論　文

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# The Use of Rankine Source to Evaluate Velocities around a Ship Hull

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## Abstract

A flow problem around a ship hull with the nonlinear free surface boundary condition has been considered within the potential flow assumption. The Green's function based on the hull boundary condition is constructed numerically and used to satisfy the free surface boundary condition. This singularity to be distributed ideally on the undulating free surface is put actually, for practical reasons, on the flat water surface with the assumption of linear variation of velocities between the two positions. The surfaces of singularity distribution are approximated by Hess and Smith type quadrilaterals.

The radiation condition is only crudely satisfied and this produced one of the major difficulties arising in the present way of attacking the problem.

## Nomenclature

$\Phi, \phi$　velocity potenial

$U$　ship speed

$u, v, w$　$x$-, $y$-, $z$- component of the disturbance velocity respectively

$g$　acceleration due to the gravity

$\zeta$　free surface elevation

$\boldsymbol{n}$　unit normal vector directing toward the flow field from the concerned surface

$n_x, n_y, n_z$　$x$-, $y$-, $z$- component of $\boldsymbol{n}$, respectively

$S_i$　area of the $i$-th source panel

$\sigma_i$　strength of singularity on the $i$-th source panel

$N_b$　number of source panels distributed on the body surface

$N_s$　number of source panels distributed on the water surface

$B\sigma_i$　source strength on the $i$-th body panel

$S\sigma_i$　source strength on the $i$-th water surface panel

In addition, a subscript usually denotes the partial derivative.

## 1. Introduction

The ship wave-making problem has been considered traditionally within the frame of linearised theory. However, with the appearance of extremely full hull forms as well as the more stringent technical demands on theoretical predictions, a full treatment of the nonlinear boundary condition on the free surface has been a subject of deep interest in recent years. The

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object of this paper is to introduce a new way of looking at the problem under the potential flow assumption with such an object in mind.

As is well known, the main difficulty of the problem lies in the fact that the free surface should satisfy a nonlinear boundary condition. If this boundary condition is linearised, it is possible to construct the Green's function wich comprises the free surface boundary condition and the radiation condition in the singularity itself, like the so-called Havelock source. With this method of approximation, the free surface boundary condition is no longer of major interest when such a singularity is employed as the basic flow generating mechanism.

However, if the nonlinear boundary condition is to be studied, it is not possible to construct a Green's function which absorbs the free surface boundary condition as an integrated part of the singularity. Hence an ordinary singularity without the property of generating free surface must be used as the basic flow generation mechanism. Then,any surface that bounds the flow domain must be represented by distributed singularities. This means that not only the body surface but also the free surface will require a singularity distribution, i.e. the space occupied by air must be treated as if it were a solid body with its geometry as a part of the problem to be sloved. In the usual case when the hull geometry is predetermined, the flow generating effect of the singularities distributed on the hull surface can eventually be merged with the flow from those on thefree surface, resulting in the Green's function based on the body boundary condition. This concept, however, is convenient in use only when the position of singularity distribution to represent the free surface is fixed. This Green's function is in contrast to the usual one based on the linearised free surface boundary condition and the radiation condition.

The problem has been considered from the above point of view in this paper using the Rankine source as the basic flow generating mechanism at the initial stage. The employment of Rankine source to tackle the flow problem of ship motion has been tried in a few previous investigations, for instance, by Gadd[1] and by Dawson[2]. However the fundamental philosophy in employing the Rankine source to be different from the present paper as the idea of double model is used in both the papers cited above, thus restricting the significance of sources distributed on the plane of mirror symmetry to absorbing or emitting whatever velocities are required from the boundary conditions rather than having the function of representing the boundary surface as in the present investigation. The merit of using the single model is that it is nearer to the reality assumed by potential flow theory.

## 2. Mathematical Statesments

The problem considered is the usual one of potential flow around a ship hull without allowing trim and sinkage. The coordinate system adopted is shown in Fig. 1.

The velocity potential can be defined and written as the following:

$$\Phi(x, y, z) = -Ux + \phi\ (x, y, z) \tag{1}$$

where $U$ is the ship speed and $\phi$ is called the disturbance velocity potential. The fluid velocity is then given by

$$\boldsymbol{v} = -\nabla\Phi$$

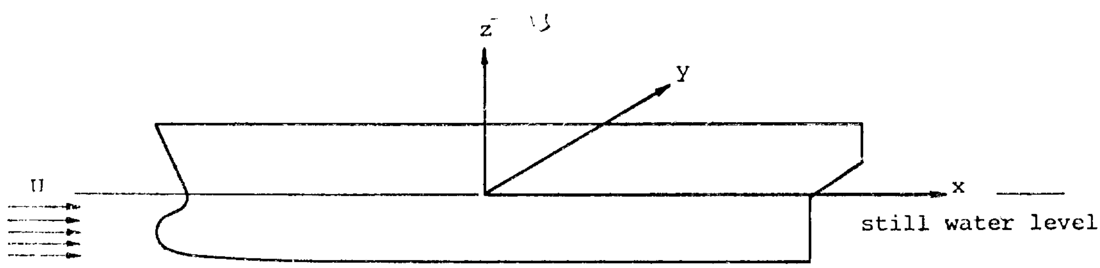



**Fig. 1.** The Coordinate System.

$$=(U+u)\boldsymbol{i}+v\boldsymbol{j}+w\boldsymbol{k} \quad (2)$$

In this expression, $(u, v, w) = -\nabla\phi$ denotes the distrbance velocity. The velocity potential satisfies the Laplace equation

