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Converse problems of $S^n \times S^n$

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§ 0. INTRODUCTION

It is well known that a hypersurafce of an odd-dimensional sphere with canonical contact structure admits the so-called (f, g, u, v, λ)-structure ([4], [13], [16] etc.).

Several results concerned with (f, g, u, v, λ) structure have been found in [2], [4], [9], [10], [13], [16] etc.

Many authors ([1], [3], [6], [7], [11]) studied hypersurfaces M of an odd -dimensional sphere under the condition that the structure tensor f induced on M and the second fundamental tensor H anticommute, and proved the hypersurface to be a sphere or a product of two spheres.

On the other hand, H. Suzuki ([9]) investigated the integrability conditions of an almost complex structure F constructed from (f, g, u, v, λ)- structure, and characterized a hypersurface of an odd-dimensional sphere by using local components of the Nijenhuis tensor of F.

In the present paper, we characterized hypersurfaces M of an odd-dimensional sphere $S^{2n+1}(1)$ under the one of integrability conditions of F above.

In §1, we recall fundamental properties and structure equations for hypersurfaces immersed in $S^{2n+1}(1)$, and define the (f, g, u, v, λ)-structure induced on M to be partially integrable.

§2 is devoted to study an Einstein hypersurface with partially integrable structure of $S^{2n+1}(1)$.

In the last §3, we examine hypersurfaces with the same kind of this structure of $S^{2n+1}(1)$ over which the Ricci curvature is covariantly constant.

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§ 1. (f, g, v, λ)-STRUCTURE INDUCED ON A HYPERSUR FACEIOF $S^{2n+1}(1)$

Let $S^{2n+1}(1)$ be a $(2n+1)$-dimensional sphere of radius 1 covered by a system of coordinate neighborhoods $\{U; y^h\}$ in a Euclidean space E, where and in the sequel, the indices h, i, j and k run over the range $\{1, 2, \cdots, 2n+1\}$. As is well known, $S^{2n+1}(1)$ admits a canonical contact metric structure (F_i^h, g_{ji}, F^h), which is induced from the natural Kaehlerian structure equipped on E. Then the structure tensors of $S^{2n+1}(1)$ satisfy

$$(1.1)\quad \begin{cases} F_j^h F_i^j = -\delta_i^h + F_i F^h, \quad F_j F_i^j = 0, \quad F_j^h F^j = 0 \\ F_j^k F_i^h g_{kh} = g_{ji} - F_j F_i \quad F_j F^j = 1, F_j = g_{ji} F_i, \end{cases}$$

and

$$(1.2)\qquad D_j F^h = F_j^h, \quad D_j F_i^h = -g_{ji} F^h + \delta_j^h F_i,$$

where D_j denotes the operator of covariant differentiation with respect to the metric g_{ji}.

Let M be a $2n$-dimensional orientable and connected hypersurface in $S^{2n+1}(1)$ covered by a system of coordinate neighborhoods $\{V; x^c\}$, where and throughout this paper, the indices a, b, c, d and e run over the range $\{1, 2, \cdots, 2n\}$ and the summation convention will be used with respect to these indices. Then the local parametric expression of M is represented by $y^h = y^h(x^a)$. If we put $B_b^h = \partial_b y^h$, $\partial_b = \partial/\partial x^b$, then B_b^h are $2n$ linearly independent vectors tangent to M. The first fundamental tensor g_{cb} of M is given by $g_{cb} = B_c^j B_b^i g_{ji}$. Since M is orientable, there is a uniquely determined unit normal vector N^h for each point of M. Denoting by ∇_c the operator of van der Waerden-Bortolotti covariant differentiation along, the hypersurface M, we obtain the equations of Gauss and Weingarten;

$$(1.3)\qquad \nabla_c B_b^h = h_{cb} N^h, \quad \nabla_c N^h = -h_c^a B_a^h,$$

respectively, where h_{cb} are components of the second fundamental tensor of M and $h_c^a = h_{cb} g^{ab}$, g^{ab} being contravariant components of g_{ab}.

