Digital Simulation of Narrow—Band Ocean Systems

(협대역 해양시스템의 Digital Simulation)

金 泳 均 *
(Kim, Young Kyun)

要 約

實際로 制限된 대역이나 협대역의 random 信號를 interpolate할 경우 sampling 理論에 근거하여 有限한 項들을 取하는 truncated expansion은 매우 有用하다. 本 論文은 海洋 시스템의 動的 分析에 있어 효과적이고도 통계학적으로 正確한 algorithm 을 얻는데 목적을 두고 있다. Truncated sampling expansion의 통계학적 正確度가 조사되어지고 간단한 海洋 시스템의 例를 들어, 실제 wave data를 가지고, 正確度를 많이 向上시키면서도 계산면에서 거의 복잡성을 주지 않는 새로운 algorithm을 보여 준다.

Abstract

Truncated expansions based upon the sampling theorem but containing only a few terms can be very useful for practical interpolations of band-limited or narrow-band random signals. The major goal of this work is to find and compare efficient and "statistically accu‸ate" algorithms for the dynamic analysis of the ocean systems.

The statistical accuracy of truncated sampling interpolations is investicated, and one simple ocean systems, which yields a Runge-Kutta simulation algorithm of improved accuracy with very little increase in computation, is indicated.

I. Introduction

In the design and evaluation of many kinds of systems and controllers, it is necessary or desirable to numerically evaluate the system performance with random inputs. One oceanic application considered involves simplified nonlinear models; namely a deep-sea platform.

Such simulations are important, for example, in *determining whether or not maximum limits on* stresses or displacements will be exceeded.

Random inputs permit the use of statistical error criteria (i.e., mean squared error). The sampling theorem for band-limited signals, which is so invaluable theoretically to signal analysis and information theory, can also be put to very practical use in the interpolation of sampled signals, if expansions of only a few terms are employed. Besides applications involving straight signal reconstruction, truncated sampling expansions can be used to improve the accuracy of digital-simulation algorithms used to determine dynamic-system response. The statistical error criterion used is the mean-squared error between the "pseudo-true" state and simulated state vector at the inter-sampling points, with the "pseudo-true" state determined at the sampling points much denser than for the simulation, which is at the Nyquist rate.

* 正会員, 大田기계창
(Taejon Machine Depot)
接受日字 : 1980年 10月 27日

For the simple nonlinear ocean system model, actual wave-force data are used. The results of the comparison study show that a modified Runge-Kutta algorithm using a few terms of the sampling expansion gives improved simulation accuracy over the standard Runge-Kutta algorithm with little increase in computation.

II. Trancated Sampling Expansions for Bandlimited Random Signals

Consider a Stationary random process of zero mean and variance σ^2, with a uniform band-limited spectrum. Let us approximate the sample function f(t) of our random process over each interval (n-1)T to nT by the truncated expansion:

$$f_a(t) = \sum_{i=j+n}^{k+n} f_i \,\mathrm{sinc}(2wt - i),\ j < 0 \qquad (1)$$

where $f_i \triangleq f_a(iT)$, and $\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$.

(w being the single-sided bandwidth in Hz.)

We may define the mean-square error function:

$$E(t) = < [f(t) - f_a(t)]^2 > \qquad (2)$$

(<·> denotes statistical expectation)

E(t) is zero at each sampling point and is a periodic time function since the random process is stationary and the points used for the interpolation are related to each interval in the same way. Hence, as an error measure, we may define the time average of E(t):

$$\overline{E(t)} = \frac{1}{T}\int_0^T E(t)\,dt \qquad (3)$$

To aid in the calculation of $\overline{E(t)}$, we introduce the following notation:

$$\phi_n(t) \triangleq \mathrm{sinc}(2wt - n) = \mathrm{sinc}(\frac{t}{T} - n) \qquad (4)$$

$$\xi_n \triangleq \frac{1}{T}\int_0^T \phi_n^2(t)\,dt \qquad (5)$$

In order to evaluate $\overline{E(t)}$ for various size expansions, we find that we need to calculate ξ_n. This can be done in terms of the since integral functions by a simple change of variable, and we obtain:

$$\xi_n = \frac{1}{\pi}[\mathrm{Si}(n2\pi) - \mathrm{Si}((n-1)2\pi)] \qquad (6)$$

$$\mathrm{Si}(x) = \int_0^x \frac{\sin y}{y}\,dy \qquad (7)$$

A short tabulation of values of ξ_n is given in Table I.

