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Stability of Asymmetric Shaft Carrying two Discs with Limited Power

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This paper presents a study of stability of the two mass rotor with limited power similar to that employed by Kononenko and also presents a study on the digital computer to obtain the unstable region of the rotor over two critical speed ranges.

The unstable region is identified by studying the Routh-Hurwitz criterion of the characteristic equation.

The results are presented in two and three-dimensional diagrams to show how the various parameters affect the unstable region.

Introduction

In many of high speed rotors (gas turbine, auxiliary power machinery, compressor, turbo-generator etc.), the design operating speed is often beyond the rotor first critical speed and under these circumstances the problem of insuring that the machine will perform with a stable low-level amplitude of vibration is often difficult to achieve.

The study of rotor dynamics has become of increasing importance in the design of power system in recent years.

This is due in part to the increased demand for reliable machine performance and variable operating conditions. Rotor dynamics consists of the study of the following major areas of concern.

1. critical speed analysis.
2. methods of calculating the residual unbalance.
3. steady state and transient response to unbalance.
4. stability analysis of rotor.

In advanced models, where stiffness asymmetry of the shaft is included, periodic coefficients appear in the equations of motion and instability speed ranges occur due to these periodic coefficients.

Solution of such equations are obtained by either the Hill method based on Floquet theory or the pertubation-variation method outlined by Hsu[1),2)].

Recent investigation such as Messal and Bonthron[3)], Tondle[4)], Kononenko[5)] and others[6),7)] have shown.

Iwatsubo[8)] have studied a problem of parametric resonance of asymmetric rotor system.

Yamamoto and Ota[9)] dealt with a simple rotor system composed of one shaft and disk, in which the asymmetry of rotational inertia, shaft stiffness were combined.

These investigations have not been treated the stability of asymmetric shaft carrying two discs with limited power systematically.

In previous works[10)], the author has treated nonstationary vibration characteristics of asym-

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metric shaft carrying two discs during passing through its critical speeds.

This paper presents an analytic study of stability of the rotor with limit power which employed at previous paper and also presents the study performed on the digital computer to obtain the stability region of the rotor over two critical speeds.

Nomenclature

$O\text{-}xyz$: fixed rectangular coordinate system

x_1, y_1, x_2, y_2: coordinate of gravitational center of the rotor

m_1, m_2: mass of the discs

I: moment of inertia of the rotor

c_{11}, c_{12}, c_{22}: stiffness of the shaft

$2\Delta c_{11}, 2\Delta c_{12}, 2\Delta c_{22}$: difference between maximum and minimum values of c_{11}, c_{12}, c_{22}

C_{e1}, C_{e2}: coefficients of the external damping

C_{i1}, C_{i2}: coefficients of the internal damping

r_1, r_2: eccentricity of the discs

ϕ: revolution angle of the rotor

β: angle between r_1 and r_2

$\omega_{1,2}$: first and second critical speed

$M-S\dot{\phi}$: driving torque

Equation of Motion

Fig. 1 illustrates the mathmatical model of the flexible rotor system.

The shaft, carrying two discs with eccentricities r_1 and r_2 rotates with angular velocity $\dot{\phi}$.

The external damping forces are assumed to be proportional to the velocity of the center of the discs, while the internal damping forces are assumed to be proportional to the velocity of the bending deformation of the shaft.

The equations of motion of the rotor are derivable from the potential, kinetic and dissipation energy functions and use of Lagrange's equation.

