The Journal of the Oceanological Society of Korea
Vol. 16, No. 2, p. 99~107, December 1981

BAROTROPIC SHELF WAVES GENERATED BY LONGSHORE WIND STRESS

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ABSTRACT

A partial differential equation for the adjusted sea level, obtained from the long wave equations in shallow water, is reduced to a simpler one by the use of physically reasonable approximations based on the observations. The similar equation for the stream function indicates that shelf waves are generated by the longshore wind stress. This indication is in good agreement with the high correlation between the adjusted sea levels and the longshore wind stress. From the dispersion relationship and the boundary conditions, there exist a countable infinite number of modes which satisfy a first-order wave equation. The adjusted sea level for a given wind stress can easily be calculated by utilizing the convolution and the Fourier transformation. Some detailed solutions are presented here for sinusoidal and exponential wind stress.

1. INTRODUCTION

According to the static theory in space and in time, sea level should respond as an inverse barometer to atmospheric pressure changes. In reality, sea level does not always show the isostatic behaviour on shelves in the low frequency domain and it has been shown that there exists a time lag between adjusted sea levels at two different stations(Hamon, 1962, 1966; Mooers and Smith, 1968; Cutchin and Smith, 1973).

Robinson(1964) first proposed a theoretical model of continental shelf waves in order to explain the observed results. Mysak(1967) extended Robinson's theory to the case of offshore stratification and offshore mean current. They suggested that the generation of shelf waves is due to resonance with pressure system. Adams and Buchwald(1969) considered the wind stress as the main cause for the phenomenon and they obtained an analytical solution for a wind stress in the form of square waves on an exponentially varying shelf. Gill and Schumann(1974) obtained the first-order wave equation by the separation of variables, assuming that the longshore wind stress is the principal driving mechanism for shelf wave generation. Crépon(1976) gave an explanation to abnormal response of the sea level based on a process of geostrophic adjustment.

In this paper, an equation for the adjusted sea level is obtained from the shallow water wave equations and then some reasonable approximations are made on the basis of the observations. This process shows importance of wind stress in the mechanism of the phenomenon more clearly than that from the direct approximations from the equations of motion. The adjusted sea level was shown to be related to the longshore wind stress more strongly than the onshore wind stress from the observations(Lie, 1979). It is shown that a countable infinite number of modes exist on a shelf from the dispersion and the boundary conditions. A simple and useful method is presented for the general solution of forced barotropic shelf waves, introducing the convolution and the Fourier transformation, and the effects of a sinusoidal wind stress and an exponential wind stress on the adjusted sea level are investigated.

2. FORMULATION OF THE PROBLEM

The linearized shallow water wave equation is considered in horizontal rectangular coordinates such that the positive x-axis is seaward from the coast($x=0$) and y-axis is along the coastline. Assuming that the fluid is incompressible and the water density is constant($\rho=$ 1g/cm^3), the equations of motion and continuity, neglecting friction and dissipation, have the form

$$\frac{\partial u}{\partial t}-fv=-g\frac{\partial(\xi'-\xi_a)}{\partial x}+\frac{\tau_x}{h} \qquad (1)$$

$$\frac{\partial v}{\partial t}+fu=-g\frac{\partial(\xi'-\xi_a)}{\partial y}+\frac{\tau_y}{h} \qquad (2)$$

$$\frac{\partial \xi'}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}=0 \qquad (3)$$

where u and v are the depth-averaged components of current velocity in the x,y directions, respectively; ξ' is the elevation of the sea surface from the equilibrium level; ξ_a is the inverse of atmospheric pressure expressed in the same unit as the sea level elevation; $h(x, y)$ is the depth; f is the Coriolis parameter; g is the acceleration due to the gravity; τ_x and τ_y are the wind stress components toward east and north, respectively.

ξ' respresents the sea level which is not influenced by the tide generating forces, and in practice its values are estimated from hourly heights recorded at a tidal station by applying an appropriate filter to the heights. The difference ξ'-ξ_a can be regarded as the non-isostatic part of the sea level changes, so-called the adjusted sea level ξ.

