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離散型 적응관측자 및 제어기의 새로운구성

A New Scheme for Discrete Implicit Adaptive Observer and Controller

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Abstract

Many different schemes of the adaptive observer and controller have been developed for both continuous and discrete systems.

In this paper we have presented a new scheme of the reduced order adaptive observer for the single input discrete linear time invariant plant. The output equation of the plant is transformed into the bilinear form in terms of system parameters and the states of the state variable filters. Using the plant output equation the discrete implicit adaptive observer based on the similar philosophy to Nuyan and Carroll is derived and the parameter adaptation algorithm is derived based on the exponentially weighted least square method.

The adaptive model following control system is also constructed according to the proposed observer scheme.

The proposed observer and controller are rather than simple structure and have a fast adaptive algorithm, so it may be expected that the scheme is suitable to the practical application of control system design.

The effectiveness of the algorithm and structure is illustrated by the computer simulation of a third order system. The simulation results show that the convergence speed is proportinal to the increasing of weighting factor alpha, and that the full order and reduced order observer have similar convergence characteristics.

1. Introduction

The adaptive abserver is a model reference adaptive scheme generating the inaccessible state variables of the unknown plant with only input and output measurement. And also the adaptive observer is essential to the adaptive model following indirect control.

Since the discretization of the continuous algorithm is not suitable to the digital computer implementation, it is desirable to develop a simple and fast convergence adaptive observer for an adaptive control system design.

A significant contributions to the design of the full order adaptive observer using Lyapunov's direct method have been made by Carroll and Lindorff[1]. Lüder, Kudva and Narendra have been reported on an adaptive observer based on the nonminimal realization of the unknown system[2],[3]. Recently Kreisselmeier has proposed the parameterized adaptive observer[4]. Nuyan and Carroll have proposed the implicit observer[5]. Narendra and Valavani have shown that the auxiliary signal is required for the minimal order realization of an adaptive observer[6]. On the other hand it is also proved that the implicit

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observer does not necessary any auxiliary signals in case of seperating the dynamics of the parameter estimation and state variable filter's dynamics[5]. In addition to these, Kudva and Narendra have proposed the discrete adaptive observer using Lyapunov's direct method[7]. Andoh and Suzuki[8] have reported the discrete adaptive observer which is based on Popov's hyperstability theorem. Suzuki, Nakamura and Koga[9] have proposed a discrete adaptive observer which is the discretized form of the Kreisselmeier's scheme[4].

In case of adaptive control systems their control scheme may be classified into two types such that direct and indirect control[10] Recently several schemes have been reported by some scholars [11]~[13]. In this paper we propose a new scheme for the discrete reduced order adaptive observer based on the similar philosophy to Nuyan and Carroll[5]. Since the proposed scheme has fast convergence characteristics, it will be able to use for the design of an indirect adaptive control system.

The problem description is given in section 2. In section 3 we describe the plant as a bilinearform in terms of artificial corresponding parameters and the state variables of the state variable filters, and then design the discrete reduced order adaptive observer. The adaptation algorithm for estimation of artificial corresponding parameters which are a one-to one mapping into the actual unknown plant parameters is also derived in section 4, and then the adaptive model following controller is formulated in section 5. Finally in section 6 we give some results of the computer simulation of a third order system to illustrate the effectiveness of the proposed scheme.

2. Problem Description

Consider the single input single output linear time invariant discrete system described by

$$x_p(k+1)=A_p x_p(k)+b_p u(k) \qquad (1)$$
$$y_p(k)=c^T x_p(k), \quad x_p(0)\triangleq x_p^0$$

where $x_p(k)$ is the nth-order state vector of the plant, $y_p(k)$ is a mearsureable scalar output of the plant, $u(k)$ is a scalar input of the plant and A_p, b_p and c are $n\times n$, $n\times 1$ and $n\times 1$ matrices respectively. Then we may assume, without loss of generality, that plant (1) has the following observable canonical form

$$A_p=\left[\begin{array}{c|c} -a_p & \begin{array}{c} I_{n-1} \\ \hline 0 \end{array}\end{array}\right], \quad a_p=\begin{bmatrix} a_{p1} \\ a_{p2} \\ \vdots \\ a_{pn}\end{bmatrix}$$

$$b_p=\begin{bmatrix} b_{p1} \\ b_{p2} \\ \vdots \\ b_{pn}\end{bmatrix}, \qquad c=\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0\end{bmatrix}$$

where the vectors a_p, b_p and x_p^0 are unknown and $I_{n-1}$is an $(n-1)st$ order unit matrix. The main idea of designing discrete adaptive implicit observer is to express the unknown output equation of nth-order plant as in the form of $y=f(p, r)$. The argument p is a set of artificial corresponding, parameters which is a one-to-one mapping into the actual unknown plant parameters, and the argument r is a set of state variables of the state variable filters which correspond to the unknown state variables of the plant. If such a function can be chosen, the estimated output $\hat{y}(k)$ of the plant is given by

$$\hat{y}(k)=f(\hat{p}(k), \ r(k)).$$

Since the estimated output error of the plant is caused by the difference between p and $\hat{p}(k)$, it is possible to find the estimated value of p. And it is also possible that the estimated value $\hat{x}_p(k)$ of the state vector of the plant can be obtained from the linear combination of the states $r_1(k)$, $r_2(k)$ of the state variable filters.

