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STRONGLY θ-CONTINUOUS FUNCTIONS

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1. Introduction

T. Noiri in [2] has defined a function $f: X \to Y$ from a topological space X into a topological space Y to be ***strongly θ-continuous*** if for each $x \in X$ and each open V containing $f(x)$ there exists an open set U containing x such that $f(\mathrm{Cl}(U)) \subset V$. Clearly such functions are always continuous. The converse need not be true, however. If the reals $\mathbf{R}$ are given the open left ray topology, then the identity function $i: \mathbf{R} \to \mathbf{R}$ is continuous, but not strongly θ-continuous. We note that if X is regular, then any continuous $f: X \to Y$ is also strongly θ-continuous. Among the concepts needed for our investigation of strongly θ-continuous functions is that of a θ-closed set. The *θ-closure* of a set $A \subset X$, denoted by $\mathrm{Cl}_\theta(A)$, is $\{x \in X$: every closed neighborhood of x meets $A\}$ [4]. The subset A is θ-closed if $\mathrm{Cl}_\theta(A) = A$. Likewise, the ***$\theta$-interior*** of A, denoted by $\mathrm{Int}_\theta(A)$ is $\{x \in X$: some closed neighborhood of x lies in $A\}$. A set A is called ***θ-open*** if $\mathrm{Int}_\theta(A) = A$. Of course θ-open sets are open and θ-closed sets are closed. Furthermore, the complement of a θ-open set is θ-closed and the complement of a θ-closed set is θ-open. Lemma 3 of [4] shows that the collection of θ-open sets in a topological space (X, T) form a topology for X which we denote by T_θ. Finally, a net (x_α) in a topological space ***θ-converges*** to x if for each open V containing x the net (x_α) is eventually in $\mathrm{Cl}(V)$ [4].

2. Basic properties

THEOREM 1. *For any $f: X \to Y$ the following are equivalent:*

(a) *f is strongly θ-continuous.*

(b) *The inverse image of a closed set is θ-closed.*

(c) *The inverse image of an open set is θ-open.*

(d) *For each $x \in X$ and each net $x_\alpha \underset{\theta}{\to} x$, then the net $f(x_\alpha) \to f(x)$.*

Proof. (a) implies (b). Let $F \subset Y$ be closed and suppose that $f^{-1}(F)$ is

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not θ-closed in X. Then there is a point $x \notin f^{-1}(F)$ such that for every open U containing x, $\mathrm{Cl}(U) \cap f^{-1}(F) \neq \phi$. Since $f(x) \notin F$, $Y-F$ is an open set containing $f(x)$ having the property that no closed neighborhood of x will map into $Y-F$ under f. Consequently f is not strongly θ-continuous at x. This contradiction implies that $f^{-1}(F)$ is θ-closed.

(b) implies (c). Let V be open in Y. Then $Y-V$ is closed and by (b) $f^{-1}(Y-V)$ is θ-closed. But $X-f^{-1}(Y-V)=f^{-1}(V)$ is θ-open.

(c) implies (d). Let $x \in X$ and let $x_\alpha \underset{\theta}{\rightarrow} x$. Let V be any open set containing $f(x)$. Then by (c), $f^{-1}(V)$ is θ-open and contains x. Thus, there exists an open set U such that $x \in U \subset \mathrm{Cl}(U) \subset f^{-1}(V)$. The θ-convergence of x_α to x now implies that x_α is eventually in $\mathrm{Cl}(U)$ so that $f(x_\alpha)$ is eventually in V. This shows that $f(x_\alpha) \rightarrow f(x)$.

(d) implies (a). Suppose f is not strongly θ-continuous for some $x \in X$. Then there is an open set V containing $f(x)$ such that for every open U containing x, $f(\mathrm{Cl}(U) \not\subset V$. New consider the directed set $\mathcal{D}=\{(x_\alpha, \mathrm{Cl}(U_\alpha)\}$ ordered by reverse inclusion where U_α contains x and $x_\alpha \in \mathrm{Cl}(U_\alpha)$ such that $f(x_\alpha) \notin V$. Then the net $g: \mathcal{D} \rightarrow X$ defined by $g(x_\alpha, U_\alpha)=x_\alpha$ θ-converges to x, but the net fg does not converge to $f(x)$. The contradiction implies that f is strongly θ-continuous at x.