The detail derivation was described in Appendix of previous paper[10] and the final equations are;

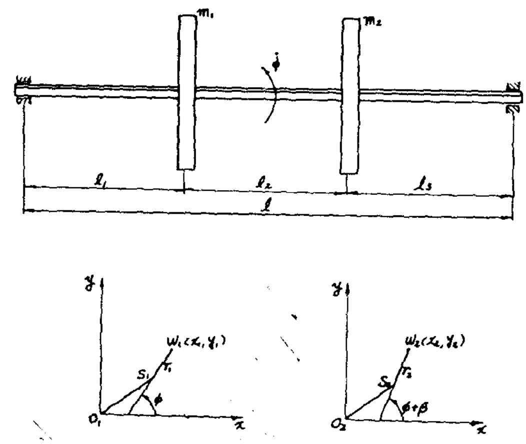

Fig. 1. A asymmetric shaft carrying two discs.

$$m_1\ddot{z}_1+c_{11}z_1+c_{12}z_2=\mathcal{E}\{m_1e_1\dot{\phi}^2e^{i\phi}-(k_1+d_1)\dot{z}_1-id_1\dot{\phi}z_1 -\Delta c_{11}\bar{z}_1e^{2i\phi} -\Delta c_{12}\bar{z}_2e^{2i\phi}\}$$

$$m_2\ddot{z}_2+c_{22}z_2+c_{12}z_1=\mathcal{E}\{m_2e_2\dot{\phi}^2e^{i(\phi+\beta)}-(k_2+d_2)\dot{z}_2 -id_2\dot{\phi}z_2-\Delta c_{22}\bar{z}_2e^{2i\phi}-\Delta c_{12}\bar{z}_1e^{2i\phi}\}$$

$$I\ddot{\phi}=\mathcal{E}\,[L(\dot{\phi})-\frac{i}{2}\{c_{11}e_1(z_1e^{-i\phi}-\bar{z}_1e^{i\phi})+c_{12}e_1(z_2e^{-i\phi}-\bar{z}_2e^{i\phi}) -c_{22}e_2(z_2e^{-i(\phi+\beta)}-\bar{z}_2e^{i(\phi+\beta)})+c_{12}e_2(z_1e^{-i(\phi+\beta)}-\bar{z}_1e^{i(\phi+\beta)}) -d_1(z_1\dot{\bar{z}}_1-\dot{z}_1\bar{z}_1)-d_2(\dot{\bar{z}}_2z_2-\dot{z}_2\bar{z}_2)\}-d_1\dot{\phi}z_1\bar{z}_1-d_2\dot{\phi}z_2\bar{z}_2+\frac{1}{2}\Delta c_{11}\{(z_1^2+\bar{z}_1^2)\sin 2\phi+i(z_1^2-\bar{z}_1^2)\cos 2\phi\}+\frac{1}{2}\Delta c_{12}\{(z_1z_2+\bar{z}_1\bar{z}_2)\sin 2\phi-(\bar{z}_1z_2+z_1\bar{z}_2)\cos 2\phi\} +\frac{1}{2}\Delta c_{22}\{(z_2^2+\bar{z}_2^2)\sin 2\phi+i(z_2^2-\bar{z}_2^2)\cos 2\phi\}] \quad \cdots\cdots(1)$$

where

$C_{e1}=\mathcal{E}k_1$, $C_{e2}=\mathcal{E}k_2$, $C_{i1}=\mathcal{E}d_1$, $C_{i2}=\mathcal{E}d_2$, $r_1=\mathcal{E}e_1$, $r_2=\mathcal{E}e_2$, $\Delta c_{11}=\mathcal{E}\,\frac{c_{x11}-c_{y11}}{2}$, $\Delta c_{12}=\mathcal{E}\,\frac{c_{x12}-c_{y12}}{2}$, $\Delta c_{22}=\mathcal{E}\,\frac{c_{x22}-c_{y22}}{2}$, $\mathcal{E}L(\dot{\phi})=M-S\dot{\phi}$

Stability Analysis

To study the stability of the stationary motions of the first and second resonance, we carry out derivation of the approximate solution and averaging operation of eq. (1) by the method of Bogoliubov's perturbation theory[11].

