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A Note on Delayed Feedback Controller for Rendezvous of Linear Time-delay Systems

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Abstract

A rendezvous method via delayed state feedback is presented for a class of linear systems with time-delays. The property of pointwise degeneracy of delay differential system is utilzed and it is shown that the controller is a minimum-time suboptimal controller.

1. Introduction

In a controlled system, the time delay is caused by various reasons. A time-delay is introduced, for example, when the values of the state variables are not immediately available to the controller. In the case of a feedback control system, this means that the controller should be a function of delayed feedback states[1]. Consider now the problem of rendezvous involving three or more systems whose controllers are of this type. Since a feedback solution is in general very difficult to obtain for time-delay systems, the rendezvous problem of this nature also seems hard to manage[2]. There exists, however, a particular class of controlled systems for which the delayed feedback solutions are available. Such solutions are possible because of the property of pointwise degeneracy of some particular delay differential systems.[3],[4] In this note, a rendezvous method is presented based on the degeneracy property for time-delay systems in which the controllers utilize the delayed state feedback.

2. Rendezvous Controller

Consider a class of time-invariant linear systems (L_i), $i=1,\cdots,l$, each being modeled by

$$(L_i)\quad \dot{x}_i(t)=A_ix_i(t)+B_ix_i(t-h)+C_ix_i(t-2h)+D_iu_i(t),\quad t>t_0$$

$$x_i(t)=\phi_i(t),\quad t\ \varepsilon\ [t_0-2h,\ t_0],$$

where $x_i\ \varepsilon\ R^n$, $u_i\ \varepsilon\ R^m$ and $\phi_i(t)$ on $[t_0-2h,\ t_0]$ are the state variable, the control variable and a given continuous initial function, respectively. A_i, B_i, C_i and D_i are constant matrices with compatible dimensions. The problem is to find controllers $u_i(t)$, $i=1, 2$, as functions of delayed feedback states $\{x_i(s)\,|\,s\leq t-h\}$, which steers the system (L_i) from the initial function $\phi_i(t)$ to a rendezvous state

$$x_1(t)=x_2(t)=\cdots=x_l(t),\quad t\geq t_1,$$

for some $t_1>0$. The following result provides an answer (i.e. a synthesis technique) for the problem.

Theorem 1.

For each $i=1,\cdots,l$, let the matrix D_i be a nonsingular. Then, the systems (L_i) $i=1,\cdots,l$, will be steered from the initial state $\phi_i(t)$ on $[t_0-2h,\ t_0]$, $i=1,\cdots,l$ to a rendezvous state

$$x_1(t)=x_2(t)=\cdots=x_l(t),\quad t\geq t_0+2h$$

by means of a feedback control law

$$\begin{aligned} u_i(t)&=u_i(t,x_t) \qquad (1)\\ &=-D_i^{-1}\Big[\frac{1}{h}e^{A_ih}+B_i\Big]x_i(t-h)\\ &\quad+D_i^{-1}\Big[\frac{1}{h}e^{2A_ih}-C_i\Big]x_i(t-2h)\\ &\quad+D_i^{-1}\Big[e^{-A_ih}y_i(t)-2y_i(t-h)\\ &\quad+e^{A_ih}y_i(t-2h)\Big],\quad t\geq 0,\ i=1,\cdots,l, \end{aligned}$$

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where $y_i(t)$ is the solution of a delay differential equation

$$(L'_i)\ \dot{y}_i(t)=A_i y_i(t)+\frac{1}{h}e^{A_i h}y_i(t-h)-\frac{1}{h}e^{2A_i h}y_i(t-2h)-\frac{1}{h^2}e^{2A_i h}x_i(t-2h),\ t>t_0$$

$$y_i(t)=0,\ [t_0-2h,\ t_0].$$

Proof

Let x be a $2nl\times n$ vector

$$x=\begin{pmatrix} x_1\\ x_2\\ \vdots\\ x_l\\ y_1\\ y_2\\ \vdots\\ y_l\end{pmatrix}$$

and let A, B_1 and B_2 be $2nl\times 2n$ matrices

$$A=\begin{pmatrix} A_1 & 0 & \cdots & 0 & e^{-A_1h} & 0 & \cdots & 0\\ 0 & A_2 & \cdots & 0 & 0 & e^{-A_2h} & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & A_l & 0 & 0 & \cdots & e^{-A_lh}\\ 0 & 0 & \cdots & 0 & A_1 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0 & 0 & A_2 & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & A_l \end{pmatrix}$$

Finally, let Q be an $n(n-1)\times 2nl$ matrix given by

$$Q=\begin{pmatrix} I & -I & 0 & \cdots & 0 & 0 & , & 0 & 0 & \cdots & 0\\ 0 & I & -I & \cdots & 0 & 0 & , & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \cdots & I & -I & , & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Here I denotes the $n\times n$ identity matrix.

