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# ON CONFORMAL $P$-KILLING VECTOR FIELDS IN ALMOST PARACONTACT RIEMANNIAN MANIFOLDS

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## 0. Introduction

A few years ago, I. Satō introduced a notion of manifolds with almost paracontact structure and studied several interesting properties of the mani fold which are closely similar to the ones of almost contact manifolds([10, 11]).

In the previous papers, we studied certain infinitesimal transformations in $P$-Sasakian manifolds and got several results about them ([4. 5. 6. 8]).

On the other hand, we defined a conformal $C$-killing vector field in an almost contact Riemannian manifold and we proved that every special conformal $C$-killing vector field in a compact Sasakian manifold (dim $>$ 5) is special $C$-Killing ([7]).

In this paper, we shall define a conformal $P$-Killing vector field in an almost paracontact Riemannian manifold and we shall consider the properties about such the vector field.

In § 1, as a preparation of the latter section, we shall recall almost paracontact Riemannian manifolds and properties of certain infinitesimal transformations in $P$-Sasakian manifolds. In § 2, we shall define a conformal $P$-Killing vector field in almost paracontact Riemannian manifolds and we now especially consider conformal $P$-Killing vector fields in $P$-Sasakian and $SP$-Sasakian manifolds.

We assume that our manifolds are connected and satisfies the axiom of second countability and all tensor fields are of class $C^\infty$.

## 1. Preliminaries

An $n$-dimensional differentiable manifold $M^n$ is called to have an almost paracontact structure if there is given triplet $(\phi_\mu{}^\lambda, \xi^\lambda, \eta_\lambda)$ of (1. 1)-tensor $\phi_\mu{}^\lambda$ contravariant vector field $\xi^\lambda$ and covariant vector field $\eta_\lambda$ in $M^n$ which satisfy the following equations;

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(1.1) $\eta_\lambda \xi^\lambda = 1,$

(1.2) $\phi_\mu{}^\gamma \phi_\gamma{}^\lambda = \delta_\mu^\lambda - \eta_\mu \xi^\lambda,$

where the indices $\nu, \mu, \cdots, \lambda$ run over the range $1, 2, \cdots, n$. We call a manifold $M^n$ with an almost paracontact structure an almost paracontact manifold.

By virtue of (1.1) and (1.2), we can easily show

(1.3) $\operatorname{rank}(\phi_\mu{}^\lambda) = n-1,$

(1.4) $\phi_\mu{}^\lambda \xi^\mu = 0, \quad \eta_\lambda \phi_\mu{}^\lambda = 0.$

Every almost paracontact manifold has a positive definite Riemannian metric $g_{\mu\lambda}$ such that

(1.5) $\eta_\mu = g_{\mu\lambda} \xi^\lambda,$

(1.6) $g_{\varepsilon\gamma} \phi_\mu{}^\varepsilon \phi_\lambda{}^\gamma = g_{\mu\lambda} - \eta_\mu \eta_\lambda.$

We call such a metric $g_{\mu\lambda}$ an associated Riemannian metric of the given almost paracontact structure and we call an almost paracontact manifold with an associated Riemannian metric an almost paracontact Riemannian manifold with an almost paracontact metric structure $(\phi_\mu{}^\lambda, \xi^\lambda, \eta_\lambda, g_{\mu\lambda})$.

In an almost paracontact Riemannian manifold, if the following equation

(1.7) $2\phi_{\mu\lambda} = \nabla_\mu \eta_\lambda + \nabla_\lambda \eta_\mu \quad (\phi_{\mu\lambda} = \phi_\mu{}^\varepsilon g_{\varepsilon\lambda})$

holds good, then we say such an almost paracontact Riemannian manifold a paracontact Riemannian manifold, where the operator $\nabla_\lambda$ denotes the covariant differentiation with respect to $g_{\mu\lambda}$.

