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論 文
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Design of Optimal Control System of Nuclear Reactor for Direct Digital Control

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Abstract

The optimal control theory is applied to the desige of a digital control system for a nuclear reactor. A linear dynamic model obtained at 85% of rated power and a quadratic perfor mance index are used. A minimal order observer used in cascade with the feedback controller is suggested as a state estimator. The total reactor power control is studied in the range of 80% to 100% of rated power, with the steady state and load-following control. The control algorithm considered is suitable for implementation in direct digital control.

1. Introduction

The design of the optimal computer control system of nuclear reactors or nuclear power plants is of great importance, because it has been completely accepted that safety, reliability and economy of their operation depend to a great extent on their control system.

There have been many studies undertaken in the past years on the control system design of the nuclear reactor by the use of modern control theory.[1]-[6] But, in almost all of these papers, concentrations are focused mainly on the studies of steady state control aspects. Furthermore, these studies are performed under the assumption that all the state variables are measurable. But, in practice, all the state variables are not always measurable. Therefore, to realize this control scheme, it is required for the optimal control system to include the estimator which estimates the unmeasurable state variables from the informations of measured state variables and of inputs. As a state estimator, observer is selected.

The objective of this study is to design an optimal control system of a nuclear reactor for both the steady state control and load-following control. The control approach adoped here is basically a regulator control coupled with a feedforward action from the load demand. The control algorithms are designed to be implemented by direct digital control(DDC).

The design approaches employed in this paper have the following steps. The first is the design of the feedforward controller which calculates reference values of the state and control variables corresponding to the power demand. The second is the design of an optimal feedback controller for the nuclear reactor assuming that all the state

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variables can be measured. The third is the construction of an observer which reconstructs the inaccessible state variables from the measured input and output informations of the reactor. Finally, the optimal digital control system is realized.

Since the optimal constant feedback gain matrix varies with the values of the weighting factors in the quadratic performance index, appropriate values of the weighting factors are selected through computer simulations and the observer is designed by Gopinath's method[7] which is simpler and more efficient than other methods.

The simulation studies for a nuclear reactor control system are performed for various operating conditions: 1) reactivity disturbances are inserted at the equilibrium conditions. 2) demand power varies with ramp change and/or with step change. Finally, comparisons are made between two cases, namely, accessible case where all state variables can be measured and inaccessible case where some state variables cannot be measured.

2. Design of Direct Digital Control System

2.1 Reactor Dynamics[8]

The point-model kinetics equations for a nuclear reactor with one-group delayed neutrons are

$$\frac{dn(t)}{dt}=\frac{\delta k(t)-\beta}{l}n(t)+\lambda c(t) \quad (1)$$

$$\frac{dc(t)}{dt}=\frac{\beta}{l}n(t)-\lambda c(t) \quad (2)$$

where

$n(t)$=nuclear power

$\delta k(t)$=reactivity

β=fraction of delayed neutrons

l=average neutron lifetime

λ=decay constant of neutron precursor

$c(t)$=equivalent delayed neutron precursor density

If the following variables are defined

$$\alpha \triangleq \frac{\beta}{l} \quad (3)$$

$$\rho(t) \triangleq \frac{\delta k(t)}{\beta} \quad (4)$$

$$z(t) \triangleq \frac{\lambda}{\alpha}c(t) \quad (5)$$

and substituted into Eqs. (1) and (2), a set of normalized equations is obtained. Thus,

$$\frac{dn(t)}{dt}=\alpha\rho(t)n(t)-\alpha n(t)+\alpha z(t) \quad (6)$$

$$\frac{dz(t)}{dt}=\lambda\{n(t)-z(t)\} \quad (7)$$

where $\rho(t)$ is reactivity in dollars, $\rho(t)$ is defined in terms of the control variable u, for which we adopt the rate of change in reactivity to prevent the occurrence of abrupt change of reactivity in control sequence;

$$\frac{d\rho(t)}{dt}=\xi u(t) \quad (8)$$

where ξ is an arbitrary given constant.

The Eqs. (6), (7) and (8) are expressed in matrix notation as;

$$\left.\begin{aligned} &\dot{\underline{x}}=\underline{f}(\underline{x},u)\\ &\underline{f}=[f_1\ f_2\ f_3]^T\\ &\underline{x}=[n\ z\ \rho]^T \end{aligned}\right\} \quad (9)$$

where

$f_1=\alpha\rho(t)n(t)-\alpha n(t)+\alpha z(t)$

$f_2=\lambda n(t)-\lambda z(t)$

$f_3=\xi u(t)$

In Eq. (9), the vector $\underline{x}^T$ and $\dot{\underline{x}}$ respectively denote the transpose and time derivative of the state variable $\underline{x}$.

The controlsystem considered here is one that controls the deviations of state vector $\underline{x}$ from the reference state vector $\underline{x}^*$ during the power transition period.