Observing that a set is θ-closed in (X, T) if and only if it is closed in (X, T_θ), Theorem 1 now allows us to conclude that $f: (X, T) \rightarrow Y$ is strongly θ-closed if and only if $f: (X, T_\theta) \rightarrow Y$ is continuous. Observe also that $i: (X, T) \rightarrow (X, T_\theta)$ in Figure 1 is continuous.

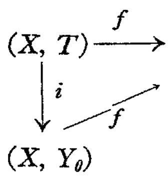

Figure 1

With these facts it is easy to obtain several results about strongly θ-continuous functions from known facts about continuous functions. An example of these is given in Theorem 2.

THEOREM 2. *Let* $f, g: (X, T) \rightarrow Y$ *be strongly* θ*-continuous and let* Y *be Hausdorff. Then the set* $A=\{x: f(x)=g(x)\}$ *is* θ*-closed in* X.

Proof. Since $f, g: (X, T_\theta) \rightarrow Y$ are continuous it is well known [1, Theorem 1.5, p. 140] that $A \subset (X, T_\theta)$ is closed. Thus $A \subset (X, T)$ is θ-closed.

THEOREM 3. *Let $f:X\to Y$ be a strongly θ-continuous injective function and let Y be Hausdorff. Then X is Urysohn.*

Proof. Let $x_1\neq x_2$ belong to X. Then $f(x_1)\neq f(x_2)$. The Hausdorff hypothesis on Y now insures the existence of disjoint open sets V_1 and V_2 containing $f(x_1)$ and $f(x_2)$, respectively. Thus, there exist open sets U_1 and U_2 containing x_1 and x_2, respectively, such that $f(\mathrm{Cl}(U_1))\subset V_1$ and $f(\mathrm{Cl}(U_2))\subset V_2$ because f is strongly θ-continuous. It follows that $\mathrm{Cl}(U_1)\cap\mathrm{Cl}(U_2)=\phi$ from which we conclude that X is Urysohn.

THEOREM 4. *Let $f:X\to Y$ be strongly θ-continuous and injective. If Y is a T_1-space, then X is Hausdorff.*

Proof. Let. $x_1\neq x_2$ belong to X. Then $f(x_1)\neq f(x_2)$ so there exists an open set V_1 containing $f(x_1)$ such that $f(x_2)\notin V_1$. Since f is strongly θ-continuous, there exists an open set U_1 containing x_1 such that $f(\mathrm{Cl}(U_1))\subset V_1$. Thus, $x_2\notin\mathrm{Cl}(U_1)$. Therefore, U_1 and $X-\mathrm{Cl}(U_1)$ are disjoint open sets separating x_1 and x_2.

THEOREM 5. *If $f:X\to Y$ is strongly θ-continuous and $g:Y\to Z$ is continuous, then the composition $gf: X\to Z$ is strongly θ-continuous.*

Proof. Let V be open in Z. Then $g^{-1}(V)$ is open in Y so that $f^{-1}(g^{-1}(V))=(gf)^{-1}(V)$ is θ-open by Theorem 1(c). Thus gf is strongly θ-continuous by Theorem 1.

It follows that the composition of two strongly θ-continuous functions is strongly θ-continuous.

LEMMA 1. *The function $f:X\to Y$ is strongly θ-continuous if and only if for each subbasic open set $V\subset Y$, $f^{-1}(V)$ is θ-open in X.*

Proof. The necessity follows from Theorem 1. Conversely, let $\{V_\alpha:\alpha\in\Delta\}$ be a subbasis for Y and assume that $f^{-1}(V_\alpha)$is θ-open for all $\alpha\in\Delta$. Then each open $V\subset Y$ can be written as

$$V=\cup\{V_{\alpha_1}\cap V_{\alpha_2}\cap\cdots\cap V_{\alpha_n}:\{\alpha_1,\alpha_2,\cdots,\alpha_n\}\subset\Delta\}$$

so that $f^{-1}(V)=\cup\{f^{-1}(V_{\alpha_1})\cap f^{-1}(V_{\alpha_2})\cap\cdots\cap f^{-1}(V_{\alpha_n})\}$.