Eliminating u and v from (1), (2) and (3), the following equation is obtained for the adjusted sea level

$$\begin{aligned}&\left[\frac{\partial^2}{\partial t^2}+f^2\right]^2\frac{\partial(\xi+\xi_a)}{\partial t}-hg\left[\frac{\partial^2}{\partial t^2}+f^2\right]\Big[\Delta\frac{\partial\xi}{\partial t}\\&-\beta\frac{\partial\xi}{\partial x}+\frac{1}{h}\frac{\partial h}{\partial x}\frac{\partial^2\xi}{\partial x\partial t}+\frac{1}{h}\frac{\partial h}{\partial y}\frac{\partial^2\xi}{\partial y\partial t}\\&+\frac{f}{h}\frac{\partial(h,\xi)}{\partial(x,y)}\Big]+2f\beta gh\left[\frac{\partial^2\xi}{\partial y\partial t}-f\frac{\partial\xi}{\partial x}\right]=\\&-\left[\frac{\partial^2}{\partial t^2}+f^2\right]\left[\frac{\partial}{\partial t}\nabla\cdot\vec{\tau}-f\vec{z}\cdot\nabla\times\vec{\tau}-\beta\tau_x\right]\\&+2f\beta\left[\frac{\partial\tau_y}{\partial t}-f\tau_x\right]\end{aligned} \qquad (4)$$

where $\beta=df/dy$; $\vec{\tau}=\tau_x\vec{x}+\tau_y\vec{y}$; $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$;

$\nabla\cdot\vec{\tau}=\frac{\partial\tau_x}{\partial x}+\frac{\partial\tau_y}{\partial y}$; $\nabla\times\vec{\tau}=\vec{z}\left[\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right]$;

$\frac{\partial(h,\xi)}{\partial(x,y)}=\frac{\partial h}{\partial x}\frac{\partial\xi}{\partial y}-\frac{\partial h}{\partial y}\frac{\partial\xi}{\partial x}$; $\vec{u}=u\vec{x}+v\vec{y}$;

$\vec{x}$, $\vec{y}$ and $\vec{z}$ are the unit vectors of x, y and z axes, respectively. The current velocity $\vec{u}$ takes the vector form as

$$\left[\frac{\partial^2}{\partial t^2}+f^2\right]\vec{u}=\left[\frac{\partial}{\partial t}-f\vec{z}\times\right]\left[\frac{\vec{\tau}}{h}-\nabla(g\xi)\right]$$

3. APPROXIMATIONS

It is very convenient to make some reasonable approximations which can reduce the equation (4).

It has been previously shown by other authors that the frequrncies (σ) of the phenomenon under consideration are much less than the Coriolis parmeter(f) when the mid-latitude and the significant spectral peaks of several days are referred to.

The exponential profile seems to be appropriate for the shelf topography. Buchwald and Adams(1968) chose this profile for the shelf off Sydney, which fitted to actual shelf. Let the depth profile be of the form

$$h(x,y)=\begin{cases} h_0(y)\exp(2\lambda x) & \text{for } 0<x<l \\ h_0(y)\exp(2\lambda l) & \text{for } x\geq l \end{cases}$$

where λ is a constant chosen from the depth variation perpendicular to the coast, and l is the width of continental shelf. The appropriate values, for instance, off La Pointe de Grave in the Bay of Biscay, are $h_0=12\times10^2$cm; $\lambda=10^{-7}$/cm; $l=3\times10^7$cm; $h(l,y)=4.8\times10^5$cm.

Considering length scales of order 10^6 or 10^7 cm and time scales of about several days, the β terms in the left side of the equation (4) can be neglected compared with the others. For the cases of the Bay of Biscay, the east sea of Australia and the east sea of Korea, we have

$$\left|\beta\frac{\partial\xi}{\partial x}\right|\Big/\left|\frac{1}{h}\frac{\partial h}{\partial x}\frac{\partial^2\xi}{\partial x\partial t}\right|\sim\left|\frac{\beta}{2\lambda\sigma}\right|\sim 0(3 \text{ or } 4\times10^{-2})$$

and

$$\left|2f\beta gh\frac{\partial^2\xi}{\partial y\partial t}\right|\Big/\left|gf^3\frac{\partial(h,\xi)}{\partial(x,y)}\right|\sim\left|\frac{\beta}{\sigma\lambda}\frac{\sigma^2}{f^2}\right|\sim 0(10^{-3})$$

The terms containing β are thus negligible in comparison with those involving the depth variation $\nabla h/h$. The β terms in the right side of (4) are also sufficently small for the above time and length scales compared with the other terms representing the divergence or curl of wind stress.