The observation process is well seperated from the adaptation process and suitable adaptation schemes can be developed in a general fashion[5].

With these adaptive algorithm and scheme of the adaptive observer, we formulate an indirect adaptive model following control system. Now we assume that such a system is described by

$$x_M(k+1)=A_M x_M(k)+b_M u_M(k) \qquad (2)$$
$$y_M(k)=c^T x_M(k)$$

where $y_M(k)$, $x_M(k)$ and $u_M(k)$ are the model output, state vector and input respectively, and the coefficient matrices of the model A_M, b_M and c are observable canonical from such that

$$A_M=\left[\begin{array}{c|c} -a_m & \begin{array}{c} I_{n-1} \\ \hline 0 \end{array}\end{array}\right], \quad a_m=[a_{m1} a_{m2} \cdots a_{mn}]^T$$

$$b_M=\begin{pmatrix} b_{m1} \\ \vdots \\ b_{mn} \end{pmatrix} \qquad c=[1\ 0\cdots 0]^T$$

In order to get the adaptive model following control we transform the model output $y_M(k)$ into the bilnear form of p and r, and then the control input $u(k)$ shall be applied to the plant for adjusting the plant parameters so that the plant's output may follow the model's.

3. The Reduced Order Discrete Adaptive Observer

Since the plant (1) is a discrete time-invariant observable system, its states can be estimated asymptotically by means of Luenburger's reduced order observer[14] such that

$$\begin{aligned} v(k+1)&=Fv(k)+gy(k)+hu(k) \\ \hat{x}_{p1}(k)&=y_p(k) \\ \hat{\bar{x}}_p(k)&=v(k)+1_0 y_p(k) \end{aligned} \tag{3}$$

where $v(k)=[v_1 v_2\cdots\cdots v_{n-1}]^T$, state vector of the reduced order discrete adaptive observer,

$\hat{\bar{x}}_p(k)=[\hat{x}_{p2}\hat{x}_{p3}\cdots\cdots\hat{x}_{pn}]^T$, the estimated value of unmeasureable state vector of the plant,

g, h and 1_0 are coefficient vectors with compatible dimension,

F is a asymptotically stable matrix whose eigenvalues are located in the unit circle and different from those of A_p,

$$F=\begin{pmatrix} -f & \begin{matrix} I_{n-2} \\ \cdots \\ 0 \end{matrix} \end{pmatrix}, \qquad f=\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_{n-1} \end{pmatrix}$$

$$g=\begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n-1} \end{pmatrix}, \qquad h=\begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_{n-1} \end{pmatrix}$$

The basic principle of the adaptive observer is to adjust g and h adaptively in real time to cause $\lim_{k\to\infty}\hat{x}_p(k)=x_p(k)$.

In formulating an adaptive observer, the unmearsurable state variables of the system are replaced by the states of the state variable filters. One of the basic question is to minimize the order of the state variable filters.

Taking the Z-transform of (1), we have

$$(z^n+a_p{}^T d(z))Y_p(z)=b_p{}^T d(z)U(z)+zd(z)x_p{}^0,$$

where $d(z)=[z^{n-1}z^{n-2}\cdots z1]$ (4)

In order to investigate the relationship between eq. (4) and the state variables of the state variables filters, we introduce $Q(z)$ be an nth-order polynomial such that

$$Q(z)=z^n+q^T d(z),$$

where $q=[q_1 q_2\cdots q_n]^T$ and $q_i(i=1,2,\cdots,n)$ will be chosen so that the roots of $Q(z)$ lie within the unit circle of z-plane.

(4) is now written as

$$(Q(z)-(q-a_p)^T d(z))Y_p(z) =b_p{}^T d(z)U(z)+zd(z)x_p{}^0 \tag{5}$$

Dividing by $Q(z)$,

$$Y_p(z)=(q-a_p)^T\frac{d(z)}{Q(z)}Y_p(z) +b_p{}^T\frac{d(z)}{Q(z)}U(z)+\frac{z.d(z)}{Q(z)}x_p{}^0 \tag{6}$$

Whenever the output of an nth-order system is to be estimated by (6), it requires two nth-order state variable filters for the input and output. The essentia property of the filters is that the linearly independent signals are needed. This property can be maintained even if the order of $Q(z)$ is reduced by one as Luenburger's reduced order observer does. Therefore a measurable output and input signal can be treated as one of the states of each state variable filter.

If we assume that $Q(z)$ has a real root α_1, then

$$Q(z)=(z-\alpha_1)M(z), \tag{7}$$

where $M(z)$ is a stable $(n-1)$st order polynomial such that

$$M(z)=z^{n-1}-m^T\bar{d}(z) \tag{8}$$

where

$$m=[m_1 m_2\cdots m_{n-1}]^T,$$
$$\bar{d}(z)=[z^{n-2}z^{n-3}\cdots z\ 1]^T$$

Plug (7) and (8) into (6), then

$$(z-\alpha_1)Y_p(z)=(q-a_p)^T\frac{d(z)}{M(z)}Y_p(z) +b_p{}^T\frac{d(z)}{M(z)}U(z)+z\frac{d^T(z)}{M(z)}x_p{}^0 \tag{9}$$