$$\nabla^2\Phi=0 \quad (3A)$$

$$\text{or } \nabla^2\phi=0 \quad (3B)$$

Application of the Bernoulli equation produces

$$\zeta(x,y)=-\frac{1}{2g}\left[2Uu+(u^2+v^2+w^2)\right]_{\boldsymbol{x}=\boldsymbol{x}_s} \quad (4)$$

for the free surface elevation, where $\boldsymbol{x}_s$ denotes the position vector of a point at the free surface. This expression will be called the "free surface generating equation".

Two boundary conditions to be satisfied are:

A) body boundary condition

$$\boldsymbol{n}(\boldsymbol{x}_b)\cdot\boldsymbol{V}(\boldsymbol{v}_b)=0 \quad (5)$$

B) free surface boundary condition

$$\boldsymbol{n}(\boldsymbol{x}_s)\cdot\boldsymbol{V}(\boldsymbol{v}_s)=0 \quad (6A)$$

$$\text{or } \zeta_x(U+u)+\zeta_y v-\mathrm{w}=0 \quad (6B)$$

where $\boldsymbol{n}$ denotes a unit vector normal to the concerned boundary and directing to the flow domain; $\boldsymbol{x}_b$ denotes the position vector of a point on the body.

Finally, the radiation condition, the main aspect of which is the fact that the wave generated by the ship motion only follows behind the vessel, should be satisfied.

## 3. Mothod of Solution

### 3.1. The Use of the Integral Solution Method and the Discretisation of the Problem

Solution of the Laplace equation can be achieved in a number of different ways, one of which is the itegral solution method based on the solution of the poisson equation $\nabla^2\psi(\boldsymbol{x})=-4\pi\sigma(\boldsymbol{x})$ given by

$$\psi(\boldsymbol{x})=\int\frac{\partial(\boldsymbol{x}')}{r(\boldsymbol{x};\boldsymbol{x}')}\,dV(\boldsymbol{x}') \quad (7)$$

where $r=|\boldsymbol{x}-\boldsymbol{x}'|$

A boundary where the Neumann condition is to be satisfied is equivalent to a surface where the sources of suitable strengths are the two boundaries where the Neumann condition is imposed, the velocity potential in the present problem may be expressed by

$$\Phi(\boldsymbol{x})=-Ux+\int_{s_b+s_s}\frac{\partial(\boldsymbol{x}')}{r(\boldsymbol{x};\boldsymbol{x}')}\,dS(x') \quad (8)$$

where $\sigma(x)$ is the source strength at the position $\boldsymbol{x}$, and $S_b$ and $S_s$ denote the surfaces of source ditribution to represent the hull and the free surface respectively. The velocity is then given by

$$\boldsymbol{V}(\boldsymbol{x})=U\boldsymbol{i}+\int_{s_b+s_s}\frac{\boldsymbol{x}-\boldsymbol{x}'}{r^3(\boldsymbol{x};\boldsymbol{x}')}\sigma(\boldsymbol{x}')ds(\boldsymbol{x}') \quad (9)$$

This integral expression may be discretised following Hess and Smith's method[3]. As is well known, the surface of source of source distribution is divided into a number of quadrilaterals on with source strengths are assumed to be uniform and each of which is designated by a point called the control point at which any flow quantity is calculated with significance. There are alternative choices for the control point but the centroid seems to be as good as any and Hess chose it in his later work[4]. A plane quadrilateral with uniform source strength is called a "source panel". When the control point of the $i$-th source panel is taken as the field point in eq. (9), the velocity is expressed in the discretised form as

$$\boldsymbol{V}(\boldsymbol{x}_i)=U\boldsymbol{i}+\sum_j\left[\boldsymbol{i}X(i,j)+\boldsymbol{j}Y(i,j)+\boldsymbol{k}Z(i,j)\right]\sigma_j \quad (10)$$

where $X(i,j)$

$$\begin{cases}=\int_{s_j}\frac{x_i-x'}{r^3(x_i;x')}\,ds(\boldsymbol{x}') & \text{when } j\neq i\\ =2\pi n_{xi} & \text{when } i=i\end{cases} \quad (11)$$

$\boldsymbol{x}_i=(x_i, y_i, z_i)$; the position vector of the control point of the $i$-th source panel

$\boldsymbol{n}(\boldsymbol{x}_i)=(n_{xi}, n_{yi}, n_{zi})$; the unit normal vector of the $i$-th source panel