With the above three approximations, the equation (4) is reduced to the form

$$\frac{\partial}{\partial t}\left[\frac{g}{f}\frac{\partial}{\partial x}\left(h\frac{\partial\xi}{\partial x}\right)-f(\xi+\xi_a)+\frac{g}{f}\frac{\partial}{\partial y}\left(h\frac{\partial\xi}{\partial y}\right)\right]+g\frac{\partial(h,\xi)}{\partial(x,y)}=\frac{1}{f}\frac{\partial}{\partial t}\nabla\cdot\vec{\tau}+\vec{z}\cdot\nabla\times\vec{\tau} \quad (5)$$

The equation (5) represents the appearance of the forcing terms, divergence and curl of wind stress.

The ratio of the second term $f(\xi+\xi_a)$ to the first term $\frac{g}{f}\frac{\partial}{\partial x}\left(h\frac{\partial\xi}{\partial x}\right)$ is of the order of $\frac{(\xi+\xi_a)}{\xi(1+2\lambda l)}\frac{l^2}{R_0^2}$ where R_0 is the barotropic radius of deformation ($R_0=(g\bar{h})^{1/2}/|f|$). Most continental shelves have small width compared with R_0. ξ and ξ_a are of the same order from the spectra of the atmospheric pressure and adjusted sea levels. Since the pressure term ξ_a is negligible in (5), the generation of shelf waves should be related to the wind stress and $\partial\xi'/\partial t$ is negligible in the equation of continuity (3). This approximation also suggests that that the gravity waves are filtered and the movement is quasi-nondivergent, which allows to introduce a stream function ψ defined as

$$u=\frac{1}{h}\frac{\partial\psi}{\partial y};\quad v=-\frac{1}{h}\frac{\partial\psi}{\partial x} \quad (6)$$

Introducing the stream function, we obtain the equation for ψ from (1), (2) and (3) using the above approximations.

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial x}\left(\frac{1}{h}\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{h}\frac{\partial\psi}{\partial y}\right)\right]-f\frac{\partial(1/h,\psi)}{\partial(x,y)}=\frac{1}{h^2}\left[\tau_y\frac{\partial h}{\partial x}-\tau_x\frac{\partial h}{\partial y}\right]-\frac{1}{h}\vec{z}\cdot\nabla\times\vec{\tau} \quad (7)$$

The adjusted sea level has been shown to be related to longshore wind stress much more than that of onshore wind stress from the correlograms between the components of wind stress and the adjusted sea levels, which implies that ξ would be explainable by the shelf waves generated by the longshore wind stress (Lie, 1979). The depth variation perpendicular to the coast is generally known to be much greater than that parallel to the coast and thus $\tau_x\frac{\partial h}{\partial y}$ can be neglected. Then, the forcing term $\frac{1}{h}\vec{z}\cdot\nabla\times\vec{\tau}$ disappears because of the scale assumptions (Adams and Buchwald, 1969; Gill and Schumann, 1974).

Using those approximations, we obtain

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial x}\left(\frac{1}{h}\frac{\partial\psi}{\partial x}\right)+\frac{1}{h}\frac{\partial^2\psi}{\partial y^2}\right]+\frac{f}{h^2}\frac{dh}{dx}\frac{\partial\psi}{\partial y}=\frac{1}{h^2}\frac{dh}{dx}\tau_y \quad (8)$$

4. FREE SHELF WAVES

Assuming that the solution of (8) for $\tau_y=0$ is

$$\psi(x,y,t)=\varphi_0\, e^{\lambda x} e^{-i(\sigma t-\vec{k}\cdot\vec{r})} \quad (9)$$

where $\vec{k}$ is the wave vector $(\vec{k}=a\vec{x}+b\vec{y})$ and $\vec{r}$ is the position vector $(\vec{r}=x\vec{x}+y\vec{y})$. The substitution of (9) in (8) gives the dispersion relation

$$a^2+b^2+\lambda^2=-2\lambda f\frac{b}{\sigma} \quad (10)$$

Solution (9) must satisfy the boundary conditions such that

$u=0$ at $x=0$,

u and v are continuous at $x=l$.