Now, we consider an $n$-dimensional differentiable manifold $M^n$ with a positive definite Riemannian metric $g_{\mu\lambda}$ which admits a unit covariant vector field $\eta_\lambda$ satisfying

(1.8) $\nabla_\mu \eta_\lambda - \nabla_\lambda \eta_\mu = 0,$

(1.9) $\nabla_\nu \nabla_\mu \eta_\lambda = (-g_{\nu\mu} + \eta_\nu \eta_\mu) \eta_\lambda + (-g_{\nu\lambda} + \eta_\nu \eta_\lambda) \eta_\mu.$

If we put

(1.10) $\xi^\lambda = \eta_\mu g^{\mu\lambda}, \quad \phi_\mu{}^\lambda = \nabla_\mu \xi^\lambda,$

then it is easily seen that the manifold in consideration becomes an almost paracontact Riemannian manifold. So we call such a manifold a $P$-Sasakian manifold.

In a $P$-Sasakian manifold, the following relations hold good:

(1.11) $R_{\omega\nu\mu}{}^\varepsilon \eta_\varepsilon = g_{\omega\mu} \eta_\nu - g_{\nu\mu} \eta_\omega, \quad R_\mu{}^\varepsilon \eta_\varepsilon = -(n-1) \eta_\mu,$

where the tensor fields $R_{\omega\nu\mu}{}^\lambda$ and $R_{\mu\lambda}$ are respectively the curvature tensor and the Ricci tensor with respect to $g_{\mu\lambda}$.

Let us consider an $n$-dimensional differentiable manifold with a positive definite Riemannian metric $g_{\mu\lambda}$ which admits a unit covariant vector field $\eta_\lambda$ satisfying

(1.12) $\nabla_\mu \eta_\lambda = \varepsilon (g_{\mu\lambda} - \eta_\mu \eta_\lambda),$

then we can easily show that the manifold in consideration is $P$-Sasakian.

So, such a manifold is called an $SP$-Sasakian manifold, where $\varepsilon$ is $+1$ or $-1$. T. Adati and T. Miyazawa proved the following

PROPOSITION 1.1. *A P-Sasakian manifold is an SP-Sasakian one if and only if the following relation holds good:*

(1.13) $\qquad \phi^2=(n-1)^2,$

*where we put* $\phi=trace(\phi_\mu{}^\lambda)$.

In a $P$-Sasakian manifold, we have from (1.9) and (1.11)

(1.14) $\qquad \nabla^\varepsilon\nabla_\varepsilon\xi^\lambda-R_\varepsilon{}^\lambda\xi^\varepsilon=0.$

Thus we have the following

PROPOSITION 1.2. *In a compact orientable P-Sasakian manifold, the vector field* $\xi^\lambda$ *is harmonic.*

Especially, if the manifold is $SP$-Sasakian, then from (1.12) and (1.13) we have $\nabla_\varepsilon\xi^\varepsilon=-(n-1)$.

Thus we have the following

PROPOSITION 1.3. *There does not exist a compact orientable SP-Sasakian manifold.*

REMARK. Recently, S. Sasaki proved that there does not exist a compact $SP$-Sasakian manifold [9].

A vector field $v^\lambda$ in an almost paracontact Riemannian manifold is said to be an infinitesimal automorphism if it leaves three tensors $\phi_\mu{}^\lambda, \eta_\lambda$ and $g_{\mu\lambda}$ invariant, that is,

(1.15) $\qquad \mathcal{L}(v)\phi_\mu{}^\lambda=0, \quad \mathcal{L}(v)\eta_\lambda=0, \quad \mathcal{L}(v)g_{\mu\lambda}=0,$

where $\mathcal{L}(v)$ denotes the Lie differentiation with respect to $v^\lambda$.

A vector field $v^\lambda$ in an almost paracontact Riemannian manifold is called an $\eta$-conformal Killing vector field if it satisfies the relation

(1.16) $\qquad \mathcal{L}(v)g_{\mu\lambda}=2\alpha(g_{\mu\lambda}-\eta_\mu\eta_\lambda),$

where $\alpha$ is a scalar function being called an associated function of $v^\lambda$. For an $\eta$-conformal Killing vector field, we proved the following

PROPOSITION 1.4. *Each* $\eta$*-conformal Killing vector field in a compact orientable P-Sasakian manifold is an infinitesimal automorphism.*