The transforms $F_i^h B_c^i$ of B_c^i by F_i^h can be expressed as linear combinations of B_c^h and N^h, that is,

$$(1.4)\qquad F_i^h B_c^i = f_c^a B_a^h - u_c N^h,$$

where f_c^a is a tensor field of type (1, 1) and u_c a 1-form on M. And the transform $F_i^h N^i$ of N^i by F_i^h, being orthogonal to N^h, can be written as

$$(1.5)\qquad F_i^h N^i = u^a B_a^h,$$

where $u^a = u_b g^{ab}$ is a vector field on M. Similarly we can put

$$(1.6)\qquad F^h = v^a B_a^h + \lambda N^h,$$

where v^a is a vector field and λ a function on M.

Applying F to the both sides of (1.4)～(1.6) respectively and using (1.1) and these equations, we find (cf. [1], [11])

$$(1.7)\quad \begin{cases} f_c{}^e f_e{}^a = -\delta_c{}^a + u_c u^a + v_c v^a, \\ u_e f_c{}^e = -\lambda v_c, \quad v_e f_c{}^e = \lambda u_c, \\ f_e{}^a u^e = \lambda v^a, \quad f_e{}^a v^e = -\lambda u^a, \\ u_e v^e = 0, \quad u_e u^e = v_e v^e = 1-\lambda^2, \\ g_{de} f_c{}^d f_b{}^e = g_{cb} - u_c u_b - v_c v_b. \end{cases}$$

Thus, M admits the so-called (f, g, u, v, λ)-structure ([13], [16]).

If we put $f_{cb} = f_c{}^e g_{eb}$, then we easily see that $f_{cb} = -f_{bc}$.

Differentiating (1.4)～(1.6) covariantly along M and taking account of (1.2) and (1.3), we find (cf. [1], [11])

$$(1.8)\quad \nabla_c f_b{}^a = -g_{cb} v^a + \delta_c{}^a v_b + h_{cb} u^a - h_c{}^a u_b,$$

$$(1.9)\quad \nabla_c u_b = \lambda g_{cb} + h_{ce} f_b{}^e,$$

$$(1.10)\quad \nabla_c v_b = f_{cb} + \lambda h_{cb},$$

$$(1.11)\quad \nabla_c \lambda = -u_c - h_{ce} v^e.$$

We note from (1.10) that the set $N = \{x \in M \mid \lambda^2(x) = 1\}$ is bordered because h_{cb} is symmetric and f_{bc} is skew-symmetric with respect to c and b.

We now define a tensor field S of type (0, 2) by

$$(1.12)\quad S_{cb} = f_c{}^e \nabla_e u_b - f_b{}^e \nabla_e u_c - (\nabla_c f_b{}^e - \nabla_b f_c{}^e) u_e - \lambda(\nabla_c v_b - \nabla_b v_c).$$

If S_{cb} vanishes identically, it is said that the (f, g, u, v, λ)-structure is partially integrable ([9]).

Substiuting (1.8)～(1.10) into (1.12), we find

$$(1.13)\quad S_{cb} = -u_c v_b + u_b v_c + (h_{ce} u^e) u_b - (h_{be} u^e) u_c.$$

Since ambient manifold is unit sphere, the equations of Gauss and Codazzi are given respectively by

$$(1.14)\quad K_{dcb}{}^a = \delta_d{}^a g_{cb} - \delta_c{}^a g_{db} + h_d{}^a h_{cb} - h_c{}^a h_{db},$$

$$(1.15)\quad \nabla_c h_{ba} - \nabla_b h_{ca} = 0,$$

$K_{dcb}{}^a$ being components of the curvature tensor of M.

Contraction (1.14) gives

$$(1.16)\quad K_{cb} = (2n-1) g_{cb} + h h_{cb} - h_{ce} h_b{}^e,$$

where K_{cb} is the Ricci tensor and $h = h_e{}^e$ the mean curvature of M. Thus the scalar curvature K of M is written in the form

$$K = 2n(2n-1) + h^2 - h_{cb}h^{cb} \tag{1.17}$$

in terms of the second fundamental form. If $\nabla_d K_{cb} = 0$, then the hypersurface M is said to be Ricci parallel. In particular, if

$$K_{cb} = \frac{K}{n} g_{cb}, \tag{1.18}$$

then M is said to be Einstein.