The following constraint comes from the boundary conditions

$$\tan(al)=-a/(\lambda+|b|) \quad (11)$$

Adams and Buchwald(1968) obtained the same condition. Gill and Schumann(1974), assuming that $v\gg u$ for $0<x<l$ and $v\ll u$ for $x>l$, obtained $\tan(al)=-a/\lambda$ which can be also obtained by neglecting $|b|$ in comparison with λ in (11).

There are infinite intersections between G_1 $[=\tan(al)]$ and $G_2[=-a/(\lambda+|b|)]$ but the dispersion relation (10) imposes a limit to the number of modes.

The relation (10) is written in a circular form

$$a^2+\left(b+\lambda\frac{f}{\sigma}\right)^2=\lambda^2\left(\frac{f^2}{\sigma^2}-1\right) \quad (12)$$

The intersections which are on a circle with the radius of $\lambda\left(\frac{f^2}{\sigma^2}-1\right)^{1/2}$ centered at $a=0$ and $b=-\lambda\frac{f}{\sigma}$ are only valid. The function G_2 shows two different curves according to the cases $b>-\lambda\frac{f}{\sigma}$ or $b<-\lambda\frac{f}{\sigma}$ because $G_2=-a\left\{\lambda+\left|-\lambda\frac{f}{\sigma}\pm\left[\left(\lambda\frac{f}{\sigma}\right)^2-(\lambda^2+\sigma^2)\right]^{1/2}\right|\right\}^{-1}$. We will call modes n the waves which correspond to the intersections for $|b|<\lambda\frac{f}{\sigma}$, and modes m the waves for $|b|>\lambda\frac{f}{\sigma}$.

Figure 1 denotes the intersections between G_1 and G_2 which satisfy the dispersion relation. Table 1 presents the values, a, b and phase velocity corresponding to the lowest six modes for $\lambda=10^{-7}$/cm and $f/\sigma=7$.

It is shown from (10) that waves propagate in the negative y direction in the Northern

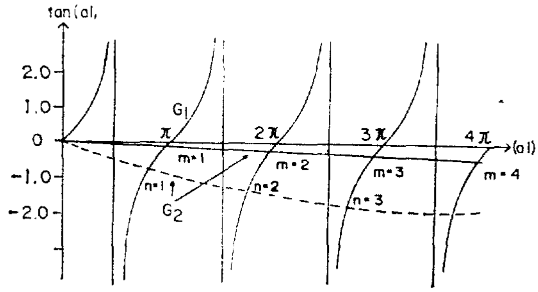

Fig. 1. Intersections between $G_1=\tan(al)$ and $G_2=-a/(\lambda+|b|)$ which satisfy the dispersion relation.

Table 1. Components of wave vector and phase velocity for $\lambda=10^{-7}$/cm and $f/\sigma=7$.

mode n	$\|a\|$ 10^{-8}/cm	b 10^{-8}/cm	phase velocity m/sec	mode m	$\|a\|$ 10^{-8}/cm	b 10^{-8}/cm	phase velocity cm/sec
1	8.308	1.218	−11.73	1	10.263	138.518	−10.31
2	17.802	3.044	− 4.69	2	20.455	136.193	−10.49
3	27.995	6.626	− 2.16	3	30.718	132.100	−10.81
4	38.397	12.331	− 1.16	4	40.911	125.913	−11.35
5	49.009	21.029	− 0.68	5	51.103	116.781	−12.23
6	59.760	34.946	− 0.41	6	61.156	102.556	−13.93

Hemisphere. The phase velocity of the first mode n corresponds well to the propagation velocity observed from the phase differences between the adjusted sea levels at different stations along the major continental shelves over the world (see Mysak, 1980, for more detail).

5. GENERAL SOLUTION OF FORCED LONG WAVES

It is assumed that the contribution of modes m to ξ is comparatively small because their velocities (of order 10 cm/sec) cannot explain the phase differences observed between the adjusted sea levels. Additionally, $\partial^2/\partial y^2$ is nearly equal to $0(b^2)$ and $\partial^2/\partial x^2$ is of order $0(a^2)$. For the first three n modes(Table 1), b^2 is very small in comparison with a^2.