The resultant equations are

$$\frac{dA_i}{dt}=(\omega-\omega_i)B_i-\frac{\mathcal{E}}{2\mu_1\omega_i}\{-a_{12}\omega^2+(b_{11}\omega_i+b_{12}\omega)A_i+kB_i\}$$

$$\frac{dB_i}{dt}=(\omega-\omega_i)A_i-\frac{\mathcal{E}}{2\mu_1\omega_i}\{a_{11}\omega^2+(b_{11}\omega_i+b_{12}\omega)B_i-kA_i\}$$

$$\frac{d\omega}{dt}=\mathcal{E}/I\ \{L(\omega)+b_{12}(A_i^2+B_i^2)(\omega_i-\omega)+d_{12}(A_i\sin\beta-B_i\cos\beta)-2A_iB_ik+3\Delta c_{12}s_iA_iB_i\}\quad (i=1,\ 2) \qquad \cdots\cdots(2)$$

where

$$s_i=\frac{\omega_i^2m_1-c_{11}}{c_{12}},\quad \mu_1=m_1+s_i^2m_2$$

$$b_{11}=k_1+d_1+s_i^2(k_2+d_2)$$

$$b_{12}=d_1+s_i^2d_2$$

$$a_{11}=m_1e_1+s_im_2e_2\cos\beta$$

$$a_{12}=s_im_2e_2\sin\beta$$

$$k=\Delta c_{11}+2\Delta c_{12}s_i+\Delta c_{22}s_i^2$$

$$d_{11}=e_1(c_{11}+s_ic_{12})+e_2(c_{12}+s_ic_{22})\cos\beta$$

$$d_{12}=e_2(c_{12}+s_ic_{22})\sin\beta$$

$$A_i=\frac{a_{12}(b_{11}\omega_i+b_{12}\omega)\omega^2-a_{11}k\omega^2-2\mu_1a_{11}\omega_i\omega^2(\omega-\omega_i)}{k^2-(b_{11}\omega_i+b_{12}\omega)^2-[2\omega_i\mu_1(\omega-\omega_i)]^2}$$

$$B_i=\frac{-a_{11}(b_{11}\omega_i+b_{12}\omega)\omega^2+a_{12}k\omega^2-2\mu_1a_{12}\omega_i\omega^2(\omega-\omega_i)}{k^2-(b_{11}\omega_i+b_{12}\omega)^2-[2\omega_i\mu_1(\omega-\omega_i)]^2}$$

This method has the advantage that the differential equations are linear with constant coefficients, so that the problem of stability reduces to the Routh-Hurwitz criterion for eq. (2).

Eq. (2) are written as

$$\frac{dA_i}{dt}=\Phi_1(A_i,B_i,\ \omega)$$

$$\frac{dB_i}{dt}=\Phi_2(A_i,\ B_i,\ \omega)$$

$$\frac{d\omega}{dt}=\Phi_3(A_i,\ B_i,\ \omega)\ (i=1,\ 2)\cdots\cdots\cdots\cdots(3)$$

where the function Φ_1, Φ_2, Φ_3 represent the right-hand sides of eq. (2).

$$A_i=a_i+\xi_i,\ B_i=b_i+\eta_i,\ \omega=\omega_0+\zeta_i\cdots\cdots\cdots(4)$$

a_i, b_i, ω_0 are values of A_i, B_i, ω_i for the stationary motion, i.e., roots of the following equation.

$$\Phi_j(a_i,\ b_i,\ \omega_0)=0 \qquad (i=1,\ 2)\ (j=1,\ 2,\ 3)\cdots\cdots(5)$$

ξ_i, η_i, ζ_i are small perturbations of the quantities A_i, B_i, ω from their values for the stationary motion.

Substituting eq. (4) into eq. (3) and expanding the expression obtained in a series of powers of the small quantities ξ_i, η_i, ζ_i taking only linear terms of the expansion,

$$\frac{d}{dt}\begin{pmatrix}\xi_i\\ \eta_i\\ \zeta_i\end{pmatrix}=\begin{pmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{pmatrix}\begin{pmatrix}\xi_i\\ \eta_i\\ \zeta_i\end{pmatrix}\qquad\cdots\cdots\cdots(6)$$