## 2. Conformal $P$-Killing vector fields

A vector field $v^\lambda$ in an almost paracontact Riemannian manifold $M^n$ is called a conformal $P$-Killing vector field with an associated function $h$ if it satisfies the relation

(2.1) $$\mathcal{L}(v)(g_{\mu\lambda}-\eta_\mu\eta_\lambda)=2h(g_{\mu\lambda}-\eta_\mu\eta_\lambda).$$

Especially, if an associated function is constant in $M^n$, then the vector field is called a homothetic one, and if a conformal $P$-Killing vector field $v^\lambda$ satisfies the condition $\eta(v)=\eta_\varepsilon v^\varepsilon=$constant, then the vector field is called a special one.

PROPOSITION 2.1. *If a P-Sasakian manifold admits a non-zero P-Killing vector field proportional to $\xi^\lambda$, then the manifold is necessarily SP-Sasakian.*

*Proof.* In general, (2.1) can be written as

(2.2) $$\mathcal{L}(v)g_{\mu\lambda}=f_\mu\eta_\lambda+f_\lambda\eta_\mu+2h(g_{\mu\lambda}-\eta_\mu\eta_\lambda),$$

where we have put

(2.3) $$f=\eta_\varepsilon v^\varepsilon, \quad f_\lambda=\nabla_\lambda f.$$

Now, let $v^\lambda=f\xi^\lambda$ be a conformal $P$-Killing vector field, then we can easily see

$$\mathcal{L}(v)g_{\mu\lambda}=f_\mu\eta_\lambda+f_\lambda\eta_\mu+2f\phi_{\mu\lambda},$$

where we put $\phi_{\mu\lambda}=\phi_\eta{}^\varepsilon g_{\varepsilon\lambda}$.

By virtue of (2.2) and the above equation, we get

(2.4) $$f\phi_{\mu\lambda}=h(g_{\mu\lambda}-\eta_\mu\eta_\lambda).$$

Transvecting (2.4) with $\phi_\nu{}^\lambda$ and taking account of (1.2) and (1.4), we have

$$f(g_{\mu\lambda}-\eta_\mu\eta_\lambda)=h\phi_{\mu\lambda}.$$

Thus we have from (2.4) and the above equation $h^2=f^2$, from which $\phi_{\mu\lambda}=\varepsilon(g_{\mu\lambda}-\eta_\mu\eta_\lambda)$, that is, the manifold is $SP$-Sasakian.

THEOREM 2.2. *In an SP-Sasakian manifold, each conformal P-Killing vector field $v^\lambda$ is decomposed into the following form:*

(2.5) $$v^\lambda=\eta(v)\xi^\lambda+w^\lambda,$$

*where $\eta(v)\xi^\lambda$ is a conformal P-Killing vector field proportional to $\xi^\lambda$, and $w^\lambda$ is a $\eta$-conformal Killing vector field.*

*Proof.* Let $v^\lambda$ be a conformal $P$-Killing vector field, then if we put (2.5), then it is clear that $\eta(v)\xi^\lambda$ and $w^\lambda$ satisfy the condition.

Conversely, let $f\xi^\lambda$ and $w^\lambda$ be a vector field proportional to $\xi^\lambda$ and a $\eta$-conformal Killing vector field respectively $f$ being a certain function in $M^n$, then it is easily shown that $f\xi^\lambda+w^\lambda$ is a conformal $P$-Killing vector field.

From (2.2) and (2.3), we can easily see

PROPOSITION 2.3. *Each special conformal P-Killing vector field is a $\eta$-conformal Killing vector field.*

Contracting $\xi^\mu$ to (2.2), we have

(2.6) $\mathcal{L}(v)\xi^\lambda=-(f_\varepsilon\xi^\varepsilon)\xi^\lambda.$

Substituting (2.2) and (2.6) into the identity

$$\mathcal{L}(v)\eta_\lambda=(\mathcal{L}(v)\xi^\varepsilon)g_{\varepsilon\lambda}+\xi^\varepsilon\mathcal{L}(v)g_{\varepsilon\lambda},$$

we obtain

(2.7) $\mathcal{L}(v)\eta_\lambda=f_\lambda.$

By virtue of (2.2) and the formula ([12])