§2. EINSTEIN HYPERSURFACES OF $S^{2n+1}(1)$

In this section we suppose that the hypersurface M of $S^{2n+1}(1)$ is Einstein and has partially integrable (f, g, u, v, λ)-structure. Then we have from (1.13), (1.16) and (1.17)

$$(h_{ce}u^e)u_b - (h_{be}u^e)u_c + u_b v_c - u_c v_b = 0, \tag{2.1}$$

$$h h_{cb} - h_{ce}h_b{}^e = \{K - 2n(2n-1)/2n\} g_{cb}. \tag{2.2}$$

Transvecting (2.1) with u^b and using (1.7), we find

$$h_{ce}u^e = -v_c + \alpha u_c \tag{2.3}$$

because the set N is bordered, where we have put

$$h_{cb}u^c u^b = \alpha(1-\lambda^2). \tag{2.4}$$

If we transvect (2.2) with $u^c u^b$ and take account of (1.7) and (2.3), we get

$$K = 2n(2n-2+h\alpha-\alpha^2) \tag{2.5}$$

because the function $1-\lambda^2$ does not vanish on M. Hence, (2.2) reduces to

$$h_{ce}h_b{}^e = h h_{cb} - (h\alpha - \alpha^2 - 1) g_{cb}. \tag{2.6}$$

Transvection u^b gives

$$h_{ce}v^e = (h-\alpha)v_c - u_c \tag{2.7}$$

with the aid of (2.3).

Differentiating this covariantly and making use of (1.9) and (1.10), we find

$$(\nabla_c h_{be})v^e + h_b{}^e(f_{ce} + \lambda h_{ce}) = \{\nabla_c(h-\alpha)\}v_b + (h-\alpha)(f_{cb} + \lambda h_{cb}) - (\lambda g_{cb} + h_{ce}f_b{}^e),$$

from which, taking the skew-symmetric part and using (1.15),

$$\{\nabla_c(h-\alpha)\}v_b - \{\nabla_b(h-\alpha)\}v_c + 2(h-\alpha)f_{cb} = 0. \tag{2.8}$$

Transvecting (2.8) with v^b and using (1.7), we find

$$(1-\lambda^2)\nabla_c(h-\alpha) = \{v^e\nabla_e(h-\alpha)\}v_c - 2(h-\alpha)\lambda u_c.$$

Therefore, the last two equations follow that

$$(h-\alpha)\{(1-\lambda^2)f_{cb} + \lambda(u_b v_c - u_c v_b)\} = 0.$$

If we transvect this with f^{cb} and take account of (1.7), we obtain $(n-1)(h-\alpha)=0$ because $1-\lambda^2$ does not vanish on M, and hence $h=\alpha$ if $n>1$. Thus (2.6) and (2.7) reduce respectively to

(2.9) $$h_{ce}h_b^e=\alpha h_{cb}+g_{cb},$$

(2.10) $$h_{ce}v^e=-u_c,$$

and hence λ is constant on M because of (1.11).

Now, differentiating (2.3) covariantly and substituting (1.9) and (1.10), we find

$$(\nabla_c h_{be})u^e+h_b^e(\lambda g_{ce}+h_{ca}f_e^a)=-f_{cb}-\lambda h_{cb}+(\nabla_c\alpha)u_b+\alpha(\lambda g_{cb}+h_{ce}f_b^e),$$

from which, taking the skew-symmetric part with respect to the indices c and b and using (1.15),

(2.11) $$2h_{be}h_{ca}f^{ea}=2f_{bc}+(\nabla_c\alpha)u_b-(\nabla_b\alpha)u_c+\alpha(h_{ce}f_b^e-h_{be}f_c^e).$$

Transvecting this with u^b and making use of (1.7), (2.3) and (2.10), we obtain

(2.12) $$(1-\lambda^2)\nabla_c\alpha=Au_c-\lambda(\alpha^2+4)v_c,$$

where we have put $A=u^e\nabla_e\alpha$.

