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—Research Paper—

Optimum Design of Oil Pipeline Network

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Abstract

The optimum design problem of a proposed oil pipeline network has been formulated as a zero-one programming model to determine the optimum sizes of pipe and pump which minimize the sum of material costs and operating costs during the 20 years of life span.

Applying to a real situation, the problem constitutes an assignment type zero-one programming with 372 zero-one variables and 13 constraints. A heuristic algorithm has been developed based on the modified Petersen algorithm utilizing the special form of the activity matrix. The results showed impressive cost savings of 37 percent of the total cost from the original proposal.

1. Introduction

Transportation of oil through pipeline can be one of the efficient...energy saving...ways for the distribution of oil from refineries to consuming areas. However, construction of a new pipeline network involves a substantial amount of initial investment...which often prohibits its realization...and thus the design of the network is usually the most critical for the project to be economically feasible.

Classical analysis of pipeline network has been to find a set of flows and pressures in the pipe network when supply and demand are known. [2] Linear programming has been applied to the water supply system [1], [4], and optimum pipeline route along with the optimum pump-pipe system has also been considered in [9], all of which are in static context.

2. Problem

The problem of pipeline design to be considered in this paper is to determine the optimum sizes of pipe & pump which will minimize the total cost of fixed and variable costs during the 20 years of life while satisfying given demand and supply, and passing the predetermined pipeline route.

The inclusion of the pipeline operating cost is critical since it is the major portion of the

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total cost especially when the energy cost is relatively high.

The predetermined network consists of 6 pipeline sections and 6 pumping stations as given in Fig. 1.

The circle denotes the pumping station or terminal, and the line denotes the pipeline section which links the two adjacent pumping stations.

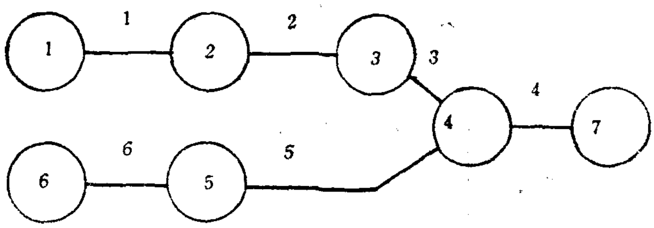

Fig. 1. Diagram for a Proposed Pipeline Network

The sources or supply points are stations 1 and 6. Station 7 functions only as a terminal while other stations are the terminals as well as pumping stations.

The problem then is to determine the pipe sizes of the 6 pipeline sections and the pump sizes of the 6 pumping stations which will minimize the total cost while satisfying the engineering and throughput requirements.

3. Model

It is assumed that the pipe sizes are chosen among the API* Specification 5LX Grade X65. It will also be assumed that the choice of pump head is restricted to the integer values with 100 feet increments.

Additional assumptions are as follows:

- the throughputs for each lines are predetermined
- the same size of pipe will be used for each line section, i.e., pipe sizes, diameter, and thickness are equal within the same section
- using engineering formula, friction head loss is assumed to be linearly proportional to the distance from the pump.

To calculate friction head loss, friction factor, maximum allowable operating pressure, and pumping horse power, the following engineering formulas were used:

Fanning Formula

$$H_f = f \cdot \frac{50.0322\, Q^2 \cdot H_r}{d^5} \tag{1}$$

where

H_f = friction head loss (ft/km)

f = friction factor

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Q=flow rate (Bbl/day)
H_r=operating hour (hr/day)
d=inside diameter (inch).

The friction factor is determined by the Reynolds number which also is a function of the pipe diameter, velocity of oil flow, density of oil, and viscosity of the fluid. The flow in the oil pipeline falls into turbulent case, and the experimental relationship between the friction factor and the Reynolds number for the turbulent flow is fitted to an equation,

$$f=0.05393R_e-0.2343 \qquad (2)$$

where "R_e" denoted the Reynolds number.