$$\mathcal{L}(v)\{{}_\nu{}^\lambda{}_\mu\}=\frac{1}{2}g^{\lambda\gamma}\{\nabla_\nu\mathcal{L}(v)g_{\gamma\mu}+\nabla_\mu\mathcal{L}(v)g_{\nu\gamma}-\nabla_\gamma\mathcal{L}(v)g_{\nu\mu}\},$$

we have

(2.8) $\mathcal{L}(v)\{{}_\nu{}^\lambda{}_\mu\}=(\nabla_\nu f_\mu)\xi^\lambda+\phi_{\nu\mu}f^\lambda+h_\nu(\delta_\mu^\lambda-\eta_\mu\xi^\lambda)+h_\mu(\delta_\nu^\lambda-\eta_\nu\xi^\lambda)$
$-h^\lambda(g_{\nu\mu}-\eta_\nu\eta_\mu)-2h\phi_{\nu\mu}\xi^\lambda,$

where $\{{}_\nu{}^\lambda{}_\mu\}$ denotes the Christoffel symbol with respect to $g_{\mu\lambda}$ and we put $h_\lambda=\nabla_\lambda h$ and $f^\lambda=g^{\lambda\varepsilon}f_\varepsilon$.

Substituting (2.6) and (2.7) into the identity

$$\mathcal{L}(v)\phi_\mu{}^\lambda=\nabla_\nu\mathcal{L}(v)\xi^\lambda+(\mathcal{L}(v)\{{}_\nu{}^\lambda{}_\varepsilon\})\xi^\varepsilon,$$

we get

(2.9) $\mathcal{L}(v)\phi_\mu{}^\lambda=(h_\varepsilon\xi^\varepsilon)(\delta_\mu^\lambda-\eta_\mu\xi^\lambda)-\phi_\mu{}^\varepsilon f_\varepsilon\xi^\lambda-(f_\varepsilon\xi^\varepsilon)\phi_\mu{}^\lambda.$

Summing up (2.8) with respect to the indices $\mu$ and $\lambda$, we obtain

(2.10) $(n-1)h_\varepsilon\xi^\varepsilon-\phi f_\varepsilon\xi^\varepsilon=0.$

On the other hand, Lie differentiation of (1.2) with respect to $v^\lambda$ gives us

(2.11) $(h_\varepsilon\xi^\varepsilon)\phi_\mu{}^\lambda-(f_\varepsilon\xi^\varepsilon)(\delta_\mu^\lambda-\eta_\mu\xi^\lambda)=0,$

from which

(2.12) $\phi h_\varepsilon\xi^\varepsilon-(n-1)f_\varepsilon\xi^\varepsilon=0.$

Thus we have from (2.10) and (2.12) the following

PROPOSITION 2.4. *For a conformal $P$-Killing vector field in a $P$-Sasakian manifold, we have*

(2.13) $f_\varepsilon\xi^\varepsilon=\varepsilon h_\varepsilon\xi^\varepsilon$

*and especially, if the manifold is not $SP$-Sasakian, then we have*

(2.14) $f_\varepsilon\xi^\varepsilon=h_\varepsilon\xi^\varepsilon=0.$

From now on, we shall consider the case of a non-$SP$-$P$-Sasakian manifold, that is, we shall consider the case of $\phi^2\neq(n-1)^2$. Then for a conformal $P$-Killing vector field we have (2.14).

Substituting (2.14) into (2.6) and (2.9), we respectively have

(2.15) $\mathcal{L}(v)\xi^\lambda=0,$

(2.16) $\mathcal{L}(v)\phi_\mu{}^\lambda=-\phi_\mu{}^\varepsilon f_\varepsilon\xi^\lambda.$