Table I.

n	ξ_n
0	.45141
1	.45141
2	.02355
3	.00824
4	.00417
5	.00251
6	.00168
7	.00120
8	.00090
9	.00070
10	.00056
11	.00046
12	.00038

With our assumption of independent samples, $\overline{E(t)}$ may be easily obtained in terms of the values of ξ_n as follows:

$$\overline{E(t)} = \frac{1}{T}\int_0^T < [f(t) - \sum_{n=j}^{k} f_n\phi_n(t)]^2 > dt \qquad (8)$$

Using the Sampling expansion for f(t), we obtain:

$$\overline{E(t)} = \frac{1}{T}\int_0^T <[\sum_{n=-\infty}^{\infty} f_n\phi_n(t) - \sum_{n=j}^{k} f_n\phi_n(t)]^2 > dt$$

(refer [2])

$$= \frac{\sigma^2}{T}\,[\int_0^T \sum_{m=-\infty}^{\infty} \phi_m(t)^2\,dt - \int_0^T \sum_{m=j}^{k} \phi_m(t)^2\,dt] \qquad (9)$$

$$\text{Since } \frac{1}{T}\int_0^T \sum_{m=-\infty}^{\infty} \phi_m(t)^2\,dt = \frac{1}{T}\int_{-\infty}^{\infty} \phi_0(t)^2\,dt = 1 \qquad (10)$$

$$\text{We have } \overline{E(t)} = \sigma^2\,[1 - \sum_{m=j}^{k} \xi_m]. \qquad (11)$$

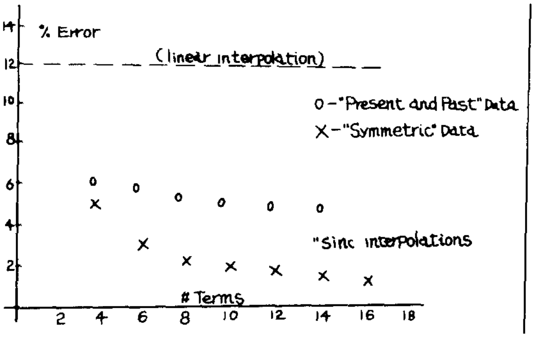

Fig. 1. $\overline{E(t)}$ as percent of signal variance.

"Symmetric" data means the same number of samples on either side of the interval being interplated.

The decrease in $\overline{E(t)}$ as the number of terms in the interpolating sum increases is indicated in Fig. 1 for "symmetric" data and for "present and past" data. The errors are expressed as a percentage of the variance σ^2. Also shown is the percent error for a linear interpolation[2], in which adjacent sample points are simply connected by a straight line.

This plot indicate that the use of more than 10-12 terms results in a negligible increase in accuracy, and this may be used as a guideline for practical interpolation applications.

III. System Modeling

There are many nonlinear system examples commonly encountered and therefore of general interest. We select one simple second-order nonlinear systems which is associated with ocean wave forces.[3]

An ocean structure model

A offshore platform structure fixed to the ocean floor is assumed to vibrate in its first mode only when subjected to random wave forces. The displacement x of the platform is assumed to be the same as the displacement for the equivalent spring-mass system shown in Fig. 2.

The mass is a composite of the platform mass, the leg mass, and a portion of the water mass which is moved by the legs during free vibrations. K is the spring constant determined from the structural stiffness.