Let $\bar{a}_p=[a_2 a_3\cdots a_n]^T$,

$\bar{b}_p=[b_2 b_3\cdots b_n]^T$,

$\bar{q}=[q_2 q_3\cdots q_n]^T$,

$\Theta(z)\triangleq Z[\theta(k)]=\dfrac{zd(z)}{M(z)}$,

and $\dfrac{z^{n-1}}{M(z)}=1-m^T\dfrac{\bar{d}(z)}{M(z)}$

Using these notations (9) is written as

$$(z-\alpha_1)Y_p(z)=(q_1-a_{p1})Y_p(z)$$
$$+(\bar{q}-\bar{a}_p-(q_1-a_{p1})m)\frac{\bar{d}(z)}{M(z)}Y_p(z)+b_{p1}U(z)$$
$$+(\bar{b}_p-mb_{p1})\frac{\bar{d}(z)}{M(z)}U(z)+\Theta(z)\bar{x}_p^0 \qquad (10)$$

Since the elements of $\bar{d}(z)$ are linearly independent, $\frac{\bar{d}(z)}{M(z)}Y_p(z)$ and $\frac{\bar{d}(z)}{M(z)}U(z)$ can be generated from $(n-1)$ st-order stable and controllable state variable filters with characteristic polynomial $M(z)$. Let $(M_1, 1_2)$ and $(M_2, 1_2)$ are the controllable system matrix pairs with the characteristic polynomical $M(z)$. Then $(n-1)$ st-order controllable state variable filters are given by

$$r_1(k+1)=M_1r_1(k)+1_1y(k)$$
$$r_2(k+1)=M_2r_2(k)+1_2u(k) \qquad (11)$$

where

M_1, M_2 : $(n-1)\times(n-1)$ matrix

$1_1, 1_2$: $(n-1)$ dim. vector

There exist $(n-1)$st order nonsingular transformation matrices T_1, T_2 if the filters are controllable. The z-transformation of filters are

$$R_1(z)=(zI-M_1)^{-1}\,1_1Y_p(z)=T_1^{-1}\frac{\bar{d}(z)}{M(z)}Y_p(z) \qquad (12)$$
$$R_2(z)=(zI-M_2)^{-1}1_2U(z)=T_2^{-1}\frac{\bar{d}(z)}{M(z)}U(z)$$

where

$R_i(z)\triangleq z(r(k)),\ i=1,2$

(12) is written as

$$(z-\alpha_1)Y_p(z)=(q_1-a_{p1})Y_p(z)+p_1^TT_1R_1(z)$$
$$+b_{p1}U(z)+p_2^TT_2R_2(z)+\Theta^T(z)\bar{x}_p^0 \qquad (13)$$

where

$$p_1=(\bar{q}-\bar{a}_p-(q_1-a_{p1})m)$$
$$p_2=\bar{b}_p-mb_{p1}$$

Taking the inverse z-transform, (13) can be written as

$$y_p(k+1)=\alpha_1y_p(k)+p^Tr(k)+\theta^T(k)\bar{x}_p^0 \qquad (14)$$

where

$$p^T=[q_1-a_{p1} : p_1^TT_1 : b_{p1} : p_2^TT_2]$$
$$r^T=[y_p : r_1^T : u : r_2^T]$$

If we choose 1_0 such that $1_0=f$ in (3), the reduced order observer can be constructed by using

$$g=\bar{q}-\bar{a}_p-(q_1-\hat{a}_{p1})f=\hat{p}_1,$$
$$h=\bar{b}_p-\hat{b}_{p1}f=\hat{p}_2, \qquad (15)$$

And det $[zI-F]=M(z)$.

Now we assume that the observer $v(k)$ can be constructed from the algebraic transformation of the states r of the state variable filter and also we assume that their operators will be given by $H_1(\hat{p}_1)$ and $H_2(\hat{p}_2)$, then the state variable of the observer is written as

$$V(z)=H_1(\hat{p}_1)R_1(z)+H_2(\hat{p}_2)R_2(z) \qquad (16)$$

From (3) the observer states can be written by

$$V(z)=(zI-F)^{-1}gY_p(z)+(zI-F)^{-1}hU(z)$$
$$+z(zI-F)^{-1}v_0 \qquad (17)$$

Eq. (16) is described by

$$V(z)=H_1(zI-M_1)^{-1}1_1Y(z)$$
$$+H_2(zI-M_2)^{-1}1_1U(z) \qquad (18)$$

In (18) we make $v_0=0$ for compuational simplicity. From (17) and (18)

$$(zI-F)^{-1}g=(zI-F)^{-1}(T_1^T)^{-1}\hat{\bar{p}}_1$$
$$=H_1(zI-M_1)^{-1}1_1$$
$$(zI-F)^{-1}h=(zI-F)^{-1}(T_2^T)^{-1}\hat{\bar{p}}_2$$
$$=H_2(zI-M_2)^{-1}1_2 \qquad (19)$$

where

$$\hat{\bar{p}}_1=T_1^T\hat{p}_1,\quad \hat{\bar{p}}_2=T_2^T\hat{p}_2$$

Using matrix properties, H_1 and H_2 are written as

$$H_1=[(T_1^T)^{-1}\hat{\bar{p}}_1 : \cdots : F^{n-2}(T_1^T)^{-1}\hat{\bar{p}}_1]$$
$$[1_1 : M_11_1 : \cdots : M_1^{n-2}1_1]^{-1},$$
$$H_2=[(T_2^T)^{-1}\hat{\bar{p}}_2 : \cdots : F^{n-2}(T_2^T)^{-1}\hat{\bar{p}}_2]$$
$$[1_2 : M_21_2 : \cdots : M_2^{n-2}1_2]^{-1} \qquad (20)$$

In addition to these results, the discrete reduced order adaptive observer can be formulated by the correct identification of the parameter p. In next section we propose such an algorithm.