Since the finite intersection of θ-open sets is θ-open and the union of θ-open sets is θ-open [4, Lemma 3], $f^{-1}(V)$ is θ-open and hence f is strongly θ-continuous by Theorem 1.

THEOREM 6. *Let $f:X\to\prod_{\alpha\in\Delta}X_\alpha$ be given. Then f is strongly θ-continuous if and only if the composition with each projection Π_α is strongly θ-continuous.*

Proof. If f is strongly θ-continuous, then $\Pi_\alpha f$ is strongly θ-continuous by the continuity of Π_α and Theorem 5.

Conversely, let V be a subbasic open set in $\prod_{\alpha\in\Delta} X_\alpha$. Then $V=\Pi_\alpha^{-1}(W)$ for some open W in X_α. Thus $f^{-1}(V)=f^{-1}(\Pi_\alpha^{-1}(W))=(\Pi_\alpha f)^{-1}(W)$ is θ-open due to $\Pi_\alpha f$ being strongly θ-continuous and Theorem 1. Thus f is strongly θ-continuous by Lemma 1.

COROLLARY TO THEOREM 6. *Let $f:X\to Y$ be a function and let $g:X\to X\times Y$, given by $g(x)=(x,f(x))$, be its graph map. Then f is strongly θ-continuous if and only if g is strongly θ-continuous. Furthermore, if $g:X\to X\times Y$ is strongly θ-continuous, then X is regular.*

Proof. Only the last statement needs verification. If g is strongly θ-continuous and $x\in X$, then for any open U containing x, $U\times Y$ is open in $X\times Y$ and contains $g(x)=(x,f(x))$. Thus, there exists an open set U_0 containing x such that $g(\mathrm{Cl}(U_0))\subset U\times Y$. Consequently, $x\in U_0\subset \mathrm{Cl}(U_0)\subset U$ showing that X is regular.

LEMMA 2. *Let $U_{\alpha_i}\subset X_{\alpha_i}$ for each $i=1,2,\cdots,n$. Then*
$$U_{\alpha 1}\times U_{\alpha 2}\times\cdots\times U_{\alpha_n}\times\prod_{\alpha\neq\alpha i}X_\alpha\subset\prod_{\alpha\in\Delta}X_\alpha$$
is θ-open if and only if U_{α_i} is θ-open in X_{α_i} for each $i=1,2,\cdots,n$.

Proof. Suppose $U_{\alpha_i}\subset X_{\alpha_i}$ is θ-open in X_{α_i} for each $i=1,2,\cdots,n$. Then for each i and each $x_i\in U_{\alpha_i}$, there exists an open V_{α_i} containing x_i such that $x_i\in V_{\alpha_i}\subset \mathrm{Cl}(V_{\alpha_i})\subset U_{\alpha_i}$. Thus, for each $\{x_\alpha\}\in U_{\alpha 1}\times U_{\alpha 2}\times\cdots\times U_{\alpha_n}\times\prod_{\alpha\neq\alpha i}X_\alpha$, $\{x\}\in V_{\alpha 1}\times V_{\alpha 2}\times\cdots\times V_{\alpha_n}\times\prod_{\alpha\neq\alpha i}X_\alpha\subset \mathrm{Cl}(V_{\alpha 1})\times \mathrm{Cl}(V_{\alpha 2})\times\cdots\times \mathrm{Cl}(V_{\alpha_n})\times\prod_{\alpha\neq\alpha i}X_\alpha\subset U_{\alpha 1}\times U_{\alpha 2}\times\cdots\times V_{\alpha_n}\times\prod_{\alpha\neq\alpha i}X_\alpha$. This shows that $U_{\alpha 1}\times U_{\alpha 2}\times\cdots\times U_{\alpha_n}\times\prod_{\alpha\neq\alpha i}X_\alpha$ is θ-open. The converse is clear.