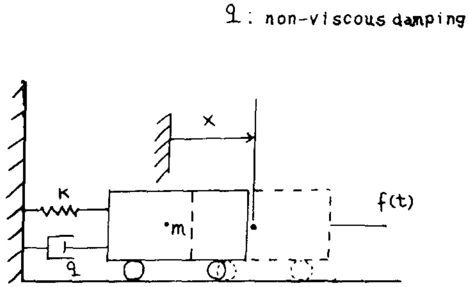

Fig. 2. Simple spring-mass system.[3]

The damping q is nonlinear.

System model:

$$m\ddot{x}(t) + \dot{x}(t)|\dot{x}(t)| + Kx(t) = f(t) \qquad (12)$$

$f(t)$: a bandlimited random force

$\dot{x}(t)|\dot{x}(t)|$: non-viscous damping force

$Kx(t)$: Spring force (assume linear)

IV. Algorithms

By examining the performance of typical methods through tests on representative systems, it is shown that for the general simulation of linear and nonlinear systems, the variable-step-size Runge-Kutta-Merson method proves to be the most accurate and the most efficient (6).

Using three basic numerical algorithms:

1. Euler method
2. Heun method
3. Fourth-order Runge-Kutta method

a comparison study has also been done for a celebrated second-order nonlinear differential equation known as the Van der Pol equation (1).

$$\frac{d^2 x(t)}{dt^2} - P[1 - x^2(t)]\frac{dx(t)}{dt} + x(t) = 0 \qquad (13)$$

The Euler method is the first-order Runge-Kutta method, and the Heun method is the second-order Runge-Kutta method.

The comparison study shows that the fourth-order Runge-Kutta method yields much greater accuracy than the other methods[1], and hence it is a reasonable choice for the simulation of nonlinear systems.

An ocean system model, eq. (12) can be vector-formed as

$$\dot{\underline{x}}(t) = g(\underline{x}(t), t) + f(t)\ , \underline{x}(o) = \underline{0} \qquad (14)$$

The fourth-order Runge-Kutta formula with Runge's coefficients uses the following equations to step the solution from kT to (k+2)T.

$$K_1 = 2T[g(\underline{x}(kT), kT) + f(kT)]$$

$$K_2 = 2T[g(\underline{x}(kT) + k_1/2, (k+1)T) + f((k+1)T)]$$

$$K_3 = 2T[g(\underline{x}(kT) + K_3, (k+2)T) + f((k+1)T)]$$

$$K_4 = 2T[g(\underline{x}(kT) + K_3, (k+2)T) + f((k+2)T)]$$

$$\underline{x}((k+2)T) = \underline{x}(kT) + (1/6)(K_1+2K_2+2K_3+K_4) \qquad (15)$$

The truncated sampling expansion can be used to give improved accuracy by the simple expedient of estimating intermediate values of the forcing function f(t), thereby allowing use of a decreased sampling internal. For example, the usual fourth-order Runge-Kutta method requires knowledge of f(t) at mid-interval points in order to obtain derivative estimates at the midpoint as well as at the end points of each simulation interval. The simplest and most obvious way of making use of the band-limited property of f(t) is to use a truncated sampling expansion to estimate the midpoint values $f((n+\frac{1}{2})T)$, thus allowing the sampling interval for the Runge-Kutta algorithm to be halved. Of course, additional interpolated values of f(t) can be estimated and the sampling interval further reduced. Whether or not this is warranted depends upon the trade-off between accuracy and efficiency for the particular case in question.

V. Simulation and Results

For an ocean platform model, the following equation is chosen:

$$\dot{x}(t) + 0.05\ \dot{x}(t)|\dot{x}(t)| + x(t) = f(t) \qquad (16)$$

The equation parameters are relaistic for a class of actual platforms. For the forcing function f(t), actual sampled wave force data was used.

The wave force data were taken every 0.2 seconds, and these forces were exerted by waves from Hurri-

As shown in the rather narrow-band spectrum, an effective (double-sided) bandwidth was about 1.25Hz, thus giving a Nyquist sampling interval of 0.8s.