The matrix of coefficients of these equation is

$$b_{jk}=\begin{pmatrix}\frac{\partial}{\partial A_i}\Phi_1, & \frac{\partial}{\partial B_i}\Phi_1, & \frac{\partial}{\partial\omega}\Phi_1\\ \frac{\partial}{\partial A_i}\Phi_2, & \frac{\partial}{\partial B_i}\Phi_2, & \frac{\partial}{\partial\omega}\Phi_2\\ \frac{\partial}{\partial A_i}\Phi_3, & \frac{\partial}{\partial B_i}\Phi_3, & \frac{\partial}{\partial\omega}\Phi_3\end{pmatrix}\qquad\cdots\cdots\cdots(7)$$

Then, the characteristic equation of the rotor system is

$$\lambda^3+B_1\lambda^2+B_2\lambda+B_3=0 \qquad\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

where

$$B_1=-(b_{11}+b_{22}+b_{33})$$

$$B_2=b_{11}b_{22}+b_{22}b_{33}+b_{33}b_{11}-b_{12}b_{21}-b_{23}b_{32}-b_{31}b_{13}$$

$$B_3=b_{11}b_{23}b_{32}+b_{12}b_{21}b_{33}+b_{13}b_{31}b_{22}-b_{11}b_{22}b_{33}-b_{12}b_{23}b_{31}-b_{13}b_{21}b_{32}$$

$$b_{11}=-\frac{\mathcal{E}}{2\mu_1\omega_i}(b_{11}\omega_i+b_{12}\omega)$$

$$b_{12}=(\omega-\omega_i)+\frac{\mathcal{E}k}{2\mu_1\omega_i}$$

$$b_{13}=B_i+\frac{\mathcal{E}}{2\mu_1\omega_i}(2a_{12}\omega-b_{12}A_i)$$

$$b_{21}=-(\omega-\omega_i)+\frac{\mathcal{E}k}{2\mu_1\omega_i}$$

$$b_{22}=-\frac{\mathcal{E}}{2\mu_1\omega_i}(b_{11}\omega_i+b_{12}\omega)$$

$$b_{23}=-A_i-\frac{\mathcal{E}}{2\mu_1\omega_i}(2a_{11}\omega+b_{12}B_i)$$

$$b_{31}=\frac{\mathcal{E}}{I}\{2b_{12}A_i(\omega_i-\omega)-d_{12}\sin\beta-B_i(2k-\Delta c_{12}s_i)\}$$

$$b_{32}=\frac{\mathcal{E}}{I}\{2b_{12}B_i(\omega_i-\omega)+d_{11}+d_{12}\cos\beta-A_i(2k-\Delta c_{12}s_i)\}$$

$$b_{33}=\frac{\mathcal{E}}{I}\{\frac{d}{d\omega}L(\omega)-b_{12}(A_i^2+B_i^2)\}$$

The necessary and sufficient condition for the stationary motion to be stable is that the characteristic equation has roots which are real and negative.

$$B_1>0,\ B_3>0,\ B_1B_2-B_3>0 \qquad\cdots\cdots\cdots\cdots\cdots(9)$$

Discussion of Results

Numerical examples illustrating the analytical

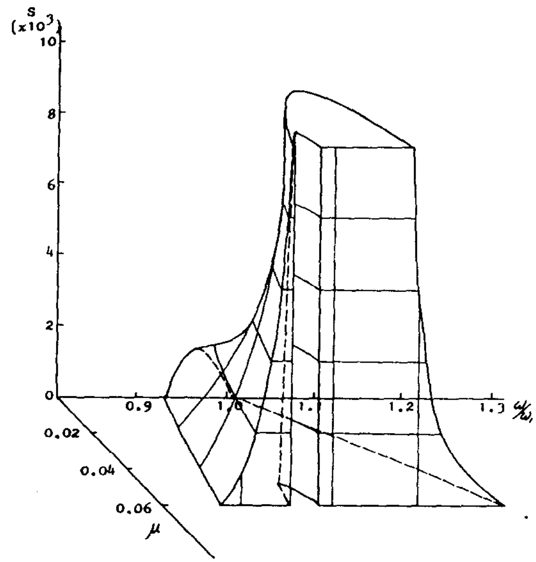

Fig. 2. Unstable region near ω_1 due to stiffness asymmetry and driving source.