$Y(i,j)$ and $Z(i,j)$ are defined in the similar way to eq. (11). For a quadrilateral source panel, these integrals can be evaluated analytically and hence these so-called influence coefficients matrices can be formed without difficulties. Substitution of the expression for the velocity into the boundary conditions(eq. (5) and eq. (6A)) yields

$$n_{xi}U+\sum_j A(i,j)\sigma_j=0 \quad (12)$$

$$\text{where } A(i,j)=n_{xi}X(i,j)+n_{yi}\,Y(i,j)+n_{zi}\,Z(i,j) \quad (13)$$

### 3.2. Still water Plane as the Position of Source Distribution to Represent the Free Surface

If the geometry of the free surface were known initially, it would be possible to distribute source panels on the free surface itself and hence the problem could be solved without any difficulty simply by calculating $\sigma$'s from eq.(12). However, as this is not the case, some way of generating the free surface, starting from an assumed one, is required.

An iterative scheme distributing source panels at the achieved wave surface and the finding the wave elevation for the next iteration immediately suggests itself. This scheme, though attractive, does not seem to offer confidence for its convergence to the right wave profile when no promising way of guiding the process can be devised, and is obviously not econom ical since the matrices of influence coefficients concerned with the water surface source panels should be reconstructed from iteration to iteration.

Hence, in this paper, with the assumption that the velocity at the free surface and at the plane $z=0$ may be approximately related by the following expressions

$$\begin{aligned} u(x,y,\zeta)&=u(x,y,0)+\zeta u_z(x,y,0)\\ v(x,y,\zeta)&=v(x,y,0)+\zeta v_z(x,y,0)\\ w(x,y,\zeta)&=w(x,y,0)+\zeta w_z(x,y,0) \end{aligned} \tag{14}$$

the still water plane $z=0$ has been chosen for the surface of source distribution to represent the free surface. This was decided because it is always possible to estimate velocities at the imaginary free surface specified by the assumed values of $\zeta$, using the velocities and the velocity derivatives evaluated at the fixed position $z=0$. These expressions are what Gadd[1] used and may indicate the degree of nonlinearity which is catered for by the present method. The remaining task is then how to generate the free surface satisfying the prescribed conditions by the use of expressions in eq. (14). This is attempted in the following sections.

### 3.3. Free Surface Elevation and the Boundary Conditions

Substituting the expressions for the velocity components at the free surface into the free surface generating equation (eq. (4))

$$\zeta=-\frac{1}{2g}\Big[2U(u_o+u_{zo}\zeta)+(u_o+u_{zo}\zeta)^2+(v_o+v_{zo}\zeta)^2+(w_o+w_{zo}\zeta)^2\Big] \tag{15}$$

where $u_o(x,y)=u(x,y,0)$, $u_{zo}(x,y)=u_{zo}(x,y,0)$, ect (16)

From this equation, we obtain the expression for the wave height

$$\zeta=(-B+D)/A \tag{17}$$

$$\begin{aligned} \text{where } A&=u_{zo}^2+v_{zo}^2+w_{zo}^2\\ B&=g+(U+u_o)u_{zo}+v_ov_{zo}+w_ow_{zo}\\ C&=2Uu_o+u_o^2+v_o^2+w_o^2\\ D&=(B^2-AC)^{\frac{1}{2}} \end{aligned} \tag{18}$$

The free surface boundary condition (eq. (6B)) becomes

$$\zeta_x(U+u_o+u_{zo}\zeta)+\zeta_y(v_o+v_{zo}\zeta)-(w_o+w_{zo}\zeta)=0 \tag{19}$$

where $\zeta_x$ and $\zeta_y$ are to be obtained by straightforward differentiation of eq. (17) with respect to $x$ and $y$ respectively. In the discretised aproximation of the surface of source distribution, every quantity of the expressions in this section should be evaluated at the particular control point where the expression is to be applied. The free surface boundary condition (eq. (19)) should be written in this way while eq. (12) is still valid for the discretised hull boundary condition. Thus we have the following set of $(N_b+N_s)$ equations

$$\zeta_{xk}(U+u_{ok}+u_{zok}\zeta_k)+\zeta_{yk}(v_{ok}+v_{zok}\zeta_k)-(w_{ok}+w_{zok}\zeta_k)=0 \qquad k=1,2,\cdots\cdots,N_s \tag{20}$$

$$\sum_{j=1}^{N_b+N_s}A(i,j)\delta_j+n_{xi}U=0 \qquad i=1,2,\cdots\cdots,N_b \tag{21}$$

for $(N_b+N_s)$ unknown source strengths. The radiation enters as a sort of constraint for the solutions of this set of nonlinear simultaneous equations in terms of the source strengths.