Let us specify a feedforward reference control $u^*(L)$ is a function of the power demand L. The $u^*(L)$ can then be used to find the steady state reference state vector, $x^*(L)$, by solving

$$\underline{f}(\underline{x}^*,u^*)=\underline{0} \quad (10)$$

Therefore,

$$x_1^*=L,\ x_2^*=L,\ x_3^*=0,\ u^*=0.$$

Assume that the feedforward controller keeps the reactor close to the reference values. This suggests that one can linearize Eq.(9) and get

$$\delta\dot{\underline{x}}=A(t)\delta\underline{x}+B(t)\delta u \quad (11)$$

where

$\delta\underline{x}\triangleq\underline{x}-\underline{x}^*,\ \delta u\triangleq u-u^*$

$$A\triangleq\left[\frac{\partial f}{\partial x}\right]\Bigg|\frac{x^*}{u^*},\quad B\triangleq\left[\frac{\partial f}{\partial u}\right]\Bigg|\frac{x^*}{u^*}$$

The matrices A and B are above designated as time variant, to reflect the fact that $x^*(t)$ and $u^*(t)$ are in general time dependent. In this paper, however, these matrices are considered time in variant with their elements evaluated at some arbitrarily specified power level (in this case 85% of rated power) in order to reduce computation time during direct digital control (DDC).

Since a DDC is considered, the linear equations above have to be transformed into discrete time difference equations.[9] If it is assumed that $\delta u(t)$ is constant between the sampling instants kT and $(k+1)T$, i.e.,

$$\delta u(t)=\delta u(kT) \text{ for } kT\leq t<(k+1)T \qquad (12)$$

where T=sampling interval,
we may interate Eq. (11) and obtain the difference state equation asfollows;

$$\delta x(k+1)=P\delta x(k)+Q\delta u(k) \qquad (13)$$

where

$$\left.\begin{aligned} P&=\exp(AT)=I+AT+\frac{1}{2!}(AT)^2+\cdots \\ Q&=(P-I)A^{-1}B \\ &=T\left\{I+\frac{1}{2!}AT+\frac{1}{3!}(AT)^2+\cdots\right\}B \\ \delta x(k)&=\delta x(kT) \end{aligned}\right\} \qquad (14)$$

I=unit matrix

Typically, $\beta=0.0065$, $l=10^{-3}$sec, and $\lambda=0.0775\text{sec}^{-1}$ for a commercial nuclear reactor[10]. By assuming that $\xi=1$ and $T=1$sec, P and Q are calculated at the 85% of the rated power level;

$$P=\begin{pmatrix} 1.3183\text{E-}02 & 9.8684\text{E-}01 & 8.9401\text{E-}01 \\ 1.1766\text{E-}02 & 9.8823\text{E-}01 & 5.5216\text{E-}02 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$$

$$Q=\begin{pmatrix} 7.3661\text{E-}01 \\ 2.4155\text{E-}02 \\ 1.0 \end{pmatrix}$$

The discrete-time linear model described by Eq. (13) will be the basis for the feedback control that is be described in the next section.

2.2 Optimal Feedback Control

Consider the deterministic feedback control problem for a system modeled by Eq. (13). A performance index is defined as follows;

$$J_N=\frac{1}{2}\sum_{k=0}^{N-1}\{\delta x^T(k)X\delta x(k)+r\delta u(k)\} \qquad (15)$$

where weighting matrix X and constant r can be chosen freely, required that $X\geq O$ and $r>O$. The problem is to find the optimal control that will minimize J_N. This problem is solved by using the discrete maximum principle[10]. The solution is

$$\delta u(k)=-H(k)\delta x(k) \qquad (16)$$

The feedback gain matrix $H(k)$ is obtained from

$$H(k)=r^{-1}Q^T(P^T)^{-1}[G(k)-X] \qquad (17)$$

where $G(k)$ is obtained by computing Riccati equation, Eq. (18) backwards, starting with $G(N)=O$.

$$G(k)=X+P^TG(k+1)[I+r^{-1}QQ^TG(k+1)]P \qquad (18)$$

Unless there is a good reason for choosing a particular value for N, one can just as well consider $N\gg1$, in which case a constant feedback gain matrix results. This gain matrix H depends on the relative magnitudes of the coefficients in the weighting matrix X and r. Conditions for convergence and uniqueness of the Riccati equation have been established by Caine and Mayne.[12]

The main reason for choosing the square performance index is the advantages resulting from dealing with a constant feedback gain. The weighting matrix X and the weighting constant r have to be assigned by trial and error and by considering the eigen values of the closed loop system, $P-QH$.