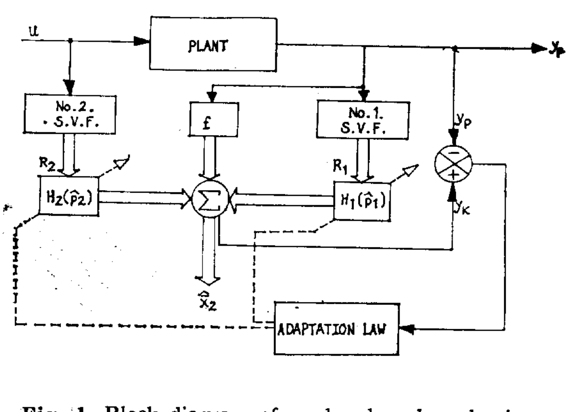

Fig. 1. Block diagram of reduced order adaptive observer

4. The Adaptation Algorithm

There are various methods in parameter identification. In this paper, we use the exponentially weighted least square method which has good convergence characteristics.

If we denote $\hat{p}(k)$ as the estimate of the parameter p at kth iteration, then from(14) the estimated plant output is written as

$$\hat{y}_{pk}(j+1)=\alpha_1 y_p(j)+\hat{p}^T(k)r(j)+cF^{j-1}v_0, \quad (21)$$

where

$j=0,1,\cdots,k,$

$v(0)\triangleq v_0.$

The output estimated error is

$$e_k(j)=\hat{y}_{pk}(j+1)-y_p(j+1) \quad (22)$$

We introduce the following criterion function to get the algorithm for error minimization.

$$J(k)=\sum_{j=0}^{k}\beta\lambda^{k-j}e_k^2(j),\ 0<\lambda<1,\ \beta=1-\lambda. \quad (23)$$

Let $v_0=0$ for computational simplicity. By taking the gradient of $J(k)$ with respect to $\hat{p}(k)$ and making zero, $J(k)$ will become minimum at each k. Therefore the following equation can be derived

$$\hat{p}(k)=[R^T(k)W(k)R(k)]^{-1}R^T(k)W(k)\Omega(k)$$
$$=\Gamma(k)R^T(k)W(k)\Omega(k) \quad (24)$$

where

$$R(k)=[r(0)\ \vdots\ r(1)\ \vdots\ \cdots\cdots\ \vdots\ r(k)]^T$$
$$\Omega(k)=[y_p(1)\ y_p(2)\cdots\cdots y_p(k+1)]^T$$
$$W(k)=\begin{pmatrix}\beta\lambda^k & & & & \\ & \beta\lambda^{k-1} & & \bigcirc & \\ & & \ddots & & \\ & \bigcirc & & \beta\lambda & \\ & & & & \beta\end{pmatrix}$$

and $\Gamma(k)=[R^T(k)W(k)R(k)]^{-1}$　(24—1)

From these equations, we can derive the following recursive equations

$$\hat{p}(k+1)=\hat{p}(k)-L(k+1)[\hat{y}_{pk}(k+1)-y_p(k+1)]$$
$$\Gamma(k+1)=\frac{1}{\lambda}[I-L(k+1)r^T(k+1)]\ \Gamma(k) \quad (25)$$

where

$$L(k+1)\triangleq\frac{\Gamma(k)}{\lambda}r(k+1)\left[\frac{1}{\beta}+r^T(k+1)\frac{\Gamma(k)}{\lambda}r(k+1)\right]^{-1}$$

Lemma: $\Gamma(k)$ defined by $\Gamma(k)=[R^T(k)W(k)R(k)]^{-1}$ is positive definite if the input sequence $u(k)$ is rich and $W(k)$ is positive definite.

Proof: We claim that $\Gamma(k)$ is positive definite if only the $R^T(k)W(k)R(k)$ is positive definite. $R(k)$ consists of the states of the state variable filters suth that

$$R(k)=[r(0)\ \vdots\ r(1)\ \vdots\ \cdots\ \vdots\ r(k)]^T$$

And the components of the $r(k)$ are given by

$$r(k)=[y_p(k)\ \ r_1^T(k)\ \ u(k)\ \ r_2^T(k)]^T \quad (L1)$$

and

$$y_p(k)=c^T[zI-A_p]^{-1}b_p\ u(k)$$
$$r_1(k)=[zI-M_1]^{-1}l_1 y(k) \quad (L2)$$
$$r_2(k)=[zI-M_2]^{-1}l_2 u(k)$$

where z is the time shift operator such that $zu(k)=u(k+1)$. Using some matrix notations (L2) may be transformed to the following eqs.

$$y_p(k)=A_p^{-1}(z)C^T[A_0z^{n-1}+A_1zn^{-2}+\cdots+A_{n-1}]b_pu(k)$$
$$r_1(k)=M^{-1}(z)[M_{10}z^{n-2}+M_{11}z^{n-3}+\cdots+M_{1n-2}]l_1y(k) \quad (L3)$$
$$r_2(k)=M^{-1}(z)[M_{20}z^{n-2}+M_{21}z^{n-3}+\cdots+M_{2n-2}]l_2u(k)$$

where

$$A_p(z)=z^n+c_p^Td(z)$$
$$A_0=I$$
$$A_i=A_pA_i+c_{pi}I\ (i=1,2,\cdots,n-1)$$
$$M(z)=z^{n-1}+m^T\bar{d}(z)$$
$$M_{j0}=I$$
$$M_{ji}=M_jM_{ji-1}+m_iI(j=1,2,\ i=1,2,\cdots,n-2)$$