### 3.4. Construction of the Induced Velocity and Velocity Derivatives Matrices

The expressions in the previous section imply that the following sixteen quantities are required at the control point of each water surface source panel:

—velocity

$u \qquad v \qquad w$

—first derivative of velocity

$u_x \qquad v_x \qquad w_x$

**Table 1.** Induced Velocity Matrices

| | by the $j$-th body panel | | | by the $j$-th water surface panel | | |
|---|---|---|---|---|---|---|
| component | $x$- | $y$- | $z$- | $x$- | $y$- | $z$- |
| induced on the $i$-th body panel | $BBU(i,j)$ | $BBV$ | $BBW$ | $BSU$ | $BSV$ | $BSW$ |
| induced on the $i$-th water surface panel | $SBU$ | $SBV$ | $SBW$ | $SSU$ | $SSV$ | $SSW$ |

**Table 2.** Induced Velocity Derivative Matrices

| | | induced on the $i$-th water surface panel | | | | | |
|---|---|---|---|---|---|---|---|
| | | by the $j$-th body panel | | | by the $j$-th water surface panel | | |
| | velocity component | $u$ | $v$ | $w$ | $u$ | $v$ | $w$ |
| differentiated with respect to | $X$ | $BUX(i,j)$ | $BVX$ | $BWX$ | $SUX$ | $SVX$ | $SWX$ |
| | $Y$ | $BUY$ | $BVY$ | $BWY$ | $SUY$ | $SVY$ | $SWY$ |
| | $XX$ | $BUXX$ | $BVXX$ | $BWXX$ | $SUXX$ | $SVXX$ | $SWXX$ |
| | $YY$ | $BUYY$ | $BVYY$ | $BWYY$ | $SUYY$ | $SVYY$ | $SWYY$ |
| | $XY$ | | | $BWXY$ | | | $SWXY$ |

$u_y$ $v_y$ $w_y$

—second derivative of velocity

$u_{xx}$ $v_{xx}$ $w_{xx}$

$u_{yy}$ $v_{yy}$ $w_{yy}$

$w_{xy}$

while the velocity components $u, v$ and $w$ only are needed at each control point on the body surface. It is to be noted that $w_x, w_y$ etc. are employed instead of $u_z, v_z$ etc. and that the subscript $o$ appearing in eq. (16) is dropped in the above list.

Each of these quantities is evaluated by the superposition of contributions from all the source panels. To do this for an assumed set of source strengths, it is necessary to construct the influence coefficient matrices and keep them for repeated use. The element at the $i$-th row and $j$-th column of any of these matrices is the quantity induced at the $i$-th control point by the $j$-th source panel with unit source strength. These matrices with their proper designations are summarized in Table 1 and Table 2.

Hess and Smith's formulae [3] have been used for the construction of the induced velocity matrices shown in Table 1 and their appropriate derivatives for the induced velocity derivatives matrices. These formulae are given in full in Lee[5].

### 3.5. Construction of the Green's Function Based on the Body Boundary Condition—The Use of Image Source Concept

To solve the set of nonlinear simultaneous equations expressed by eq. (20) (together with eq. (21)), it is both desirable and necessary to single out explicitly the effects of source strengths on the water surface because the change of source strength of a body panel will be a dependent variable (due to the boundary condition at the fixed body surface) while that of a water surface panel is an independent one, in the course of determining them. Suppose there is only one source panel, say the $j$-th, with unit source strength on the water surface in an otherwise zero velocity field. A set of source strengths on the body panels should react to satisfy the body boundary condition in the velocity field created by this water surface panel alone. Let them be $\lambda(1,j)$, $\lambda(2,j), \cdots$, $\lambda(N_b,j)$. They can be determined from

$$C(m,j)+\sum_{i=1}^{Nb}\lambda(i,j)A(m,i)=0 \quad (m=1,2,\cdots, N_b) \tag{22}$$

where
$$C(m,j)=n_{xm}BSU(m,j)+n_{ym}BSV(m,j)+n_{zm}BSW(m,j) \tag{23}$$

$$A(m,i)=n_{xm}BBU(m,i)+n_{ym}BBV(m,i)+n_{zm}BBW(m,i) \tag{24}$$

The solutions for $\lambda$'s constitute the image sources on the body panels for the $j$-th water surface panel with unit source strength and this set of source strengths will be called the "$j$-th basic image system". Considering all the water surface panels in turn, it can be easily seen that one image system corresponds to each water surface panel and all these basic image systems may be determined from the following collective expression;

$$\sum_{m=1}^{Nb} A(i,m)\ \lambda(m,j) = -C(i,j) \quad (i=1,2,\cdots\cdots,N_b) \quad (j=1,2,\cdots\cdots,N_s) \tag{25}$$

The velocity potential induced at a field point by the $j$-th water surface panel with unit source strength and its basic image system will then be the sum of velocity potentials coming from the both origins, as the following

$$G(\boldsymbol{x};j) = \phi(x)_{j\ \mathrm{own}} + \phi(\boldsymbol{x})_{j\ \mathrm{image}} = \int_{S_j} \frac{dS(x')}{r(\boldsymbol{x};\boldsymbol{x}')} + \sum_{m=1}^{Nb} \lambda(m,j) \int_{S_m} \frac{dS(\boldsymbol{x}')}{r(\boldsymbol{x};\boldsymbol{x}')} \tag{26}$$

This velocity potential $G(x,j)$ is what is meant in this paper by "Green's function based on the body boundary condition".