With the above approximations, the equation (8) is reduced to

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial x}\left(\frac{1}{h}\frac{\partial \psi}{\partial x}\right)\right]+\frac{f}{h^2}\frac{dh}{dx}\frac{\partial \psi}{\partial y}=\frac{1}{h^2}\frac{dh}{dx}\tau_y(y,t) \qquad (13)$$

where the wind stress is assumed a function of t and y. Taking a solution $\psi(x,y,t)=\sum_n \varphi_n(x)G_n(y,t)$, a first order wave equaton is obtained by the same process of Gill and Schumann(1974)

$$\frac{1}{c_n}\frac{\partial}{\partial t}G_n(y,t)+\frac{\partial}{\partial y}G_n(y,t)=\frac{p_n}{f}\tau_y(y,t) \qquad (14)$$

where $\varphi_n(x)=A_n e^{\lambda x}\sin(a_n x)$, $P_n=\int_0^l \frac{1}{h^2}\frac{dh}{dx}\varphi_n(x)dx=-\frac{a_n c_n A_n}{f h_0}$ and $A_n^2=h_0/\lambda[l+\lambda/(\lambda^2+a_n^2)]$

The problem of Cauchy for the equation (14) consists in finding for $t<0$ a solution $G_n(y,t)$ for the right part of (14) and $G_n(y,0)=G_{n0}(y)$ given. We assume the motions start from the rest at $t=0$ to solve this equation (14). We consider that the wind stress starts action at time $t=0$ and that $G_n(y,t)$ and $\tau_y(y,t)$ are zero for $t<0$. Call $\overline{G}_n$ and $\overline{Q}$ the extended functions of G_n and Q for $t<0$, where $Q(y,t)=p_n\tau_y(y,t)/f$.

Applying the Fourier transformation to (14) in y for fixed t with the definitien of $\overline{G}_n$ and $\overline{Q}$, we have

$$\frac{1}{c_n}\frac{\partial \overline{V}_n}{\partial t}+2\pi ik\,\overline{V}_n=\overline{Z}(k,t) \qquad (15)$$

where $\overline{V}_n(k,t)=\int_{-\infty}^{+\infty}\overline{G}_n(y,t)e^{-2\pi iky}\,dy$ and $\overline{Z}(k,t)=\int_{-\infty}^{+\infty}\overline{Q}(y,t)e^{-2\pi iky}\,dy$. The solution $\overline{G}_n(y,t)$ can be also obtained by the Fourier transformation in t for fixed y or by the Laplace transformation. The equation (15) is expressed in the form of convolution in t for fixed k (Schwartz, 1965):

$$\left[\frac{1}{c_n}\frac{\partial\delta(t)}{\partial t}+2\pi ik\delta(t)\right]\underset{(t)}{*}\overline{V}_n(k,t)=\overline{Z}(k,t) \qquad (16)$$

where the symbol * represents the convolution product and $\delta(t)$ is the Dirac distribution, defined by

$$\delta(t)=\int_{-\infty}^{+\infty}e^{-2\pi i\lambda t}d\lambda$$

The elementary solution $c_nY(t)e^{-2\pi ikc_nt}$ of $\overline{V}_n(k,t)$ is obtained directly from the inverse of $\left[\frac{1}{c_n}\frac{\partial\delta}{\partial t}+2\pi ik\delta\right]\cdot\overline{V}_n(k,t)$ is therefore

$$\overline{V}_n(k,t)=\overline{Z}(k,t)\underset{(t)}{*}c_nY(t)e^{-2\pi ikc_nt} \qquad (17)$$

where $Y(t)=\begin{cases}0 & \text{for } t<0\\ 1 & \text{for } t>0\end{cases}$

$\overline{G}_n(y,t)$ is obtained from the inversion of the Fourier transformation on $\overline{V}_n(k,t)$:

$$\overline{G}_n(y,t)=c_n\int_{-\infty}^{+\infty}\overline{Z}(k,t)\underset{(t)}{*}Y(t)e^{-2\pi ik(c_nt-y)}dk \qquad (18)$$

$\overline{G}_n(y,t)$ has to be always zero for $t<0$ by the definition and the expression (18) is the solution in question.