THEOREM 7. *Define $\prod_\alpha f_\alpha:\prod_\alpha X_\alpha\to\prod_\alpha Y_\alpha$ by $\{x_\alpha\}\to\{f_\alpha(x_\alpha)\}$. Then $\prod_\alpha f_\alpha$ is strongy θ-continuous if and only if each $f_\alpha:X_\alpha\to Y_\alpha$ is strongly θ-continuous.*

Proof. Let $V=V_{\alpha 1}\times V_{\alpha 2}\times\cdots\times V_{\alpha_n}\times\prod_{\alpha\neq\alpha i}X_\alpha$ be a basic open set in $\prod_\alpha Y_\alpha$. Then if $f_{\alpha_i}^{-1}(V_{\alpha_i})$ is θ-open in X_{α_i} for each α_i, we have
$$(\prod_\alpha f_\alpha)^{-1}(V)=f^{-1}(V_{\alpha 1})\times f^{-1}(V_{\alpha 2})\times\cdots\times f^{-1}(V_{\alpha_n})\times\prod_{\alpha\neq\alpha i}X_\alpha$$
is θ-open in $\prod_\alpha X_\alpha$ by Lemma 2. This implies that $\prod_\alpha f_\alpha$ is strongly θ-continuous.

Conversely, suppose $\prod_\alpha f_\alpha$ is strongly θ-continuous. Let $V_{\alpha_i}\subset Y_{\alpha_i}$ be open.

Then $V = V_{\alpha_i} \times \prod_{\alpha \neq \alpha_i} Y_\alpha$ is subbasic open in $\prod_\alpha Y_\alpha$ and $(\prod_\alpha f_\alpha)^{-1}(V) = f_{\alpha_i}^{-1}(V_{\alpha_i}) \times \prod_{\alpha \neq \alpha_i} X_\alpha$ is θ-open. Thus $f^{-1}(V_{\alpha_i})$ is θ-open in X_{α_i} which implies that f_{α_i} is strongly θ-continuous by Theorem 1.

3. Sufficient conditions for strong θ-continuity

THEOREM 8. *Let $f: X \to Y$ be continuous. If Y is regular and T_1, then f is strongly θ-continuous.*

Proof. Let $x \in X$ and let V be an open set in Y containing $f(x)$. Since Y is regular, there exists an open set W such that $f(x) \in W \subset \mathrm{Cl}(W) \subset V$. This fact, along with the continuity of f, implies $x \in f^{-1}(W) \subset \mathrm{Cl}(f^{-1}(W)) \subset f^{-1}(\mathrm{Cl}(W)) \subset f^{-1}(V)$. Now let $U = f^{-1}(W)$. Then $f(\mathrm{Cl}(U)) \subset V$ showing that f is strongly θ-continuous.

If $f: X \to Y$ is a function and $G(f) = \{(x, f(x)) : x \in X\}$ denotes the graph of f, we define $G(f)$ to be *θ-closed with respect to $X \times Y$* if for each $(x, y) \notin G(f)$ there exist open sets U and V containing x and y respectively, such that $(\mathrm{Cl}(U) \times \mathrm{Cl}(V)) \cap G(f) = \phi$. With this definition we are now ready to prove another sufficient condition for strong θ-continuity.

THEOREM 9. *Let $f: X \to Y$ have a θ-closed graph with respect to $X \times Y$. If Y is minimal Hausdorff, then f is strongly θ-continuous.*

Proof. We use the fact that a minimal Hausdorff space is semi-regular and H-closed [6, 17M, p. 129]. Thus, let $x \in X$ and let V be a regular-open set containing $f(x)$. Then $Y - V$ is regular closed and for each $y \in Y - V$, $(x, y) \notin G(f)$. The hypothesis now asserts the existence of open sets $U_y(x)$ and $W(y)$ containing x and y, respectively, such that $(\mathrm{Cl}(U_y(x)) \times \mathrm{Cl}(W(y))) \cap G(f) = \phi$ or that $f(\mathrm{Cl}(U_y(x))) \cap \mathrm{Cl}(W(y)) = \phi$. The collection $\{W(y) : y \in Y - V\}$ forms an open cover of the regular-closed. hence H-closed, subset $Y - V$. Consequently, there is a finite number $\{W(y_i) : i = 1, 2, \cdots, n\}$ such that $Y - V \subset \bigcup_{i=1}^{n} \mathrm{Cl}(W(y_i))$. Now let $U = \bigcap_{i=1}^{n} U_{y_i}(x)$. Then $f(\mathrm{Cl}(U)) \subset V$ showing that f is strongly θ-continuous.