Using the above nonlinear second-order equation, we investigated the mean-squared error between the "pseudo-true" state and simulated state vector using two different methods – i.e., the fourth-order Runge-Kutta method and the modified Runge-Kutta method, using a Nyquist sampling interval. In this case, the word, "pseudo-true" state was meant by using all the

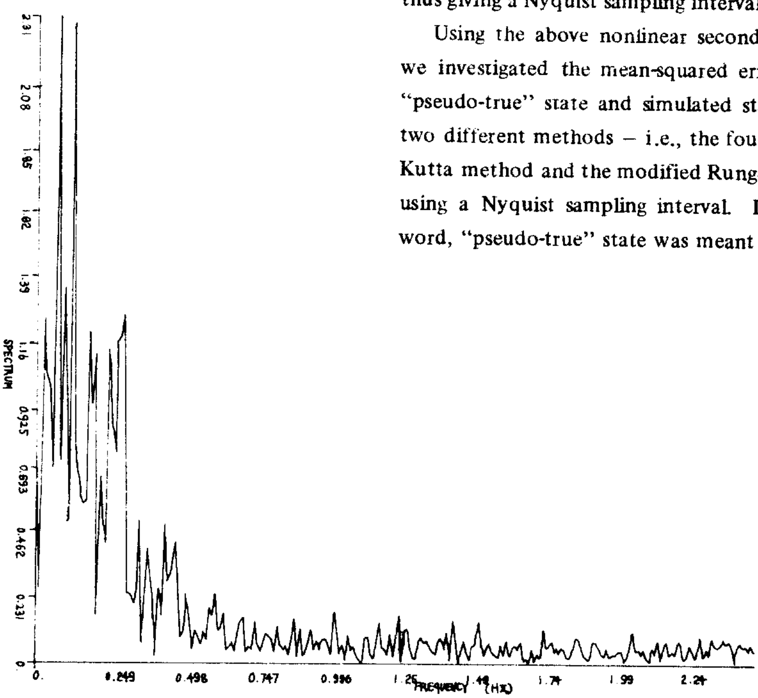

Fig. 3. Wave power spectrum.

available wave force data, which were actually four times denser than a Nyquist sampling. The midpoint values of f(t) in the Nyquist intervals were estimated using ten terms of the truncated sampling expansion, and these midpoint values were used in the modified Runge-Kutta method, with the sampling interval T=0.8s. In the fourth-order Runge-Kutta method, the sampling interval of 1.6s was used.

In the computer experiments, an estimate of the mean-squared error between the so called, "pseudo-true" state and the simulated state vector was obtained, and the percent error reduction achieved with the modified Runge-Kutta method was about 57 percent for each of the state variables $x(t)$ and $\dot{x}(t)$ -- a significant improvement.

In the simulation process with the IBM-360 computer, we found very little increase in computation time with new algorithm. And in the actual dynamic system analysis, more emphasis will be put on the accuracy improvement over the conventional methods rather than the computational loads. In these respects, the new, developed algorithm might be the most efficient and "statistically accurate" one for the dynamic system which has band-limited or narrow-band random signals.

References

1. Athens, Dertozos, Spann and Mason, *Systems, Networks, and Computation*, McGraw-Hill, 1977.
2. R.B. Kerr, "Polynomial Interpolation Errors for Band-limited Random Signals", IEEE Trans. on Syst., Man and Cybern., Nov. 1976.
3. B.J. Muga and J.F. Wilson, *Dynamic Analysis of Ocean Structures*, New York: Plenum, 1970.
4. Young Kim, "Digital Simulation of Narrow-band Ocean System", IEEE Ocean '80 International Conference, 1980.
5. Losch F., *Tables of Higher Functions*, Sixth Edition, McGraw Hill, 1960.
6. Martens H.R., "A Comparative Study of Digital Integration Methods", Simulation, Vol. 12, Feb. 1969.