6. ELEVATION AND LONGSHORE CURRENT VELOCITY

The difference in elevation between any two points across the shelf due to the continental shelf waves is obtained from (1) with the approximation $\sigma b \ll fa$

$$\xi_n(x_2,y,t)-\xi_n(x_1,y,t)=-\frac{f}{g}\int_{x_1}^{x_2}\frac{1}{h}\frac{\partial\psi_n}{\partial x}dx$$
$$=-\frac{f}{g}\frac{A_n\bar{G}_n(y,t)}{h_0(\lambda^2+a_n^2)}\{[(a_n^2-\lambda^2)\sin(a_nx)-2\lambda a_n\cos(a_nx)]e^{-\lambda y}\Big|_{x=x_1}^{x=x_2}$$

For instance, the difference in elevation between the coast and the limit of shelf is

$$\xi_n(0,y,t)-\xi_n(l,y,t)\sim-\frac{a_nc_nA_n\bar{G}_n(y,t)}{gh_0} \quad (19)$$

This difference is proportional to a_n, c_n, A_n and $\bar{G}_n$ but it diminishes with the depth at the coast h_0. The elevation at l is negligible compared with that at the coast because $\xi_n(l)/\xi_n(0)$ is of order e^{-3}. Typical adjusted sea level changes are of order 0.1 m and then $\xi_n(l,y,t)$ can be considered as zero.

Figure 2 shows $\xi_n(x,y,t)$ as a function of x across the shelf for the first three modes n.

The longshore current at any x is with the help of (6)

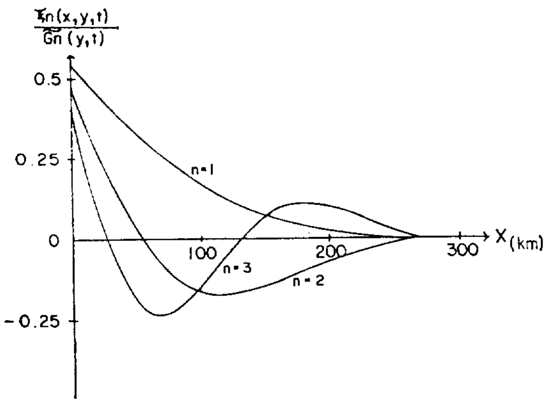

Fig. 2. Elevations ξ_n of the first three modes n across the shelf, assuming that $f/(g\sqrt{h_0})=1$.

$$V_n(x,y,t)=-\frac{A_n\bar{G}_n(y,t)}{h_0e^{\lambda x}}[\lambda\sin(a_nx)+a_n\cos(a_nx)] \quad (20)$$

The current at $x=0$ is therefore

$$V_n(0,y,t)=-\frac{a_nA_n}{h_0}\bar{G}_n(y,t)\sim\frac{g}{c_n}[\xi_n(0,y,t)-\xi_n(l,y,t)]$$

$V_n(0,y,t)$ depends thus on the difference in elevation across the shelf like the geostrophic current, and on the phase velocity c_n. And the current at $x=l$ is enough small to be neglected because $V_n(l)$ is of order $V_n(0)e^{-3}$.

7. SOLUTION FOR THE SINUSOIDAL WIND STRESS AS A FUNCTION OF t AND y

Consider the wind stress which propagates parallel to the coast with a frequency ω_0 and a wavenumber b_0

$$\vec{\tau}(y,t)=\vec{y}\,Y(t)\tau_0e^{i(\omega_0t-b_0y)} \quad \text{for } -\infty<y<+\infty \quad (21)$$

where $Y(t)$ is the step function.

The solution $\bar{G}_n(y,t)$ is obtained from (18) and a characteristic of Dirac function

$$\bar{G}_n(y,t)=\frac{p_nc_n\tau_0}{f(\omega_0-b_0c_n)}[e^{i(\omega_0t-b_0y)-i\pi/2}-e^{ib_0(c_nt-y)-i\pi/2}] \quad (22)$$

The first term in (22) represents the direct effect of the progressive wave in the same form as that related to wind stress but with a phase difference $\pi/2$. The second term would be the part of the response due to the continental shelf waves with a phase velocity c_n.