Since λ is constant, if we differentiate this covariantly and use (1.9) and (1.10), then we have

$$(1-\lambda^2)\nabla_c\nabla_b\alpha=(\nabla_cA)u_b+A(\lambda g_{cb}+h_{ce}f_b^e)-2\alpha\lambda(\nabla_c\alpha)v_b-\lambda(\alpha^2+4)(f_{cb}+\lambda h_{cb})$$

or, take the skew-symmetric part with respect to c and b,

(2.13) $$(\nabla_cA)u_b-(\nabla_bA)u_c+A(h_{ce}f_b^e-h_{be}f_c^e)-2\alpha\lambda\{(\nabla_c\alpha)v_b-(\nabla_b\alpha)v_c\}$$
$$-2\lambda(\alpha^2+4)f_{cb}=0.$$

Transvecting this with u^b and making use of (1.7), (2.3) and (2.10), we get

(2.14) $$(1-\lambda^2)\nabla_cA=(u^e\nabla_eA)u_c-\lambda\{3\alpha A+2\lambda(\alpha^2+4)\}v_c.$$

Substituting (2.12) and (2.14) into (2.13), we have

$$A(1-\lambda^2)(h_{ce}f_b^e-h_{be}f_c^e)-2\lambda(1-\lambda^2)(\alpha^2+4)f_{cb}-\lambda\{\alpha A+2\lambda(\alpha^2+4)\}(v_cu_b-v_bu_c)=0,$$

from which, transvecting f^{cb} and using (1.7), (2.3), (2.10) and the fact that $h=\alpha$,

$$-A\alpha\lambda^2-\lambda(\alpha^2+4)\{2n-2(1-\lambda^2)\}+\lambda^2\{\alpha A+2\lambda(\alpha^2+4)\}=0$$

bacause $1-\lambda^2$ does not vanish on M, and consequently $\lambda=0$ if $n>1$. Therefore (2.12) and (2.14) become respectively

$$(1-\lambda^2)\nabla_c\alpha=Au_c,\quad (1-\lambda^2)\nabla_cA=(u^e\nabla_eA)u_c.$$

Using these facts, (2.13) reduces to $A(h_{ce}f_b^e-h_{be}f_c^e)=0$. Thus (2.11) implies $A(h_{be}h_a{}^e f_c{}^a - f_{bc}) = 0$. Transvection f^{bc} yields

$$A(h_{cb}h^{cb} - \alpha^2 + 2n - 4) = 0 \tag{2.15}$$

with the aid of (1.7), (2.3) and (2.10).

On the other side, we have from (2.9) that $h_{cb}h^{cb} = \alpha^2 + 2n$ because $h - \alpha = 0$. Thus, (2.15) means that $A = 0$ if $n > 1$ and consequently α is a constant on M. Hence we see from (2.9) that h_c^a has exactly two constant pricipal curvatures $\sigma_1 = (\alpha + \sqrt{\alpha^2 + 4})/2$ and $\sigma_2 = (\alpha - \sqrt{\alpha^2 + 4})/2$

Since α is constant, by differentiating (2.9) covariantly, we find

$$(\nabla_d h_{ce})h_b{}^e + h_c{}^e(\nabla_d h_{be}) = \alpha \nabla_d h_{cb}, \tag{2.16}$$

from which, taking the skew-symmetric part with respect to the indices d and c, and using (1.15),

$$h_c{}^e(\nabla_d h_{be}) - h_d{}^e(\nabla_c h_{be}) = 0,$$

from which, using (1.15)

$$h_c{}^e(\nabla_d h_{be}) - h_b{}^e(\nabla_d h_{ce}) = 0. \tag{2.17}$$

Substituting (2.17) into (2.16), we get

$$2h_c{}^e(\nabla_d h_{be}) = \alpha \nabla_d h_{cb}.$$

Transvecting this with $h_a{}^c$ and using (2.9), we easily verify that $\nabla_d h_{cb} = 0$ (cf. [2]). Thus the eigenspaces corresponding σ_1 and σ_2 define respectively p-dimensional and $(2n-p)$-dimensional distributions D_1 and D_2 over M which are both parallel and integrable. Moreover each integral manifold of D_1 is totally geodesic in M and so does each integral manifold of D_2.

Assuming the hypersurface is complete, in usual way ([5]), M is a product of two spheres $S^p \times S^{2n-p}$ and since M is Einstein, we have $p = n$.

Thus we have

Theorem 1(cf. [5]). Let M be a complete and connected Einstein hypersurface of a $(2n+1)$ dimensional $(n > 1)$ sphere $S^{2n+1}(1)$. If the induced (f, g, u, v, λ)-structure on M is partially integrable, then M is a product of two spheres $S^n \times S^n$.