From (L3) we will get the following equation;

$$r_1(k)=M^{-1}(z)A_p^{-1}(z)\ \left[\sum_{i=0}^{n-2}\sum_{j=0}^{n-1}M_{1n-2-i}l_1c^T A_{n-1}b_p z^{i+j}\right]U(k) \quad (L4)$$

If we define that the coefficient vector of the element z^i $(i=0,1,2,\cdots,2n-3)$ is given by Si, then we have the following equation;

$$R^T(k)=\begin{pmatrix}0 & c^TA_0b_p & c^TA_1b_p\cdots c^TA_{n-1}b_p & 0 & \cdots\cdots & 0\\ 0 & 0 & S_{2n-3}\cdots\cdots\cdots Sn\cdots\cdots\cdots\cdots & & S_1 & S_0\\ 1 & 0 & 0\ \cdots\cdots\cdots 0\cdots\cdots\cdots\cdots\cdots\cdots & & & 0\\ 0 & M_{20}l_2 & M_{21}l_2\cdots M_{2n-2}l_2 & 0 & \cdots\cdots & 0\end{pmatrix}$$
$$\times\begin{pmatrix}u(0) & u(1)\cdots u(2n) & u(2n+1)\cdots u(k)\\ 0 & u(0)\cdots u(2n-1) & \cdots\cdots\cdots\cdots u(k-1)\\ \cdots\cdots & \cdots\cdots & \cdots\cdots\\ 0 & 0\cdots u(0) & u(1)\ \cdots\cdots u(k-2n)\end{pmatrix} \quad (L5)$$

$$\triangleq S\times U$$

S is $2n\times 2n$ matrix and U is $2n\times(k+1)$ matrix. If the rank of the $R(k)$ is $2n$ and the weighting matrix $W(k)$ is positive definite, then $R^T(k)W(k)R(k)$ is definite.

Since the state variable filter and the plant are controllable system, the rank of matrix S is $2n$. Therefore the rank of matrix U must be $2n$ in order to satisfy the fact that the rank of $R(k)$ is $2n$.

If the input sequence $u(k)$ satisfy above condition, we call it that the input $u(k)$ is rich. QED.

Theorem: The value ef $\Gamma(k)$ calculated by (24) is equal to $[R^T(k)W(k)R(k)]^{-1}$ and then $\hat{p}(k)$ approaches to p asymptotically for any initial conditions.

Proof:

If we let an initial value of $\Gamma(k)$ be given by Γ_0, $\Gamma(k)$ can be described by the following equation

$$\Gamma(k)=\left[\beta\lambda^k\Gamma_0^{-1}+\sum_{j=0}^{k}\beta\lambda^{k-j}r(j)r^T(j)\right]^{-1}$$

Now we assume that $0<\lambda<1$ and each element of Γ_0 are relatively large value, the effectiveness of Γ_0 can be negligible as k approaches to infinite. Therefore the value $\Gamma(k)$ will be expressed by $[R^T(k)W(k)R(k)]^{-1}$. If $\Gamma(k)$ expressed by $[R^T(k)W(k)R(k)]^{-1}$ and positive definite, $\hat{p}(k)$ approach to p asymptotically according to the (24) for any initial conditions. Q.E.D.

5. Adaptive Model Following Control

In this section we describe about an indirect control scheme for model following discrete adaptive control [10] using the proposed discrete adaptive observer described in section 3.

At first we express the output error in terms of bilinear form of p and r, and then choose the control input $u(k)$ so that $e_1(k)$ may approach to zero as $\hat{p}(k)\to p$. Finally we expect that the plant output will follow the model's one.

In order to apply the scheme of the prescribed reduced order observer to a model following adaptive control system for simplicity, we choose a new matrix Q whose characteristic polynomial is given by $Q(z)$, where

$$Q=\left(-q\begin{array}{c}I_{n-1}\\ \cdots\cdots\\ 0\end{array}\right),\quad q=[q_1q_2\cdots q_n]^T$$

$$Q(z)=\det[zI-Q].$$

From (1) and (2)

$$x_p(k+1)=Qx_p(k)+(q-a_p)y_p(k)+b_pu(k)$$
$$x_M(k+1)=Qx_M(k)+(q-a_M)y_M(k)+b_Mu_M(k) \tag{26}$$

Output error is given by

$$e_1(k)=y_M(k)-y_p(k) \tag{27}$$

Taking Z-transform the output error equation is written by

$$\begin{aligned}e_1(z)=&(q-a_M)^T\frac{d(z)}{Q(z)}Y_M(z)\\&+b_M{}^T\frac{d(z)}{Q(z)}U_M(z)-(q-a_p)^T\frac{d(z)}{Q(z)}Y_p(z)\\&-b_p{}^T\frac{d(z)}{Q(z)}U(z)+\frac{z.d(z)}{Q(z)}e_0\end{aligned} \tag{28}$$