The function (22) is plotted in Figure 3 as a function of ω_0/b_0 The response function $|G_n(y,t)|$ diminishes exponentially with ω_0/b_0 symmetrically around the resonant frequency $(\omega_0/b_0=c_n)$. When ω_0/b_0 is equal to c_n, resonance occurs and $\bar{G}_n(y,t)$ increases linearly in time t:

$$\bar{G}_n(y,t)=\frac{p_nc_nt}{f}\tau_0e^{ib_0(c_nt-y)} \quad (23)$$

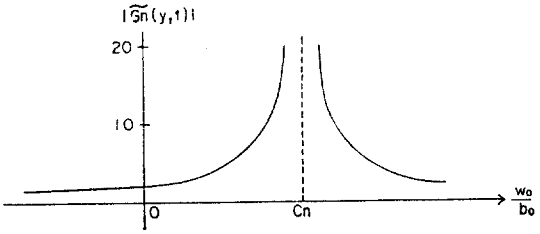

Fig. 3. $|\bar{G}_n(y,t|)$ as a function of ω_0/b_0 for the sinusoidal wind stress of the form $\tau(y,t)=\vec{y}\tau_0\exp[-i(\omega_0 t-b_0 y)]$, assuming $|b_0 f|=b_n{}^-{}_0$ and $\sin[(\omega_0-b_0c_n)t/2]=1$.

The difference of elevation across the shelf is obtained from the expressions (19) and (22)

$$\xi_n(0,y,t)-\xi_n(l,y,t)\sim\frac{M_n e^{-ib_0 y}}{(\omega_0 b_0 c_n)}$$

$$\sin\left[\frac{1}{2}(\omega_0+b_0c_n)t\right]e^{i(\omega_0+b_0c_n)t/2} \qquad (24)$$

where $M_n=-\dfrac{2a_n{}^2c_n{}^3\tau_0}{h_0 g f^2\lambda(l-c_n/2f)}$

When M_2 and M_3 are compared with M_1 for a_n and c_n given in Table 1, $M_2/M_1\sim0.325$; $M_3/M_1\sim0.082$; $0(M_1)\sim5\times10^{-4}\tau_0/\omega_0$ The contribution of the second and third modes to the elevation should be equal to 32% and 8%, respectively, of the contribution of the first mode.

8. SOLUTION FOR THE EXPONENTIAL WIND STRESS AS A FUNCTION OF t AND y

Now consider the wind stress $\vec{\tau}(y,t)$ which varies exponentially with time t and axis y of the form

$$\vec{\tau}(y,t)=\vec{y}\,Y(t)\tau_0 e^{-(\alpha t+\gamma|y|)} \qquad (25)$$

where α and γ are positive.

The solution $\bar{G}_n(y,t)$ is obtained from the expression (18)

$$\bar{G}_n(y,t)=Y(y)\frac{p_n\tau_0}{f(\gamma-\alpha/|c_n|)}$$

$$\left[\frac{2\gamma e^{-\alpha(y+|c_n|t)/|c_n|}}{\gamma+\alpha/|c_n|}-e^{-\alpha t}e^{-\gamma y}\right]$$

$$-Y(y+|c_n|t)\frac{p_n\tau_0}{f(\gamma-\alpha/|c_n|)}$$

$$\left[\frac{2\gamma e^{-\alpha(y+|c_n|t)/|c_n|}}{\gamma+\alpha/|c_n|}-e^{-\gamma(y+|c_n|t)}\right] \qquad (26)$$

The response function (26) shows the effects of wind stress (α,γ) and shelf waves (c_n) and it can be divided into two parts, the first part for the region $y>0$ and the second part for the other region $y<0$ and $y>-|c_n|t$.

(i) region $y>0$

$$\bar{G}_n(y,t)=\frac{p_n\vec{\tau}(y,t)}{f\Delta\gamma}[e^{-\Delta\gamma|c_n|t}-1] \qquad (27)$$

where $\Delta\gamma=\gamma-\alpha/|c_n|$

When the phase velocity $|c_n|$ is greater than α/γ, $|\bar{G}_n(y,t)|<\left|\dfrac{p_n\vec{\tau}(y,t)}{f\Delta\gamma}\right|$, while for $|c_n|<\alpha/\gamma$, $|\bar{G}_n(y,t)|\sim\left|\dfrac{p_n\vec{\tau}(y,t)}{f\Delta\gamma}e^{-\Delta\gamma|c_n|t}\right|$

The maximal value of $|\bar{G}_n(y,t)|$ is therefore more important for $|c_n|<\alpha/\gamma$ than that for $|c_n|>\alpha/\gamma$.