Maximum Allowable Operating Pressure

This formula calculates the allowable design stress of the pipe.

$$\text{MAOP}=2s\cdot(T/D) \qquad (3)$$

where

$s=0.72E$
s=allowable stress (psi)
E=joint or seam strength which is equal to 1.0 for the pipe manufactured in accordance with API−5LX
T=thickness of pipe (inch)
D=outside diameter (inch)

Operating pressure and pumping horse power are calculated by the equations,

$$P=0.434S_g\cdot H \qquad (4)$$

and

$$\text{BHP}=0.000017\cdot\frac{Q\cdot P}{E_p} \qquad (5)$$

where

S_g=specific gravity, 0.84 was used for the light oil
H=head (ft)
BHP=pumping horse power
Q=flow rate (Bbl/day)
P=pressure (psi)
E_p=pump efficiency

Using the previous assumptions and the formulas, a zero-one programming model for the optimum pipe and pump size selection problem can be formulated as follows:

Objective Function

minimize

$$Z=\sum_{i=1}^{6}L_i\sum_{i=1}^{K}(PC_j+T\cdot OPPC)X_{ij}$$

$$+\sum_{i=1}^{6}\sum_{j=1}^{J}[HEDC_i\{HL_i+100(j-1)\}]HEDF_i\times H_{ij} \qquad (6)$$

where

"i" denotes the line section and pumping station

K=total number of alternative pipe sizes

J=total number of alternative pump heads

T=life span of the pipeline

L_i=length of the i-th line section

PC_j=pipe cost of the j-th assignment

$OPPC$=operating (maintenance) cost of pipeline per unit length

$HEDC_i$=head cost at the i-th station,

HL_i=lowest allowable head limit at the i-th station

$HEDF_i$=the number of actual operating years expressed in full utilization of the pipeline section during the life.

X_{ij}=zero-one variable for pipe size assignment

$X_{ij}=1$, if the j-th size in the i-th section is selected

$=0$, otherwise

H_{ij}=zero-one varialbe for head assignment

$H_{ij}=1$, if the j-th head in the i-th station is selected

$=0$, otherwise

Constraints

1) Head Constraints

$$\sum_{j=1}^{K} S_{1j}X_{1j}+HS_1+Y_1=H_1 \tag{7}$$

$$\sum_{j=1}^{K} S_{2j}S_{2j}+HS_2+Y_2=H_2+Y_1$$

$$\sum_{j=1}^{K} S_{3j}X_{3j}+HS_3+Y_3=H_3+Y_2$$

$$\sum_{j=1}^{K} S_{4j}X_{4j}+HS_4+Y_{41}=H_4+Y_3$$

$$\sum_{j=1}^{K} S_{4j}X_{4j}+HS_4+Y_{42}=H_4+Y_5$$

$$\sum_{j=1}^{K} S_{5j}X_{5j}+HS_5+Y_5=H_5+Y_6$$

$$\sum_{j=1}^{K} S_{6j}X_{6j}+HS_6+Y_6=H_6$$

$Y_i \geqslant 100$ for $i=1, 2, 3, 5, 6$

$Y_{41} \geqslant 100$

$Y_{42} \geqslant 100$

where

S_{ij}=friction head loss of the i-th line for the j-th alternative pipe size

HS_i=static head of the i-th station

Y_i=suction pressure at the i-th station

Y_{41} = suction pressure of line section 3 at station 4
Y_{42} = suction pressure of line section 5 at station 5
H_i = total differential head (TDH) at the i-th station

$$H_i=\sum_{j=1}^{J}[HL_i+100(j-1)]H_{ij}$$

2) Pipe Pressure Constraints

$$\sum_{j=1}^{K}2s(T_j/D_j)X_{1j}\geqslant 0.434\cdot S_g\cdot H_1\cdot(1+\text{PALW}) \tag{8}$$

$$\sum_{j=1}^{K}2s((T_j/D_j)X_{2j}\geqslant 0.434\cdot S_g\cdot(H_2+Y_1)\cdot(1+\text{PALW})$$