Let $Q(z)$ have one real root α_1 as previous section, then

$$Q(z)=(z-\alpha_1)M(z)$$

As previous section, (28) can be rearranged

$$\begin{aligned}(z-\alpha_1)e_1(z)=&(z-\alpha_1)Y_M(z)-(q_1-a_{p1})Y_p(z)\\&-(\bar{q}-\bar{a}_p-(f_1-a_{p1})m)^T\frac{\bar{d}(z)}{M(z)}Y_p(z)\\&-b_{p1}U(z)-(\bar{b}_p-mb_{p1})^T\frac{\bar{d}(z)}{M(z)}U(z)\end{aligned} \tag{29}$$

From (29) we choose the control input $u(z)$ for the model following discrete control as follows:

$$\begin{aligned}U(z)=&\frac{1}{\hat{b}_{p1}}((z-\alpha_1)Y_M(z)-(q_1-a_{p1})Y_p(z))\\&-(\bar{q}-\hat{\bar{a}}_p-(q_1-\hat{a}_{p1})m)^T\frac{\bar{d}(z)}{M(z)}Y_p(z)\\&-(\hat{\bar{b}}_p-m\hat{b}_{p1})\frac{\bar{d}(z)}{M(z)}U(z))\end{aligned} \tag{30}$$

Taking the inverse Z-transform and combining with (16)

$$\begin{aligned}u(k)=&\frac{1}{\hat{b}_{p1}}(y_M(k+1)-\alpha_1y_M(k)\\&-(q-\hat{a}_{p1})y_p(k)-\hat{p}_1{}^TT_1r_1(k)\\&-\hat{p}_2{}^TT_2r_2(k))\end{aligned} \tag{31}$$

In order to investigate the stability of the prescribed adaptive model following control system we plug (30) into (29), then

$$(z-\alpha_1)e_1(z)=z[(\hat{p}(k)-p)^Tr(k)]$$

The estimate value $\hat{p}(k)$ approaches to p as k increases because of the stable adaptation algorithm. Therefore output error $e_1(z)$ approaches to zero for α_1 lies in unit circle of z-plane. Fig. 2 shows block diagram of the proposed control system. One of the feature of the controller has relatively simple structure by using $\hat{p}(k)$ directly.

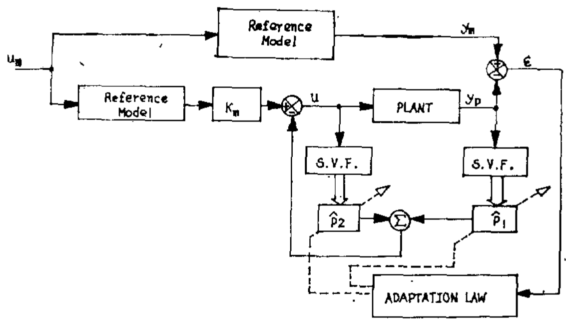

Fig. 2. Block diagram of adaptive model following control system

6. Simulation and Discussions

A third order plant with the following state equations was chosen for simulation. Fig 3 shows the flow chart of the simulation. The plant is

$$x_p(k+1)=\begin{pmatrix}-1 & 1 & 0\\ -0.31 & 0 & 1\\ -0.03 & 0 & 0\end{pmatrix}x_p(k)+\begin{pmatrix}1\\1\\1\end{pmatrix}u(k)$$

$$y_p(k)=[1\ 0\ 0]x_p(k)$$

And the model is

$$x_M(k+1)=\begin{pmatrix}-0.08 & 1 & 0\\ 0.104 & 0 & 1\\ -0.0096 & 0 & 0\end{pmatrix}x_M(k)+\begin{pmatrix}0.8\\1.0\\1.2\end{pmatrix}u_M(k)$$

$$y_M(k)=[1\ 0\ 0]x_M(k)$$

The reference input is stair case wave form shown in Fig. 4. In the adaptation algorithm the initial value of $\Gamma(k)$ is diagonal form such that diag. [100 100 100 100 100 100] and λ is given by 0.95.

In order to evaluate the effect of state variable filter's characteristic polynomial, we take the following cases for digital simulation by IBM 360.

First; the poles of the filter are located at the origin of z-plane, the deadbeat type observer.

Second; the poles of the filter and that of the model are located at the same place.

Third; the poles of the filter are located near the inside of unit circle. The state variable filters are given by (11) such that