The velocity components or any of their derivatives induced at the control point of the i-th water surface panel by the j-th water surface panel with unit source strength and its basic image system can be calculated from the appropriate derivatives of the above expression taking the control point of the i-th panel as the field point. This can in fact be achieved through the use of matrices constructed already. Considering the x-component of velocity as an example, the result is

$$SSU(i,j) + \sum_{m=1}^{N_b} \lambda(m,j) SBU(i,m) \tag{27}$$

which is denoted by $NU(i,j)$. Matrices constructed in this way will be called the N-matrices and those corresponding to each of the sixteen quantities of velocity and velocity derivatives are summarized in Table 3. The use of these matrices will enable the source strengths on the water surface panels only to be the independent variables in the process for determination of their values. This is clear because the source strengths of the body panels can be shown as the linear combination of those of the water surface panels as the following;

**Table 3.** The N-Matrices

| velocity | x-derivative | y-derivative | xx-derivative | yy-derivative | xy-derivative |
|---|---|---|---|---|---|
| NU | NUX | NUY | NUXX | NUYY | |
| NV | NVX | NVY | NVXX | NVYY | |
| NW | NWX | NWY | NWXX | NWYY | NWXY |

$$B\sigma_i = B\sigma_{oi} + \sum_{j=1}^{N_s} \lambda(i,j) S\sigma_j \tag{28}$$

where $S\sigma_j$ and $B\sigma_i$ are the source strengths of the j-th water surface panel and that of the i-th body panel respectively. $B\sigma_{oi}$ is the source strength of the i-th body panel to satisfy the body boundary condition in the uniform onset velocity field when every source strength of the water surface panel is zero and may be calculated from the following equation;

$$\sum_{j=1}^{N_b} A(i,j) B\sigma_{oi} = -Un_{xi} \qquad (i=1,2,\cdots\cdots N_b) \tag{29}$$

It is to be noted that $B\sigma_o$'s are constant so long as the free stream velocity $U$ does not vary.

Consequently any component of velocity or velocity derivatives at the control point of a water surface panel can be expressed as a linear function of the source strengths of water surface panels only. Consider x-component of velocity, for instance, induced at the i-th water surface panel;

$$u_i = \sum_{m=1}^{N_b} SBU(i,m) B\sigma_m + \sum_{j=1}^{N_s} SSU(i,j) S\sigma_j = \sum_{m=1}^{N_b} SBU(i,m) B\sigma_{om} + \sum_{j=1}^{N_s} SSU(i,j) + \sum_{m=1}^{N_b} \lambda(m,j) SBU(i,m)\ S\sigma_j$$

$$= u_{oi} + \sum_{j=1}^{N_s} NU(i,j) S\sigma_j \tag{30}$$

where

$$u_{oi} = \sum_{m=1}^{N_b} SBU(i,m) B\sigma_{om} \tag{31}$$

This expression shows that $u_i$ is a linear function of $S\sigma$'s. It is to be noted that the subscript $o$ of eq.(31) has a different meaning from the same subscript in eq. (16).

The use of matrices in Table 3 in connection with eq. (28) allows us to ignore the body boundary condition in the process to determine the source strengths of water surface panels.

### 3.6. Generation of the Free Surface Configuration

A set of source strengths on the water surface panels is sought at this stage so that the free surface boundary condition eq. (20) should be satisfied. For this numerical adjustment of the values for a large number of independent variables, it is convenient to rewrite eq.(20) as

$$R_i(S\sigma_1, S\sigma_2, \cdots\cdots, S\sigma_{N_s}) = \xi_{xi}(U + u_i + w_{xi}\xi_i) + \xi_{yi}(v_i + w_{yi}\xi_i) - w_i + (u_{xi} + v_{yi})\xi_i \quad (i = 1, 2, \cdots\cdots N_s) \tag{32}$$

making use of irrotationality

$$u_z = w_x$$

$$v_z = w_y$$

and continuity

$$w_z = -(u_x + v_y)$$

of the velocity field and neglecting the subscript $o$ in eq. (20). For an assumed set of source strengths of the water surface panels, every quantity in the right hand side of eq. (32) can be calculated by the use of eqs. (17), (30) and the like. Hence the residuals, $R$'s, of the left hand side of eq. (22) can be evaluated. The object would be then to reduce the magnitude of these residuals through an iterative process.

However, although the two boundary conditions may be satisfied by the way described so far, there has been nothing in the process to say that the radiation condition is satisfied. An additional plan should be provided for this end. No readily usable method is available for the purpose but a crude intuitive scheme that is the prescription of a reasonable nonoscillating wave elevation on the water surface panels ahead of the body may be an acceptable improvisation with the present particular way of attacking the problem.