Furthermore, by virtue of (2.2) and (2.16), we get

(2.17) $\mathcal{L}(v)\phi_{\mu\lambda}=-2h\phi_{\mu\lambda}.$

Next, substituting (2. 7) into the formula ([12])

$$\mathcal{L}(v)R_{\omega\nu\mu}{}^{\lambda}=\nabla_{\omega}\mathcal{L}(v)\{_{\nu}{}^{\lambda}{}_{\mu}\}-\nabla_{\nu}\mathcal{L}(v)\{_{\omega}{}^{\lambda}{}_{\mu}\},$$

we have

$$\begin{aligned}(2.18)\quad \mathcal{L}(v)R_{\omega\nu\mu}{}^{\lambda}=&-R_{\omega\nu\mu}{}^{\varepsilon}f_{\varepsilon}\xi^{\lambda}-(g_{\omega\mu}\eta_{\nu}-g_{\nu\mu}\eta_{\omega})f^{\lambda}+\phi_{\nu\mu}\nabla_{\omega}f^{\lambda}-\phi_{\omega\mu}\nabla_{\nu}f^{\lambda}\\&+\phi_{\omega}{}^{\lambda}\nabla_{\nu}f_{\mu}-\phi_{\nu}{}^{\lambda}\nabla_{\omega}f_{\mu}-h_{\nu}(\eta_{\mu}\phi_{\omega}{}^{\lambda}-\phi_{\omega\mu}\xi^{\lambda})+h_{\omega}(\eta_{\mu}\phi_{\nu}{}^{\lambda}-\phi_{\nu\mu}\xi^{\lambda})\\&-h_{\mu}(\eta_{\nu}\phi_{\omega}{}^{\lambda}-\eta_{\omega}\phi_{\nu}{}^{\lambda})+h^{\lambda}(\eta_{\nu}\phi_{\omega\mu}-\eta_{\omega}\phi_{\nu\mu})+(\nabla_{\omega}h_{\mu})(\delta_{\nu}{}^{\lambda}-\eta_{\nu}\xi^{\lambda})\\&-(\nabla_{\nu}h_{\mu})(\delta_{\omega}{}^{\lambda}-\eta_{\omega}\xi^{\lambda})-(\nabla_{\omega}h^{\lambda})(g_{\nu\mu}-\eta_{\nu}\eta_{\mu})+(\nabla_{\nu}h^{\lambda})(g_{\omega\mu}-\eta_{\omega}\eta_{\mu})\\&+2h(g_{\omega\mu}\eta_{\nu}-g_{\nu\mu}\eta_{\omega})\xi^{\lambda}-2h(\phi_{\nu\mu}\phi_{\omega}{}^{\lambda}-\phi_{\omega\mu}\phi_{\nu}{}^{\lambda}).\end{aligned}$$

REMARK. Taking account of (2. 18), if the vector field $w^{\lambda}$ in Proposition 2. 2 preserves the scalar curvature invariant, then $R=-(n-1)$ or the vector field $w^{\lambda}$ is an infinitesimal automorphism in $M^n$, where $R$ denotes the scalar curvature with respect to $g_{\mu\lambda}$.

Now, contracting $\eta_{\lambda}$ to (2. 18) and taking account of (1. 11), (2. 2), (2. 7) and (2. 14), we can get

$$\begin{aligned}(2.19)\quad &-\phi_{\nu\mu}\phi_{\omega\lambda}f^{\lambda}+\phi_{\omega\mu}\phi_{\nu\lambda}f^{\lambda}+h_{\nu}\phi_{\omega\mu}-h_{\omega}\phi_{\nu\mu}+\phi_{\omega\lambda}h^{\lambda}(g_{\nu\mu}-\eta_{\nu}\eta_{\mu})\\&-\phi_{\nu\lambda}h^{\lambda}(g_{\omega\mu}-\eta_{\omega}\eta_{\mu})=f_{\omega}\eta_{\nu}\eta_{\mu}-f_{\nu}\eta_{\omega}\eta_{\mu}+g_{\omega\mu}f_{\nu}-g_{\nu\mu}f_{\omega}.\end{aligned}$$

Transvecting (2. 19) with $g^{\nu\mu}$ and $\phi^{\nu\mu}$, respectively, we have

$$(n-1)f_{\omega}-\phi\phi_{\omega}{}^{\varepsilon}f_{\varepsilon}+(n-1)\phi_{\omega}{}^{\varepsilon}h_{\varepsilon}-\phi h_{\omega}=0,$$

$$-(n-1)\phi_{\omega}{}^{\varepsilon}f_{\varepsilon}+\phi f_{\omega}+\phi\phi_{\omega}{}^{\varepsilon}h_{\varepsilon}-(n-1)h_{\omega}=0.$$