Then, by adjoining the equation (L'_i) to (L_i) in which $u_i(t)$ is replaced by Eq. (1), one can obtain an augmented system of order $2nl$

$$\dot{x}(t)=Ax(t)+B_1x(t-h)+B_2x(t-2h)$$

$$x(t)=\begin{pmatrix}\phi_1(t)\\ \vdots\\ \phi_l(t)\\ 0\\ \vdots\\ 0\end{pmatrix},\quad t_0-2h\leq t\leq t_0$$

and

$$Qx(t)=[x_1(t)-x_2(t),\ x_2(t)-x_3(t),\cdots,\ x_{l-1}(t)-x_l(t)]^T$$

Since $Qx(t)=0$ if and only if $x_1(t)=x_2(t)=\cdots,\ x_l(t)$, it is sufficient to show that

$$Qx(t)=0\quad t\geqq t_0+2h$$

As in [1] (also see [3]), define

$$B_1=\begin{pmatrix} -\frac{1}{h}e^{A_1h} & 0 & \cdots & 0 & -2I & 0 & \cdots & 0\\ 0 & -\frac{1}{h}e^{A_2h} & \cdots & 0 & 0 & -2I & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & -\frac{1}{h}e^{A_lh} & 0 & 0 & \cdots & 2I\\ 0 & 0 & \cdots & 0 & \frac{1}{h}e^{A_1h} & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0 & 0 & \frac{1}{h}e^{A_2h} & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & 0 & 0 & 0 & \vdots & \frac{1}{h}e^{A_lh} \end{pmatrix}$$

$$B_2=\begin{pmatrix} \frac{1}{h}e^{2A_1h} & 0 & \cdots & 0 & e^{A_1h} & 0 & \cdots & 0\\ 0 & \frac{1}{h}e^{2A_2h} & \cdots & 0 & 0 & e^{A_2h} & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & \frac{1}{h}e^{2A_lh} & 0 & 0 & \cdots & e^{A_lh}\\ -\frac{1}{h^2}e^{2A_2h} & 0 & \cdots & 0 & -\frac{1}{h}e^{2A_1h} & 0 & \cdots & 0\\ 0 & -\frac{1}{h^2}e^{2A_2h} & \cdots & 0 & 0 & -\frac{1}{h}e^{2A_2h} & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & -\frac{1}{h^2}e^{2A_lh} & 0 & 0 & \cdots & -\frac{1}{h}e^{2A_lh} \end{pmatrix}$$

$$Z=\begin{pmatrix} e^{A_1h} & 0 & \cdots & 0 & hI & 0 & \cdots & 0 \\ 0 & e^{A_2h} & \cdots & 0 & 0 & hI & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & e^{A_lh} & 0 & 0 & \cdots & hI \\ -\frac{1}{h}e^{2A_1h} & 0 & \cdots & 0 & -e^{A_1h} & 0 & \cdots & 0 \\ 0 & -\frac{1}{h}e^{2A_2h} & \cdots & 0 & 0 & -e^{A_2h} & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & -\frac{1}{h}e^{2A_lh} & 0 & 0 & \cdots & e^{-A_lh} \end{pmatrix}$$

and let

$$z(t)=x(t)+Zx(t-h),\quad t\geq t_0+h.$$

Then it can be easily shown that $z(t)$ for $t\geq t_0+h$ satisfies the homogeneous differential equation

$$\dot{z}(t)=Az(t),\quad t\geq t_0+h.$$

Hence

$$\begin{aligned} z(t)&=e^{A(t-t_0-h)}z(t_0+h) \\ &=e^{Ah}e^{A(t-h-t_0-h)}z(t_0+h) \\ &=e^{Ah}z(t-h) \end{aligned}$$

Replace $z(t)$ by $x(t)+Zx(t-h)$ in the above equation and premultiply the result by Q to obtain

$$Qx(t)+QZx(t-h)=Qe^{Ah}x(t-h)$$
$$+Qe^{Ah}zx(t-2h)$$

But since

$$QZ=Qe^{Ah}$$

and

$$Qe^{Ah}Z=QZ^2=0,$$

it follows that

$$Qx(t)=0\quad t\geq t_0+2h.$$ Q.E.D.

3. Concluding Remark

For a class of time-invariant linear systems for which only delayed state information is available, a simple feedback law for rendezvous was derived. The controller thus obtained is a minimum-time suboptimal controller in the sense that no feedback controllers of the same form can achieve the required rendezvous in less than the derived rendezvous time $t_1=3h$. This is a direct consequence of[4].

References

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