As the reasonable wave elevation, the pressure heads for the associated double model on a few leading water surface panels are intended to be used in the present paper. These heads may be determined as the following: the flow field of the associated double model can be generated by a set of source strengths on the water surface panels determined from the condition that z-component of velocity at the control point of every water surface panel should be zero, i.e. from the similar expression to eq.(30)

$$w_i - w_{oi} + \sum_{j=1}^{N_s} NW(i,j) S\sigma_j = 0$$

$$i = 1, 2, \cdots\cdots, N_s \tag{33}$$

and the pressure heads are, with the use of source strengths determined from the above equation, then given by

$$\xi_{pk} = (-B_k + D_k)/A_k$$

$$k = 1, 2, \cdots\cdots, L \tag{34}$$

where $A_k$, $B_k$ and $D_k$ are calculated from eq. (18) with the help of eq. (30) and the like, and $L$ is the number of leading water surface panels where the water elevations are to be prescribed. Then the prescription of wave elevation means the definition of residuals

$$R_k(S\sigma_1, S\sigma_2, \cdots\cdots, S\sigma_{N_s}) = \xi_{pk} - \xi_k$$

$$k = 1, 2, \cdots\cdots, L \tag{35}$$

in addition to $N_s$ residuals in eq. (32).

Because of the radiation condition, the number of equations to be solved increases from $N_s$ to $(N_s + L)$. Since there are only $N_s$ unknowns in this set of $(N_s + L)$ equations, the solutions should be sought in the sense of least sum of squares of the residuals.

There are a number of standard methods to solve a system of nonlinear simultaneous equations by a computing machine. Many of these methods require the means to evaluate Jacobian matrix. This is possible in the present formulation since the partial derivative of a residual with respect to an independent variable can be obtained by the straightforward differentiation of eq. (32) and eq. (35).

### 3.7. Initial Solutions

To solve the set of simultaneous equations eq.(32) and eq. (35) by iterative procedure, a set of starting values for the independent variables in needed. The importance of these initial solutions lies in the fact that not only do they influence overall computational efficiency but even the success of a mothod can depend on their adequateness.

The immediate response to this need is the use of linearised formulation to get the initial solution. However experience has shown that this approach was not as good as expected. Instead, the simplified nonlinear approach proved to work quite satisfactorily, this being the full nonlinear process but with

$$u(x,y,\xi)=u(x,y,0)$$
$$v(x,y,\xi)=v(x,y,0)$$
$$w(x,y,\xi)=w(x,y,0) \qquad (36)$$

instead of eq. (14). Consequently the expression for the wave elevation, typically, takes the form

$$\xi=-\frac{1}{2g}(2Uu+u^2+v^2+w^2) \qquad (37)$$

in contrast with eq. (15) and those corresponding to eqs. (18), (32) and (35) become simpler.

As the initial soutions for this simplified formulation, the set of source strengths on the water surface panels achieved by solving eq. (33) was sufficiently good.

## 4. Computation and the Results

The flows over a container ship hull and the DTMB Model Series 57 No. 4202 have been analysed by the present method. A plane quadrilateral was employed as the basic shape for the source panels. A typical distribution of these panels on the flat water surface is shown in Fig. 2.

The steps to get the final solution were computations, first of the set of source strengths on the water surface panels satisfying the condition of flat water surface, then of the set satisfying the simplified nonlinear equations and finally of the set satisfying the full nonlinear equations, in that order. The second and third steps took the major part of computing time. The nonlinear simultaneous equations concerning these two steps were solved by Marquardt's method [6], the basic feature of which is that the correction vector $\delta$ is estimated from the current vector $S\sigma$ by solving

$$(J^TJ+KD)\underline{\delta}=-J^T\underline{R} \qquad (38)$$

where $\underline{R}$ is the current residual vector; $J$ is the Jacobian matrix evaluated at the current point; $K$ is a positive parameter; and $D$ is a positive diagonal matrix (set to the unit matrix in the present use). When $K$ is zero the method is the same as Newton's one where the convergence is rapid but less reliable, and when $K$ is large, an incremental step along the direction of the steepest descent, where the progress is assured though slow, for the sum of squares of the residuals is calculated for the correction vector $\delta$. Thus the parameter provides a bias from the efficient course towards the more reliable direction whenever the process appears to diverge.

When the final solution for the source strengths on the water surface panels is achieved, the source str-