Thus we have from the above two equations

$$(2.20)\qquad f_{\omega}+\phi_{\omega}{}^{\varepsilon}h_{\varepsilon}=0.$$

Hence we have

PROPOSITION 2. 5. *For a conformal P-Killing vector field in a non-SP-P-Sasakian manifold, a necessary and sufficient condition for the vector field to be a homothetic conformal P-Killing vector field is the vector field to be special.*

By virtue of Propositions 1. 1, 1. 2, 1. 3, 1. 4, 2. 1 and 2. 5, we have

THEOREM 2. 6. *Each homothetic conformal P-Killing vector field in a compact orientable P-Sasakian manifold is identically an infinitesimal automorphism.*

Now, we assume that the conformal $P$-Killing vector field $v^{\lambda}$ preserves the curvature tensor invariant, that is, the left hand side of (2. 18) is equal to zero. Then, transvecting that equation with $\xi^{\omega}$, we have

$$-f_{\nu}\eta_{\mu}\xi^{\lambda}+f_{\mu}(\delta_{\nu}{}^{\lambda}-\eta_{\nu}\xi^{\lambda})+2\mathrm{h}(\eta_{\nu}\eta_{\mu}-g_{\nu\mu})\xi^{\lambda}=0,$$

from which, we can show $h=0$.

Thus we have

PROPOSITION 2.7. *If a conformal P-Killing vector field in a P-Sasakian manifold preserves the curvature tensor invariant, then the vector field is an infinitesimal automorphism.*

Summing up (2.18) with respect to $\omega$ and $\lambda$, we obtain

$$(2.20)\quad \mathcal{L}(v)R_{\nu\mu}=-4(f_\nu\eta_\mu+f_\mu\eta_\nu)+(\nabla_\omega f^\omega)\phi_{\nu\mu}-(n-5)\nabla_\nu h_\mu +\phi\nabla_\nu f_\mu-\phi(h_\nu\eta_\mu+h_\nu\eta_\mu)-(\nabla_\omega h^\omega)(g_{\nu\mu}-\eta_\nu\eta_\mu)-2h\phi\phi_{\nu\mu}.$$

We assume that the vector field preserves the Ricci tensor invariant, then we have from (2.20)

$$(2.21)\quad -4(f_\nu\eta_\mu+f_\mu\eta_\nu)+(\nabla_\omega f_\omega{}^\omega)\phi_{\nu\mu}-(n-5)\nabla_\nu h_\mu+\phi\nabla_\nu f_\mu -\phi(h_\nu\eta_\mu+h_\mu\eta_\nu)-(\nabla_\omega h^\omega)(g_{\nu\mu}-\eta_\nu\eta_\mu)-2h\phi\phi_{\nu\mu}=0.$$

Transvecting (2.21) with $g^{\nu\mu}$ respectively, we have

$$\phi\nabla_\omega f^\omega-(n-3)\nabla_\omega h^\omega-\phi^2h=0,$$

$$(2.22)\quad (n-3)\nabla_\omega f^\omega-\phi\nabla_\omega h^\omega-(n-1)\phi h=0.$$

By virtue of (2.22), we have

$$(2.23)\quad \{\phi^2-(n-3)^2\}\nabla_\omega h^\omega+2\phi^2h=0.$$

Hence we get the following

PROPOSITION 2.7. *Let $v^\lambda$ be a Ricci preserving conformal P-Killing vector field in a P-Sasakian manifold $M^n$. Then we have*

(i) *If $\phi^2-(n-3)=0$ and $n\neq 3$, then the vector field is an infinitesimal automorphism.*

(ii) *If $\nabla_\omega h^\omega=0$, then $\phi=0$ or $h=0$.*

(a) *In the case of $\phi=0$, the vector field is homothetic and $n$=odd.*

(b) *In the case of $h=0$, the vector field is an infinitesimal automorphism.*

(iii) *If $\phi=0$ and $n\neq 3$, then the vector field is homothetic and $n$=odd.*

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