§3. HYPERSURFACES WITH PARALLEL RICCI TENSOR OF $S^{2n+1}(1)$

In the present section we assume that the hypersurface M of $S^{2n+1}(1)$ has parallel Ricci tensor and partially integrable (f, g, u, v, λ)-structure. Then we have (2.3).

Moreover, if we suppose that the Sasakian structure vector F^h is tangent to M, that is, $\lambda = 0$, then we get (2.10).

Transvecting (1.16) with u^c and v^c respectively and using (2.3) and (2.10), we find

(3.1) $$K_{be}u^e = (2n-2+h\alpha-\alpha^2)u_b + (\alpha-h)v_b,$$

(3.2) $$K_{be}v^e = 2(n-1)v_b + (\alpha-h)u_b.$$

Differentiating (3.2) covariantly along M and subsituting (1.9) and (1.10) with $\lambda=0$, we find

(3.3) $$K_{be}f_c{}^e = 2(n-1)f_{cb} + \{\nabla_c(\alpha-h)\}u_b + (\alpha-h)h_{ce}f_b{}^e,$$

because of $\nabla_a K_{cb} = 0$, from which, transvecting u^b and taking account of (1.7) with $\lambda=0$ and (3.1), $\nabla_c(\alpha-h)=0$ and hence $\alpha-h$ is constant on M. Thus (3.3) becomes

$$\{(2n-1)g_{be} + hh_{be} - h_{ba}h_e{}^a\}f_c{}^e = 2(n-1)f_{cb} + (\alpha-h)h_{ce}f_b{}^e$$

with the help of (1.16), or equivalently

(3.4) $$h_{ba}h_e{}^a f_c{}^e = f_{cb} + h(h_{ce}f_b{}^e + h_{be}f_c{}^e) - \alpha h_{ce}f_b{}^e.$$

Since the Ricci tensor of M is parallel, by differentiating (3.1) covariantly, we obtain

(3.5) $$K_{ce}h_{ba}f^{ea} = \{\nabla_b(h\alpha-\alpha^2)\}u_c + (\alpha-h)f_{bc} + (2n-2+h\alpha-\alpha^2)h_{be}f_c{}^e$$

because of (1.9), (1.10) and the fact that $\alpha-h$ is a constant. Transvecting this with u^c and making use of (1.7) with $\lambda=0$ and (3.1), we also find $\nabla_c(h\alpha-\alpha^2)=0$. Thus (3.5) reduces to

$$-h_b{}^a K_{ce}f_a{}^e = (2n-2+h\alpha-\alpha^2)h_{be}f_c{}^e + (\alpha-h)f_{bc}.$$

Substituting (3.3) with $\nabla_c(\alpha-h)=0$ into this, we obtain

$$-h_b{}^a\{(2n-2)f_{ac} + (\alpha-h)h_{ae}f_c{}^e\} = (2n-2+h\alpha-\alpha^2)h_{be}f_c{}^e + (\alpha-h)f_{bc},$$

or, using (3.4)

$$(\alpha-h)^2(h_{ce}f_b{}^e + h_{be}f_c{}^e) = 0.$$

Since $\alpha-h$ is a constant, by continuity, we have $\alpha=h$ or

(3.6) $$h_{ce}f_b{}^e + h_{be}f_c{}^e = 0$$

on M.

First of all, if $\alpha=h$ on M, (3.1) ~ (3.3) reduces respectively to

(3.7) $$K_{be}u^e = 2(n-1)u_b,\quad K_{be}v^e = 2(n-1)v_b,$$

(3.8) $$K_{be}f_c{}^e = 2(n-1)f_{cb}.$$

Transvecting (3.8) with f_a^c and using (3.7), we find $K_{ba}=2(n-1)g_{ba}$. Thereby M is an Einstein space. According to Theorem 1, by completeness, M is $S^n\times S^n$.