$$r_1(k+1)=M_1r_1(k)+l_1y(k)$$
$$r_2(k+1)=M_2r_2(k)+l_2u(k) \qquad (11)$$

Case 1. $\alpha_1=0.2,\ \alpha_2=\alpha_3=0$

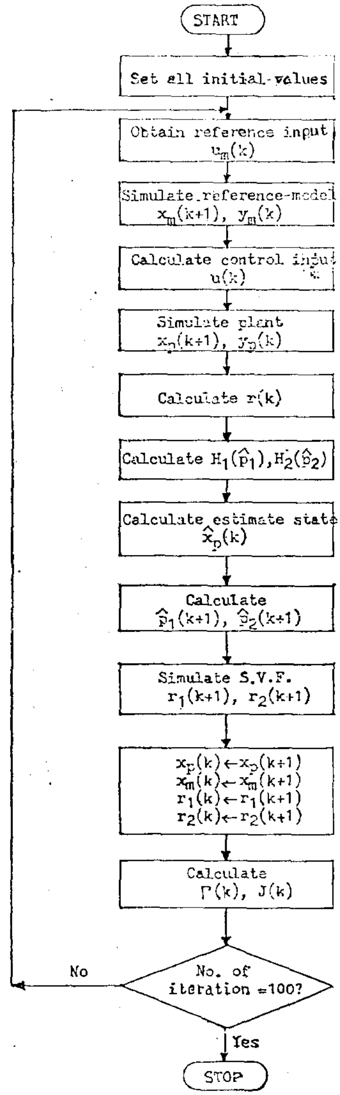

Fig. 3. Flow chart of the simulation algorithm

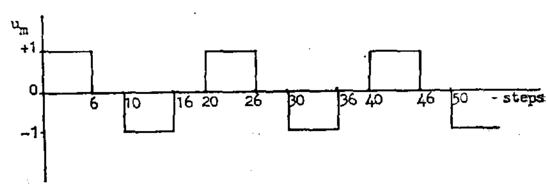

Fig. 4. Input waveform

$$M_1=M_2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\ l_1=\begin{bmatrix}0\\1\end{bmatrix},\ l_2=\begin{bmatrix}1\\1\end{bmatrix}$$

Case 2. $\alpha_1=0.2,\ \alpha_2=0.12,\ \alpha_3=-0.4$

$$M_1=M_2=\begin{bmatrix}-0.28 & 1\\ 0.048 & 0\end{bmatrix},\ l_1=\begin{bmatrix}0\\1\end{bmatrix},\ l_2=\begin{bmatrix}1\\1\end{bmatrix}$$

Case 3. $\alpha_1=0.9,\ \alpha_2=-0.8,\ \alpha_3=+0.7$

$$M_1 = M_2 = \begin{bmatrix} 0.1 & 1 \\ 0.72 & 0 \end{bmatrix}, \quad l_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad l_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

In order to illustrate the convergence characteristics of the adaptation algorithm we simulate the behavior of the performance criterion $J(k)$ according to three cases in Fig. 5. The fact that the values of $J(k)$ are very low in initial 6 steps means that the number of output are less than that of the coefficient to be adapted.

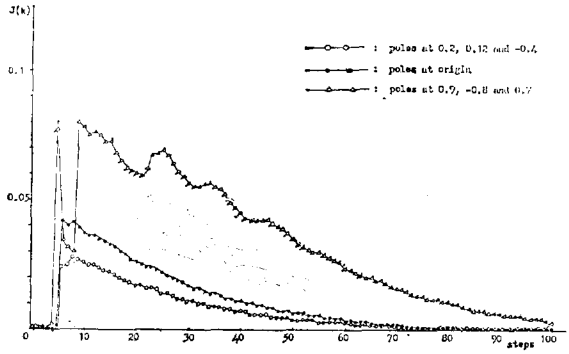

Fig. 5. Behavior of performance criterion due to the poles of state variable filter in adaptation algorithm

In adaptive observer simulation, we investigate the state error distance between the plant state and estimated state to show the convergence characteristics. The state error distance is difined by

$$\sigma = \left[\sum_{j=1}^{n} (\hat{x}_{pj}(k) - x_{pj}(k))^2 \right]^{1/2}$$

The state error distance of the reduced order adaptive observer are shown in Fig. 6. In adaptive model following control system simulation, the output of the reference model and that of plant are shown in Fig. 7. In addition to these simulation we observe the effect of the weighting coefficient and compare these results with the adaptive parameterized full order observer[8]. Fig. 8 and 9 show the state error distance variation according to λ of the reduced and

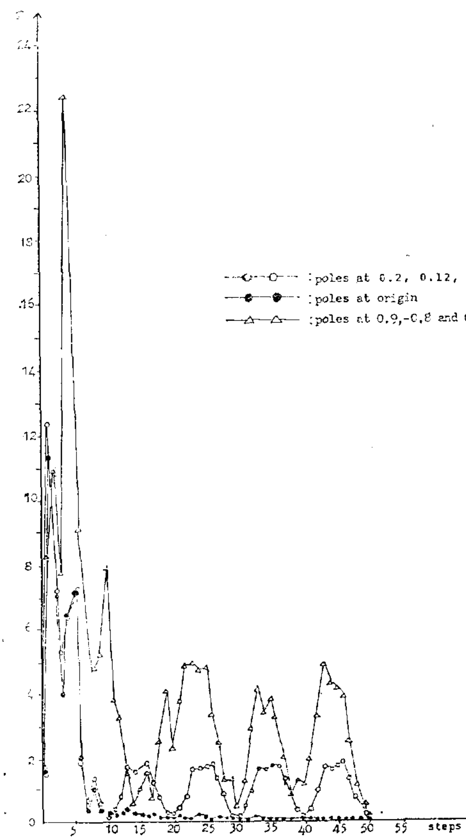

Fig. 6. State Error Distance Variation due to the poles location of SVF in Adaptive Reduced Order Observer with $\lambda=0.95$