The graph of $f: X \to Y$ is called *θ-closed with respect to X* if for each $(x, y) \notin G(f)$ there exist open sets U and V containing x and y, respectively, such that $(\mathrm{Cl}(U) \times V) \cap G(f) = \phi$.

A function $f: X \to Y$ is called *θ-continuous* if for each $x \in X$ and each open

V containing $f(x)$ there exists an open set U containing x such that $f(\mathrm{Cl}(U))\subset\mathrm{Cl}(V)$. Of course a strongly θ-continuous function is θ-continuous. The next theorem shows when a θ-continuous function will also be strong θ-continuous.

THEOREM 10. *If Y is rim-compact and $f:X\to Y$ is a θ-continuous function whose graph is θ-closed with respect to X, then f is strongly θ-continuous.*

Proof. Let $x\in X$ and let W be any openset containing $f(x)$. Since Y is rim-compact, there exists an open set V such that $f(x)\in V\subset W$ whose boundary $\mathrm{Bd}(V)$ is compact. For each $y\in\mathrm{Bd}(V)$, $(x,y)\notin G(f)$ so there are open sets $U_y(x)$ and $S(y)$ such that $(\mathrm{Cl}(U_y(x))\times(S(y))\cap G(f)=\phi$ or that $f(\mathrm{Cl}(U_y(x))\cap S(y)=\phi$ because $G(f)$ is θ-closed with respect to X. The compactness of $\mathrm{Bd}(V)$ now implies there are a finite number of open sets $S(y_1), S(y_2)\cdots, S(y_n)$ from the open cover $\{S(y):y\in\mathrm{Bd}(V)\}$ which cover $\mathrm{Bd}(V)$. Since f is θ-continuous, there is an open set $U_0(X)$ such that $f(\mathrm{Cl}(U_0))\subset\mathrm{Cl}(V)$. Consider $U=U_0(x)\cap[\bigcap_{i=1}^{n} U_{y_i}(x)]$. It follows that U is open and $\mathrm{Cl}(U)=\mathrm{Cl}(U_0\cap[\bigcap_{i=1}^{n} U_{y_i}(x)])\subset\mathrm{Cl}(U_0)\cap[\bigcap_{i=1}^{n}\mathrm{Cl}(U_{y_i}(x)]$.
Thus,

$$f(\mathrm{Cl}(U))\cap(Y-V)=f(\mathrm{Cl}(U))\cap\mathrm{Bd}(V)$$
$$\subset\bigcup_{i=1}^{n}[f(\mathrm{Cl}(U))\cap S(y_i)]\subset\bigcup_{i=1}^{n}[f(\mathrm{Cl}(U_{y_i}(x))\cap S(y_i)]=\phi.$$

Therefore, $f(\mathrm{Cl}(U))\subset V\subset W$ showing that f is strongly θ-continuous.

THEOREM 11. *Let Y be compact. If $f:X\to Y$ has a graph which is θ-closed with respect to X, then f is strongly θ-continuous.*

Proof. Let $x\in X$ and let V be open and contain $f(x)$. Then for each y in the compact set $Y-V$, we have $(x,y)\notin G(f)$. Theree there exist opensets $U_y(x)$ and $W(y)$ containing x and y, respectively, such that $f(\mathrm{Cl}(U_y(x)))\cap W(y)=\phi$ because $G(f)$ is θ-closed with respect to X. Thus, there exists a finite subcover $\{W(y_i):\ i=1,2,\cdots,n\}$ of $Y-V$ and the corresponding $U_{y_i}(x)$ have the property that $f(\bigcap_{i=1}^{n}\mathrm{Cl}(U_{y_i}(x)))\cap[\bigcup_{i=1}^{n} W(Y_i)]=\phi$. But $\mathrm{Cl}(\bigcap_{i=1}^{n} U_{y_i}(x))\subset\bigcap_{i=1}^{n}\mathrm{Cl}(U_{y_i}(x))$, so if we let $U=\bigcap_{i=1}^{n} U_{y_i}(x)$ then we have $f(\mathrm{Cl}(U))\cap[\bigcup_{i=i}^{n} W(y_i)]=\phi$. Consequently, $f(\mathrm{Cl}(U))\subset V$ showing that f is strongly θ-continuous at x.