Secondly, if (3.6) holds, (3.4) becoms

$$h_{ba}h_e^a f_c^e=f_{cb}+\alpha h_{be}f_c^e.$$

Transvecting this with f_d^c and using (1.7), we find

$$h_{ba}h_e^a(-\delta_d^e+u_du^e+v_dv^e)=-g_{bd}+u_du_d+v_bv_d+\alpha h_{be}(-\delta_d^e+u_du^e+v_dv^e),$$

from which, using (2.3) and (2.10),

$$(3.9)\qquad h_{ce}h_b^e=g_{cb}+\alpha h_{cb}.$$

As in the proof of Theorem 1, we have from (2.3) $\nabla_c\alpha=Au_c$ because of $\lambda=0$. Differentiation covariantly gives

$$\nabla_b\nabla_c\alpha=(\nabla_bA)u_c+Ah_{be}f_c^e$$

with the aid of (1.9) with $\lambda=0$. If we take tke skew-symmetric part of this and use (3.6), we get

$$(\nabla_bA)u_c-(\nabla_cA)u_b+2Ah_{be}f_c^e=0.$$

Transvecting this equation with u^c and using (1.7) with $\lambda=0$, we find $\nabla_bA=(u^e\nabla_eA)u_b$. Thus, it follows that $Ah_{be}f_c^e=0$. Transvection f_a^c yields

$$(3.10)\qquad A\{h_{ab}+(u_av_b+v_au_b)-\alpha u_au_b\}=0$$

because of (2.3) and (2.10).

Transvecting (3.10) with g^{ab} and (1.7) with $\lambda=0$, we obtain

$$(3.11)\qquad A(h-\alpha)=0.$$

If we transvect also (3.10) with h^{ab} and use (2.3), (2.10) and (3.9), we have

$$A(2n+\alpha h-2-\alpha^2)=0.$$

The last two relationships give $A=0$ for $n>1$ and hence α is a constant on M. Denoting by σ the principal curvature corresponding to an eigenvector h_c^a, the equation (3.9) implies $\sigma^2-\alpha\sigma-1=0$. Since α is a constant on M, the second fundamental tensor h_c^a has exactly two constant principal curvatures $\sigma_1=(\alpha+\sqrt{\alpha^2+4})/2$ and $\sigma_2=(\alpha-\sqrt{\alpha^2+4})/2$.

On the other hand, taking account of $\sigma_1^2=\alpha\sigma_1+1$, we see from (2.3) and (2 10) that

$$(3.13)\qquad h_e^a(\sigma_1u^e-v^e)=\sigma_1(\sigma_1u^a-v^a).$$

Therefore $\sigma_1 u^a - v^a$ is an eigenvector of h_c^a corresponding σ_1. If we assume that there exists another eigenvector X^a orthogonal to u^a and v^a corresponding σ_1, then we find from (3.6)

$$h_c^a (f_e^c X^e) = \sigma_1 (f_e^a X^e).$$

Thus, it follows that $f_e^a x^e$ is also an eigenvector of h_c^a corresponding σ_1, which is orthogonal to X^a and $\sigma_1 u^a - v^a$. Hence the mutipicity of the principal curva'ure σ_1 is odd. Since dim $M = 2n$, so does σ_2. As in the proof of Theorem 1, by completeness, M is a product of two spheres $S^p \times S^{2n-p}$, p being an odd number.

Developed above, we conclude

Theorem 2. Let M be a complete and connected hypersurface with parallel Ricci tensor of an odd-dimensional sphere $S^{2n+1}(1)$, $(n > 1)$. If the induced (f, g, u, v, λ)-structure is partially integrable and the Sasakian structure vector F of $S^{2n+1}(1)$ is tangent to M, then M is a product of two spheres $S^p \times S^{2n-p}$, p being an odd number or $S^n \times S^n$.

From (1.9) and (1.11) we have

$$\nabla_c(\lambda u^c) = -(u_c + h_{ce} v^e) u^c + 2n\lambda^2,$$

or, substitute (2.3), $\nabla_c(\lambda u^c) = 2n\lambda^2$. By compactness, we see that $\lambda = 0$ on M.

Thus we have

Corollary 3. Let M be a compact and connected hypersurface with parallel Ricci tensor of an odd-dimensional sphere $S^{2n+1}(1)$, $(n > 1)$. If the induced (f, g, u, v, λ)-structure is partially integrable, then M is a product of two spheres $S^p \times S^{2n-p}$, p being an odd number or $S^n \times S^n$.

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