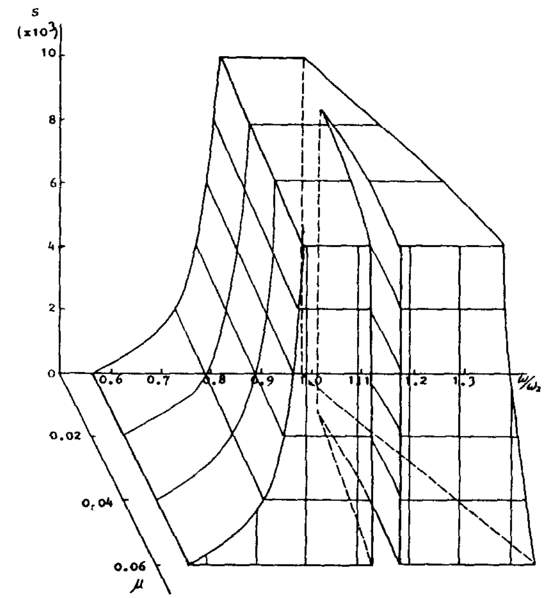

Fig. 3. Unstable region near ω_2 due to stiffness asymmetry and driving source.

developments are shown in Fig. 2—6.

It is clear that the first condition of stability $B_1>0$ is always satisfied for an energy source characteristics which is falling $d/d\omega\ L(\omega)<0$.

In Fig. 2, the unstable region in the vicinity of the first critical speed narrows with the increasing of gradient of driving torque.

The unstable region widens with the increasing of shaft asymmetry.

If the shaft asymmetry is large, a narrower stable region appears in the vicinity of the critical speeds.

Similarly, Fig. 3 shows unstable region in the vicinity of the second critical speed.

Since the interaction of driving torque and resisting torque appear clearly, unstable region gets wider than that in Fig. 2.

The effect of eccentricity near ω_2 are shown in Fig. 4.

In case of $B_1B_2-B_3>0$, the unstable region exists in the vicinity of the critical speed, not depending on the amount of eccentricity and damping coefficient.

In case of $B_3>0$, when damping coefficient increases, unstable region increases and then

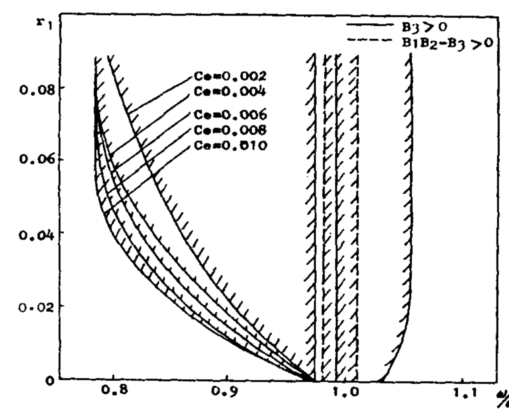

Fig. 4. The effect of eccentricity near $\omega_2(r_2=r_1)$.

becomes nearly constant, disregarding increasing of eccentricity.

Fig. 5 shows the characteristics of the driving torque for $B_1B_2-B_3>0$.

Unstable region increases only with shaft asymmetry, disregarding the characteristics of the driving torque.

Fig. 6(a), (b) shows the effect of phase for $B_3>0$, $B_1B_2-B_3>0$.

For $B_3>0$, if damping coefficient increases,

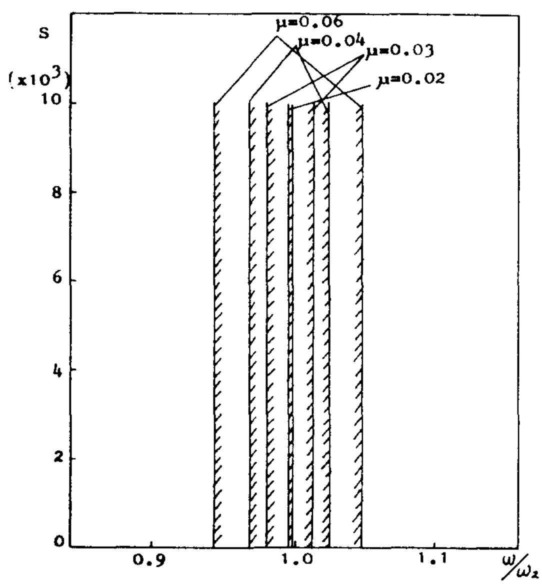

Fig. 5. The effect of characteristics of the driving driving torque($B_1B_2-B_3>0$).