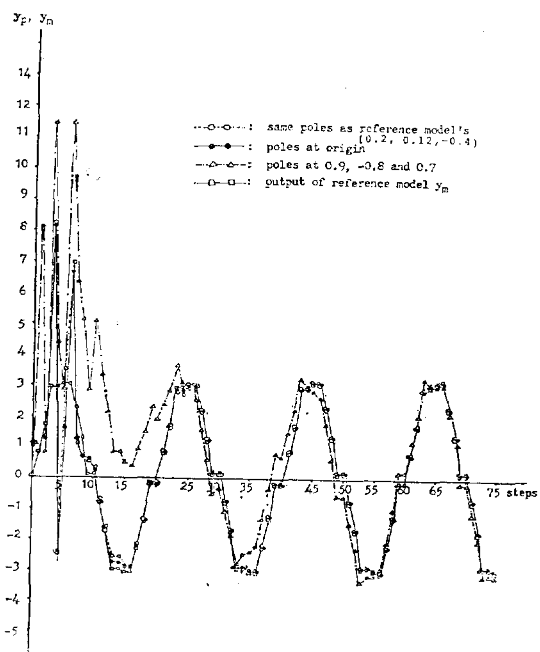

Fig. 7. Plant and reference model output y_p and y_m due to the pole location of state variable filter in adaptive model following control system with $\lambda=0.95$

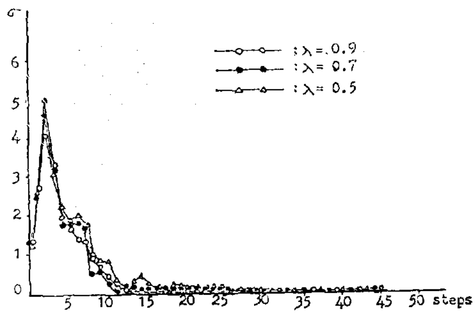

Fig. 8. State error distance variation due to λ in adaptive reduced order observer

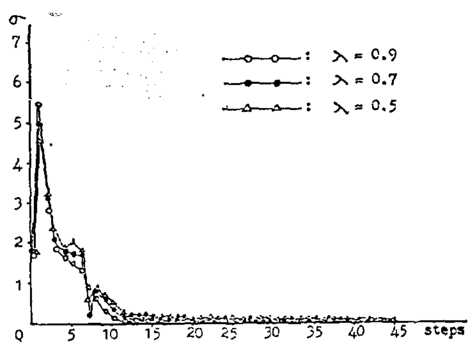

Fig. 9. State error distance variation due to λ in adaptive full order observer

full order observer of case 1 respectively.

Above simulation results are compared with state errer distances of the adaptive full order observer, which are shown in Fig. 10. From these simulation, we may state that the poles of $M(z)$ is important in convergence characteristics. In adaptive observer, the case 1 which has the poles of the state variable filter at the origin shows the best convergence characteristics. The state error distance of the adaptive observer converges to 5% of the initial distance in 40 steps approximately.

In adaptive model following control system, the case 2, which has the poles of the state variable filter at the pole locations of the reference model. And in case 2, the error between reference model and the plant reaches within 5% of initial error in 40 steps. And we can state that the convergence speed and the peak error are increased in accordance to the weighting factor λ. And also the convergence characteristics of the reduced order adaptive observer is similar to that of the full order adaptive abserver.

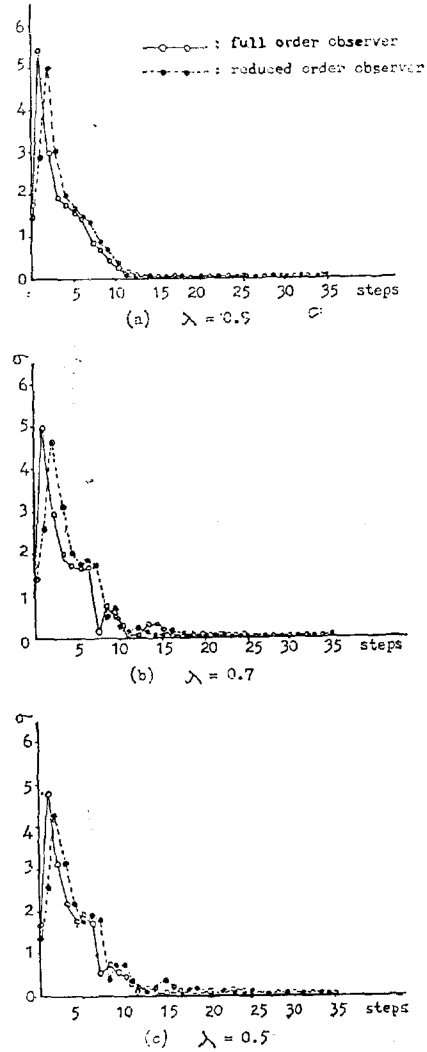

Fig. 10. Comparison of the state error distance variation between the reduced order observer and the full order observer when λ is varied

7. Conclusion

In this paper we have presented the discrete version of an adaptive reduced order observer without auxiliary signals. The implicit observer accomplishes the property through seperating the dynamics of the parameter adaptation and that of the state variable filters.

The adaptation law is derived based on the exponentially weighted least square method which has good convergence characteristics in deterministic system. And we derive the adaptive model following control system which is a similar structure to the reduced order adaptive observer.

The effectiveness of these proposed algorithm has been illustrated by computer simulation carried out for a third order system.

From these results we can state that the reduced order discrete adaptive implicit observer has simple structure and has fast convergence characteristics. Therefore it is desirable to apply this algorithm to one of the practical design problems. And this scheme can be expended into multi-input case.

And also the adaptive model following control algortithm can be applied to the practical control system design due to its simple structure and fast convergence characteristics. But there are some problems to apply these algorithms to the stochastic system whose input and output accompany with random noise. The optimal choice of $M(z)$ and the biasing problem in adaptation algorithm are the future problems. It would be possible to apply the other adaptation algorithms.

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