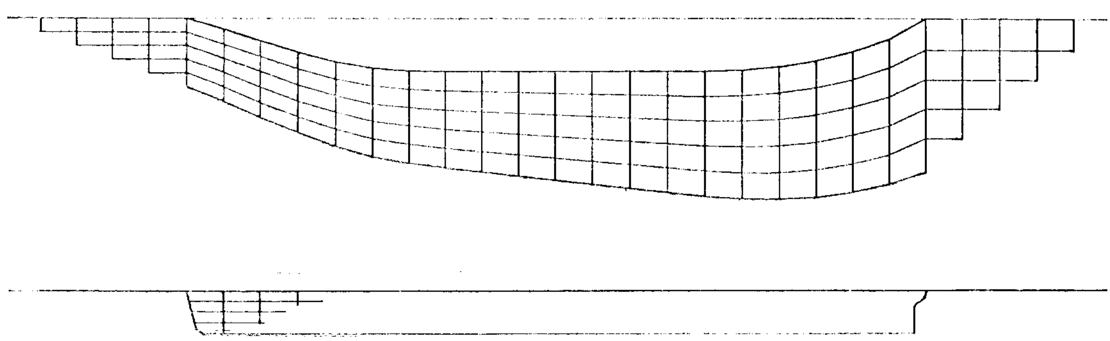
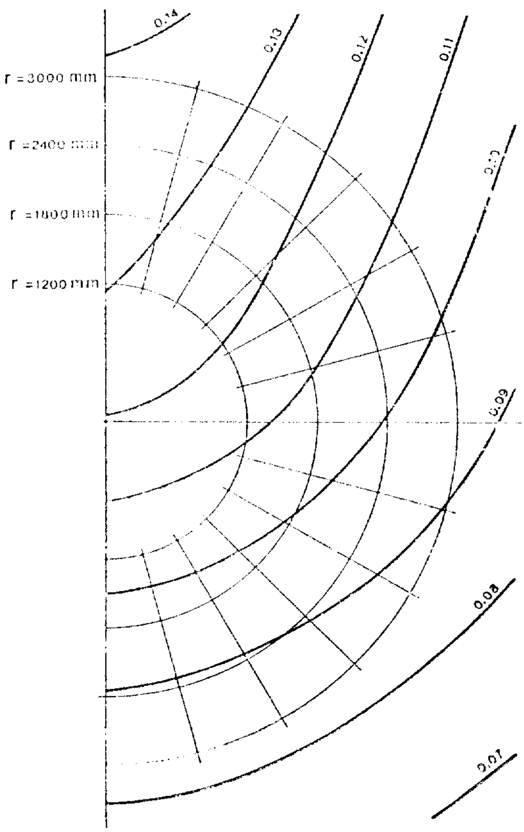


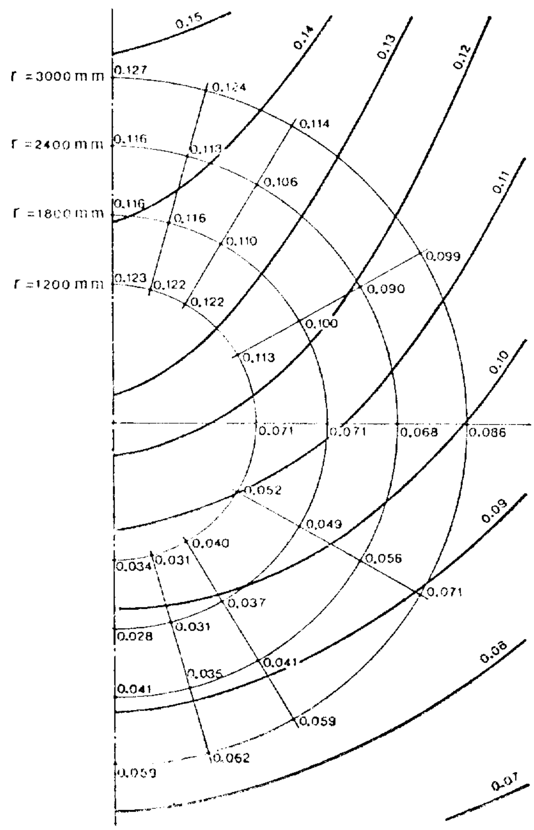
**Fig.** 2. Distribution of Source Panels engths on the body panels can be calculated by eq. [28]. The pressure coefficients at the control points of body panels can also be calculated, which may be useful for the estimation of fore and aft component of force exerting on the body. The wave elevation at each control point on the water surface is calculated from eq. [17]. However, if the control points are located some distance from the body surface as in Fig. 2, the wave profile along the hull is not directly available but should be obtained by extrapolation. Some information on the analyses of the two models is shown in Table 4. The results are presented in the forms of Talor wake fractions by Fig. 3 and Fg.i 4, and of wave profile along the hull projected to the center plane by Fig. 5, Fig. 6 and Fig. 7.

In Fig. 3, an experimental measurement of the wake fraction which was obtained by towing astern (to measure potential wake as nearly as possible) is plotted together with the predicted curves. It is not easy to draw lines of constant wake fration from the experimental measurement because of the limited number of points measured but there some wriggling which does not appear in the result predicted by the present method. A general tendency is that the predicted values are considerably higher compared to those measured. Marked differences seem to occur at the region directly below the propeller axis. Distrib-

**Table 4.** Information on the Analyses of the Two Models

| Model | $C_b$ | No. of panels | $F_n$ | CPU (IBM 370) |
|---|---|---|---|---|
| Container Ship | 0.591 | $N_s=124$ | 0.518 | 437 sec. |
| | | $N_b=147$ | 0.225 | 831 |
| DTMB Model | 0.7 | $N_s=120$ | 0.208 | 1026 |
| | | $N_b=129$ | 0.223 | 1348 |