minimum unstable region moves from $\pi/4$ to 2π (0°), while maximum unstable region appears at π. For $B_1B_2-B_3>0$, maximum unstable region appears at 0.5π and 1.5π.

Conclusions

The dynamics of the rotor system with shaft asymmetry carrying two discs have been studied with various factors.

It can be seen that the criteria of stability depends on the shaft asymmetry and the characteristics of the driving torque $L(\omega)$.

Damping force decreases unstable region caused by shaft asymmetry, but increases unstable region caused by driving torque, and self-excited vibration are likely to occur with it.

Angle between eccentricities were found to have a great importance on the trend of the unstable region.

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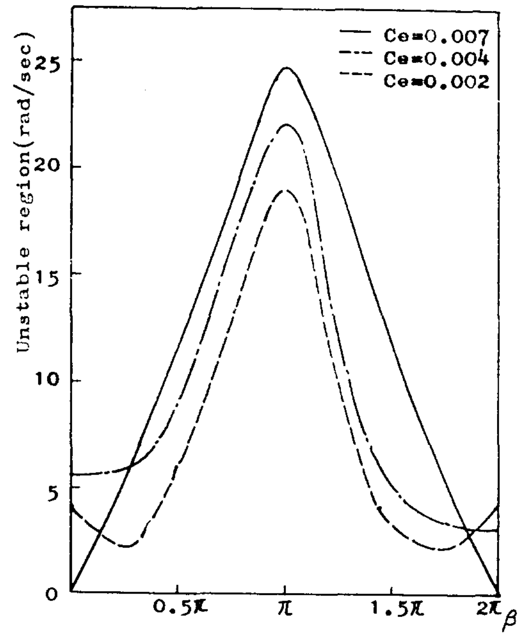
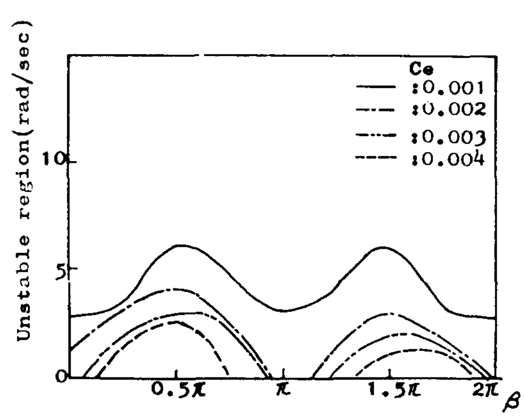


(a) In case of $B_3>0$



(b) In case of $B_1B_2-B_3>0$

Fig. 6. The effect of phase.

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有限驅動力을 갖는 2個圓板이 附着된 非對稱軸의 安定性에 관한 研究

梁　保　錫

回轉軸系의 모델로서 驅動源과 相互作用을 고려한 2個圓板이 附着된 非對稱軸系의 安定性을 究明하기 위해 安定條件을 유도하고 數値計算을 행하여 第一, 二共振領域의 不安定區間을 구하였다.

非對稱軸系의 危險速度 通過時의 不安定은 軸의 非對稱度에 의한 不安定과 驅動 Torque 特性에 의해 發生하는 不安定의 2종류로 분류된다.

편심의 位相角이 불안정에 미치는 영향은 매우 크며, 이는 後者에 의한 不安定에 기인한다.

減衰는 전자에 의해 생기는 불안정을 減少시키나, 후자에 의한 불안정은 오히려 증가시켜서 자려진동을 유발하므로 주의를 요한다.