THEOREM 12. *If $f:X\to Y$ is strongly θ-continuous and Y is Hausdorff, then $G(f)$ is θ-closed with respect to X.*

Proof. Let $x\in X$ and $y\neq f(x)$. Then there are open disjoint sets W and V containing $f(x)$ and y, respectively. Since f is strongly θ-continuous, there is an open set U containing x such that $f(\mathrm{Cl}(U))\subset W$. Therefore $f(\mathrm{Cl}(U))\cap V=\phi$. This shows that $G(f)$ is θ-closed with respect to X.

THEOREM 13. *Let Y be a compact Hausdorff space. Then $f:X\to Y$ is strongly θ-continuous if and only if $G(f)$ is θ-closed with respect to X.*

Proof. Theorems 11 and 12.

4. Properties preserved by strongly θ-continuous functions

A set A in a topological space X is defined to be an *H-set* [4] if for each cover of A with open sets in X, there exists a finite number of the covering sets whose closures cover A.

THEOREM 14. *Let $f:X\to Y$ be strongly θ-continuous. If $A\subset X$ is an H-set, then $f(A)$ is compact.*

Proof. Let A be an H-set in X and let $\mathcal{V}$ be an open cover of $f(A)$. For each $a\in A$ there is an open set $V_a\in\mathcal{V}$ such that $f(a)\in V_a$. Since f is strongly θ-continuous, there exists an open set U_a containing a such that $f(\mathrm{Cl}(U_a))\subset V_a$. The collection $\{U_a:a\in A\}$ now forms an open cover of A so there exists a finite subcollection $U_{a_1}, U_{a_2}\cdots, U_{a_n}$ such that $A\subset\bigcup_{i=1}^{n}\mathrm{Cl}(U_{a_i})$ because A is an H-set. Thus, $f(A)\subset f(\bigcup_{i=1}^{n}\mathrm{Cl}(U_{a_i}))=\bigcup_{i=1}^{n}f(\mathrm{Cl}(U_{a_i}))\subset\bigcup_{i=1}^{n}V_{a_i}$ so that $\mathcal{V}$ has a finite subcollection $\{V_{a_i}:i=1,2,\cdots,n\}$ which covers $f(A)$. Consequently, $f(A)$ is compact.

COROLLARY to THEOREM 14. *If $f:X\to Y$ is strongly θ-continuous, surjective and X is H-closed, then Y is compact.*

COROLLARY to THEOREM 14. *A strongly θ-continuous real valued function defined on an H-closed space X is bounded.*

A function $f:X\to Y$ is defined to be *regular-open* [3, Definition 3.1] if the image of every regular-open set is open.

THEOREM 15. *Let $f:X\to Y$ be a regular-open and strongly θ-continuous function of X onto Y. If X is locally H-closed and Y is Hausdorff, then Y is locally compact.*

Proof. Let $y \in Y$ and let $x \in X$ such that $f(x) = y$. Since X is locally H-closed, there exists a regular-openset H such that $x \in H$ and $\mathrm{Cl}(H)$ is an H-set. By Theorem 14, $f(\mathrm{Cl}(H))$ is compact, hence closed in the Hausdorff space Y. Now since f is regular-open, the open set $f(H)$ contains $f(x) = y$ and $\mathrm{Cl}(f(H)) \subset f(\mathrm{Cl}(H))$ is compact. Therefore, Y is locally compact.

A Hausdorff space X is called *C-compact* if each closed set in X is an H-set [5].

THEOREM 16. *Let $f: X \to Y$ be strongly θ-continuous and let X be a C-compact Hausdorff space and let Y be Hausdorff. If f is bijective, then X is homeomorphic to Y and both X are compact.*

Proof. Since f is strongly θ-continuous, f is continuous. Furthermore, if $A \subset X$ is closed, then A is an H-set so that $f(A)$ is compact by Theorem 14 and hence closed in the Hausdorff space Y. This shows that f is a homeomorphism from X onto Y. Now since X is itself an H-set, $f(X) = Y$ is compact again by Theorem 14. It follows that both X and Y are compact since they are homeomorphic.

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