• ; Point Measured ——; Calculated by the Present Method

**Fig.** 3. Distribution of Potential Wake, $F_n=0.158$

**Fig.** 4. Distribution of Potential Wake, $F_n=0.225$

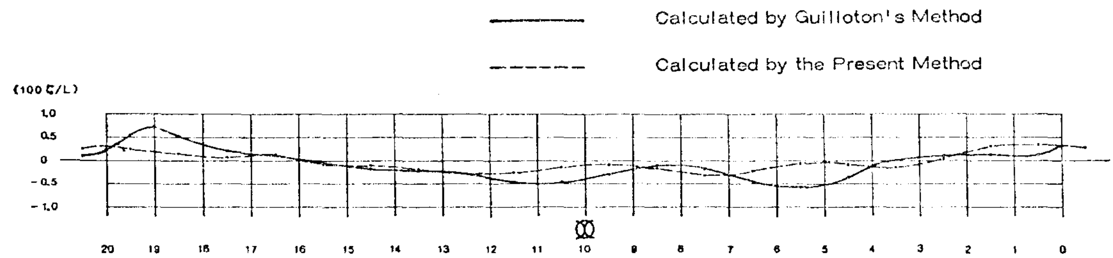



**Fig. 5.** Wave Profile of the DTMB Model, $C_b=0.7$, $F_n=0.208$

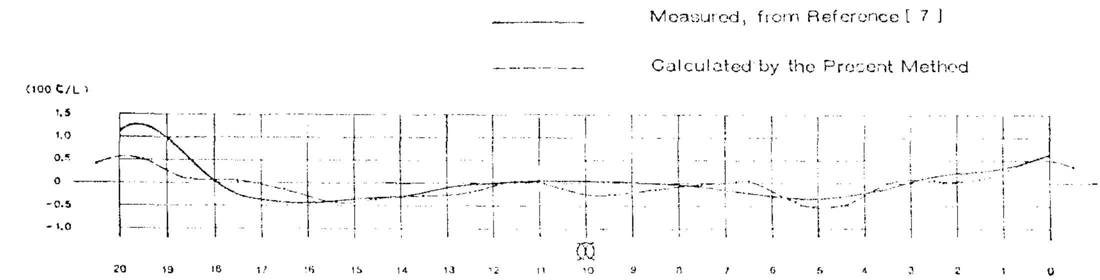



**Fig. 6.** Wave Profile of the DTMB Model, $C_b=0.7$, $F_n=0.223$

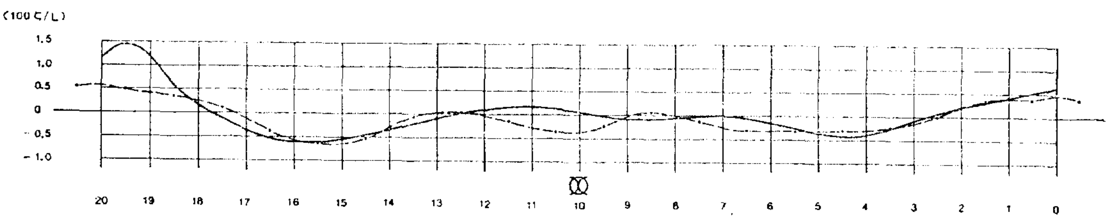



**Fig. 7.** Wave Profile of the Container Ship, $C_b=0.591$, $F_n=0.225$

ution of wake fraction values of the same vessel for a higher Froude number does not differ much from that for the Froude number condidered although a minor difference can be detected at the region above the propeller axis. This difference may be accounted for by the disparity of source strengths on the water surface panels above the region in the two cases.

Fig. 5 and Fig. 6 show the wave profiles for the DTMB Model used by Korvin-Kroukovsky and Jacobs 7. The measured profiles taken from the reference and the predicted ones by the present method are drawn together in the diagrams. As can be seen, the present method has produced rather poor results at the bow region in both cases but, with this exception, the agreement seems to be reasonable. Another recognizable discrepancy occurs at the region around amidship. The present method has predicted a marked hollow just behind and in front amidship which is absent or not so conspicuous in the actual measurements.

Fig. 7 shows the result of the container ship. Instead of measured wave profile calculated by Guilloton's method is plotted in this diagram for the comparison. It is believed that the prediction by Guilloton's method should be quite accurate for a hull form with such a block coefficient as the present one. Again the wave elevation at the bow region is rather poorly calculated and, an addition, there seems to be some forward phase shift of humps and hollows in the profile by the present method compared with that by the other method.

In spite of these deficiencies, the prospective strength of the present method may be attributed to its conformity with the idea of the integral solution method in the form involving no predetermined assumption. The flow around a ship hull as a nonliner boundary value problem, which is one of the main interests in current ship hydrodynamics, is more like ly to be determined by the use of the method with this principle than by some variation of the linearised formulation. From this point of view the present method of attacking the problem seems to be worth pursuing further.

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**과학으로 싸우자 기술로 건설하자**

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