$$\sum_{j=1}^{K}2s(T_j/D_j)X_{3j}\geqslant 0.434\cdot S_g\cdot(H_3+Y_2)\cdot(1+\text{PALW})$$

$$\sum_{j=1}^{K}2s(T_j/D_j)X_{4j}\geqslant 0.434\cdot S_g\cdot\left(H_4+\frac{Y_3+Y_5}{2}\right)\cdot(1+\text{PALW})$$

$$\sum_{j=1}^{K}2s(T_j/D_j)X_{5j}\geqslant 0.434\cdot S_g\cdot(H_5+Y_6)\cdot(1+\text{PALW})$$

$$\sum_{j=1}^{K}2s(T_j/D_j)X_{6j}\geqslant 0.434\cdot S_g\cdot H_6\cdot(1+\text{PALW})$$

where

s = allowable stress
T_j = thickness of the j-th pipe size
D_j = outside diameter of the j-th pipe size
S_g = specific gravity, 0.84
PALW = allowance for pipe pressure

3) Variable Restrictions

$$X_{ij}=0 \text{ or } 1, \text{ for all i, j} \tag{9}$$

$$H_{ij}=0 \text{ or } 1, \text{ for all i, j}$$

$$\sum_{j=1}^{K}X_{ij}=1 \quad \text{for all i}$$

$$\sum_{j=1}^{J}H_{ij}=1 \quad \text{for all i}$$

4. A Heuristic Algorithm

The model has been applied to the real situation as given in [6]. With the numbers of alternative pipe and pump sizes to be 42 and 20, respectively, the model contains 372 zero-one variables (X_{ij}'s and H_{ij}'s). The resulting "assignment-type" zero-one programming model can be reformulated as

$$\text{minimize } Z=\sum_{i\in I}\sum_{j\in J}C_{ij}x_{ij} \tag{10}$$

$$\text{Subject to } \sum_{i\in I}\sum_{j\in J}a^k_{ij}x_{ij}\leqslant b_k \quad \text{for } k\in K \tag{11}$$

$$x_{ij}=0 \text{ or } 1 \qquad \text{for } i\in I,\ j\in J \tag{12}$$

$$\sum_{i\in I} x_{ij}=1 \qquad \text{for } j\in J \tag{13}$$

where $I=\{I_p, I_h\}$, $J=\{J_p, J_h\}$

and

I_p=index set for pipe sizes

I_h=index set for pump sizes

J_p=index set for pipeline sections

J_h=index set for pumping stations

K=index set for constraints

With the specific applications given in [6], the overall structure of the model can be summarized as in Fig. 2 where the cost coefficients are rearranged in increasing order within each group of assignments, i.e., within one pipeline section or one pumping station, then the resulting activity matrix can be decomposed into four parts; two with respect to the strict or relative orderings of magnitude in the elements of the matrix, and two with respect to the strictness of infeasibility, i.e., the first 7 strong constraints and the rest 6 weak constraints.

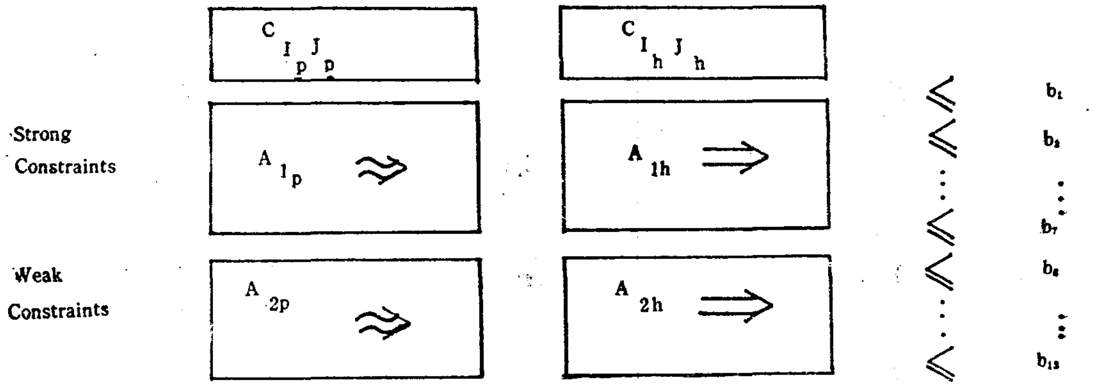

Fig. 2. Structure of the Pipe-pump Size Assignment Problem

Utilizing the specific structure of the model, a heuristic algorithm has been developed as following:

Step 0 : The model was rearranged by sorting C_{ij} in ascending orders for $j\in J_i$ of each $i\in I$.
Set L_i as the last elements of x_{ij}, $j\in J_h$, for each $i\in I_h$.

Step 1 : Assign 1 to the x_{ij}, $j\in L_i$ for each $i\in I_h$, which will provide the maximum feasibility of strong constraints with less costs relative to x_{ij}, $j\in J_p$, $i\in I_p$.

Step 2 : **Feasibility Test**
Delete the variable set of x_{ij}, $i\in I_p$, $j\in J_p$, which will violate the strong constraints if 1's are assigned to them. Obtain a free variable set N_i for each $i\in I_p$, which excludes the above deletion set.

Sept 3 : Assign 1 to the first elements of x_{ij}, $j\in N_i$ for each $i\in I_p$. If current assignments are feasible, i.e., satisfy weak constraints also, go to Step 5.

Step 4 : If L_i is the first elements of $j \in J_h$ for each $i \in I_h$, and there was no current optimum, then the problem is infeasible. Otherwise go to Step 6.

Step 5 : **Fine Tuning**

Fine-tuning the assignments of x_{ij}, $j \in J_h$, $i \in I_h$ by choosing the largest decrements in costs while satisfying the feasibility.

Update the "current optimum" using the current assignments.

Step 6 : If L_i is the first elements of $j \in J_h$ for each $i \in I_h$, the current optimum is the global optimum.

Step 7 : Select a new set L_i by moving one j of the assignment to the left which provides the maximum feasibility if infeasibility exists or selects one which will reduce the cost in maximum if no infeasibility exists.

Go to step 1.

For practical purposes a stopping criteria can also be added in Step 5, where the algorithm will terminate when the ratio of the changes in the "previous" current optimum to current optimum is less than a certain limit.

The key for the algorithm to be complete is the way to update the "new" L_i. The algorithm remains to be heuristic for this specific structure of the problem at this point. Nevertheless, the efficiency of the algorithm can be envisaged by considering the substantial reductions in the total number of combinations, i.e., from the Balasian combinations of 2^{372} to 15^6 in this particular case of 42 alternatives for each of 6 pipe sizes and 15 alternatives for each of 6 pump sizes.

5. Computational Results

Using the data from the real situation as given in [6], the heuristic algorithm solved the problem efficiently in less than 100 seconds of CPU time on CYBER 174.

Table 1 shows the optimum pipe sizes, i.e., outside diameter and thickness, and pump sizes in total differential head (TDH).

〈**Table 1**〉 Optimum Pipe and Pump Size

Station or Line Number	Pipe Size		Pump Size
	Outside Diameter (inch)	Thickness (inch)	TDH (feet)
1	10	0.219	1600
2	16	0.188	1900
3	14	0.188	2600
4	22	0.219	1600
5	18	0.188	2100
6	22	0.219	1400

6. Concluding Remarks

The paper illustrated a real application of O.R. technique (zero-one programming to be specific) to the large scale investment project.

The results were remarkable. It showed impressive cost savings of 37 percent of the total cost of the objective function while satisfying all the engineering constraints from the original proposal designed by a famed engineering company in this field relying heavily on the engineering handbooks.

Unquestionably, the results improved the economical and financial feasibilities of the project greatly, which eventually made the project favorable in this particular case.

A mixed integer formulation with the pump sizes to be continuous, a case of variable pipe sizes within each line section, routing problem with optimum number of pumping stations, completeness of the algorithm remain to be the areas for further study.

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