$\sum_{n=0}^{\infty}C_n(y^{r_1})t^n=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-)^n(2n+1)_k 2^{2n}y^{r_1 n}[(d_e)]_n t^{n+k}}{n!k!\,[(f_a)]_n}\tag{3.7}$$

then

$$C_n(y^{r_1})=\sum_{h=0}^{n}\frac{2^{2n}[(d_e)]_n(\alpha+b_1)_n(v/x)^{n-h}(-)^h}{[(f_a)]_n(\alpha+b_1)_h(n-h)!(\mu E)^n}$$

$$\times\frac{[(N_1(i,j))]_{n+h}[1-(N_2(i,j))]_{n+h}}{[(M_1(i,j))]_{n+h}[1-(M_2(i,j))]_{n+h}}\overline{R}^{\,1:-:-:-:1}_{h:1:0:0}(x,y)$$

$$\times F\left[\begin{matrix}-n,\ \Delta(2:-2n),\ 1-(M_1(i,j))-n,\ (M_2(i,j))-n,\\ 1-(f_a)-n: \\ -2n,\ 1-(d_e)-n,\ 1-(N_1(i,j))-n,\ (N_2(i,j))-n: \\ \dfrac{(-)^{(e-a+\sum_{2}^{q+1}B_j-\sum_{1}^{p}A_j-1)}(\mu Ex)}{v}\end{matrix}\right] \tag{3.8}$$

PROOF. Proceeding exactly similar as above we can find (3.8).

Special cases of (3.8)

As usual, we get the following results;

(i) $$C_n(y)=\sum_{h=0}^{n}\frac{\left(\frac{1}{2}\right)_n 2^{2n}[(d_e)]_n(-)^h H_{2h}(y)}{[(f_a)]_n\left(\frac{1}{2}\right)_h(n-h)!2^{2n}h!}$$

$$\times F\left[\begin{matrix}-n,\ \Delta(2:-2n),\ 1-(f_a)-n; \\ -2n,\ 1-(d_e)-n,\ \frac{1}{2}-n;\end{matrix}\ (-)^{e-a-1}\right]$$

(ii) $$C_n(y)=\sum_{h=0}^{n}\frac{2^{2n}[(d_e)]_n(1+\lambda)_n(-)^h L_h^{(\lambda)}(y)}{[(f_a)]_n(-)^n(n-h)!(1+\lambda)_h}$$

$$\times F\left[\begin{matrix}-n,\ \Delta(2:-2n),\ 1-(f_a)-n: \\ -2n,\ 1-(d_e)-n,\ -\lambda-n:\end{matrix}\ (-)^{e-a}\right]$$

Similarly the expansions of Hermite, Laguerre, Jacobi polynomials etc. in terms of $\overline{R}_n(x,y)$ and vice-versa can be easily deduced.

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