(ii) region $-|c_n|t<y<0$

$$\bar{G}_n(y,t)=\frac{p_n\tau_0}{f\Delta\gamma}e^{-\gamma(y+|c_n|t)}$$

$$\left[1-\frac{2\gamma}{2\gamma-\Delta\gamma}e^{\Delta\gamma(y+|c_n|t)}\right] \qquad (28)$$

For $|c_n|>\alpha/\gamma$, $|\bar{G}_n(y,t)|$ is always less than $\left|\dfrac{2p_n\tau_0}{f\Delta\gamma}\right|$, while for $|c_n|<\alpha/\gamma$, the maximal value of $|\bar{G}_n(y,t)|$ is smaller than $\left|\dfrac{p_n\tau_0}{f\Delta\gamma}e^{-\gamma(y+|c_n|t)}\right|$ because $\dfrac{2\gamma}{2\gamma-\Delta\gamma}$ and $e^{\Delta\gamma(y+|c_n|t)}$ are less than 1.

At the place where the shelf wave arrives $(y\sim-|c_n|t)$, the solution is approximately equal to $-\dfrac{p_n\tau_0}{f(2\gamma-\Delta\gamma)}$. The elevation depends on γ and the anomaly $\Delta\gamma$.

The response function (26) for the exponential wind stress depends on two factors, the phase velocity of shelf waves and the anomaly $\Delta\gamma$. It presents also the possibility of resonance when the phase velocity $|c_n|$ is equal to α/γ.

9. RESULTS AND DISCUSSIONS

The generation of shelf waves does not seem to be directly driven by the moving pressure forcing term from the fact that ξ_a is negligible in the equation (5) compared with the other terms. Shelf waves are most likely generated by the wind stress. The partial differential equation for the stream function (7) indicates that shelf waves are generated by the longshore wind stress. Also, high correlations between adjusted sea levels and longshore wind stress support this possibility(Lie, 1979).

Equation (5) represents appearance of two wind forcing terms on the right side; divergence and curl of wind stress. The forcing term, proportional to $\nabla\times\vec{\tau}$, has been assumed to be negligible according to scale assumption, but it may play an important part as forcing mechanism when the wind varies rapidly across the shelf.

Gill and Schumann(1974) gave an explanation that it is the boundary condition at the coast which introduces the wind forcing term. But no forcing terms in their equation are rather due to the assumption $\frac{\partial}{\partial y}\ll\frac{\partial}{\partial x}$ in the equations of motion. Equations (5) and (7) present both the appearance of two wind forcing terms without consideration of the boundary condition at the coast. If there does not exist the boundary condition at the coast, the wind forcing terms produce waves which satisfy only the dispersion relation (10), but the boundary condition at the coast permits only a countable infinite number of modes to exist on the shelf, which satisfy the first-order wave equation.

Equation (7) can be written in the form

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial x}\left(\frac{1}{h}\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{h}\frac{\partial\psi}{\partial y}\right)\right.$$
$$+f\left[\left(\frac{\partial\psi}{\partial x}+\frac{\tau_x}{f}\right)\frac{\partial}{\partial y}\left(\frac{1}{h}\right)\right.$$
$$\left.-\left(\frac{\partial\psi}{\partial y}+\frac{\tau_y}{f}\right)\frac{\partial}{\partial x}\left(\frac{1}{h}\right)\right]$$
$$=-\frac{1}{h}\vec{z}\cdot\nabla\times\vec{\tau} \qquad (29)$$

The equation (29) describes well the physical mechanisms concerning total and Ekman transports. Distinction between the two mechanisms was discussed in detail by Gill and Schumann (1974).

The method used in obtaining the general solution of forced waves is very easy and usefull one; it gives directly an elementary solution and sea level changes for a given wind stress can be easily calculated by the Fourier transformation and the convolution. The elementary solution has a similar form of a standing wave. The integration in (18) implies the possibility of resonance when the wind stress varies with time and y-axis. The response function (22) for the sinusoidal wind stress presents resonance when the phase velocity of shelf waves c_n is equal to the propagation velocity of wind. The solution for an exponential wind stress which is not in the form of progressive waves also gives the possibility of resonance at $|c_n|=\alpha/\gamma$.

ACKNOWLEDGEMENTS

The author wishes to present his gratitude to Professor Saint-Guily and Dr. M. Crépon for their helpful and constant discussions during this work.

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