2.3 Nuclear Performance Index

The general performance index given by Eq. (15) can be written in expanded form as a function of the nuclear power deviation, reactivity, and reactivity rate;

$$J_N=\frac{1}{2}\sum_{k=0}^{N-1}\{\delta n^2(k)+a\rho^2(k)+bu^2(k)\} \qquad (19)$$

where a and b are weighting coefficients. By comparing Eq. (15) with Eq. (19), the weighting matrix X and weighting constant r can be obtained as follows;

$$X=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{pmatrix},\quad r=b$$

The optimal control law obtained by using the performance index, Eq. (19), will minimize the sum of the squares of nuclear power deviation and the reactivity at the sampling instants.

2.4 State Estimator

In the nuclear reactor system, the delayed neutron precursor density and reactivity cannot be measured. Therefore, for the optimal control of the nuclear reactor these state variables are to be estimated from the informations of measured nuclear power and input.

We choose the observer as a state estimator. Basically, there are two kinds of observers, namely, full order observer which reconstructs all the state variables and minimal order observer which reconstructs only the unmeasurable state variables.[13]. To reduce transient errors caused by parameter perturbations, we select the minimalorder observer, which is designed by the use of Gopinath's method.[7]

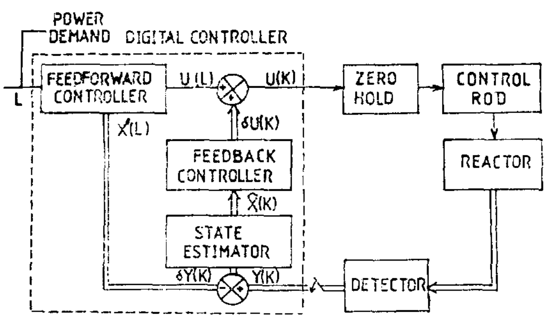

Fig. 1. Digital control system of a nuclear reactor

Since only nuclear power is measurable, the measurement variable $y(t)$ is related to the state vector as

$$y(t)=Cx(t) \tag{20}$$

where,

$$C=[1\ 0\ 0]$$

Thus,

$$\delta y(k)=C\delta x(k) \tag{21}$$

Consider the reactor dynamic equations described by Eq. (13) and Eq. (21)

$$\delta x(k+1)=P\ \delta x(k)+Q\ \delta u(k) \tag{22}$$

$$\delta y(k)=C\delta x(k)$$

Assume that C is of the form $C=[I_p, 0]$, $p\leq n$ where I_p is the identity matrix, p is the number of measurable state variables and n is the dimension of state variable vector. Then the state vector can be partitioned as follows;

$$\delta x(k)=\begin{bmatrix}\delta x_1(k)\\ \delta x_2(k)\end{bmatrix}$$

where

$\delta x_1(k)$=measurable state vector $(p\times 1)$

$\delta x_2(k)$=unmeasurable state vector $((n-p)\times 1)$

Therefore, the original system Eq. (22) can be represented as twosub-systems:

$$\begin{bmatrix}\delta x_1(k+1)\\ \delta x_2(k+1)\end{bmatrix}=\begin{bmatrix}p_{11} & p_{12}\\ p_{21} & p_{22}\end{bmatrix}\begin{bmatrix}\delta x_1(k)\\ \delta x_2(k)\end{bmatrix}+\begin{bmatrix}Q_1\\ Q_2\end{bmatrix}\delta u(k) \tag{23}$$

Now, the design procedure will be described in the form of two theorems for clarity.

Theorem 1 : If (C, P) is completely observable, then (P_{12}, P_{22}) is also completely observable.

Theorem 2 : There exists an observer of dimension $(n-p)$ for the system of Theorem 1.

Proofs of these theorems are omitted because they are well presented in Gopinath's work.[7] From these two theorems, observer dynamics can be obtained as follows.

$$\delta\hat{x}_2(k+1)=(p_{22}-p_{12})\ \delta\hat{x}_2(k)-Dp_{11}\delta x_1(k)$$
$$+p_{21}\delta x_1(k)+(p_{22}-Dp_{12})\ \delta x_1(k)-(DQ_1-Q_2)$$
$$\delta u(k) \tag{24}$$

where $\delta\hat{x}_2(k)$ is estimated state vector of $\delta x_2(k)$. Then, the design problem is only to find a matrix D such that $(P_{22}-DP_{12})$ has arbitrary eigenvalues.

2.5 Description of Digital Control System

The digital control system of a nuclear reactor for DDC is shown in Fig. 1. The digital controller consists of two parts. The feedforward controller calculates reference values of the state and control variables corresponding to the power demand. The feedback controller with observer finds an optimal control correction term to minimize the deviation between the reference and actual state values.

The reactor is initially operating at a constant power level with the optimal control input $u^*(L)$. When state variables deviate from the reference values on account of some external reactivity disturbances (in the case of steady state control) or power demand change from one power level to another (in the case of load-following control), the digital controller computes the deviation and modifies the control input from u^* to $u^*+\delta u$ in such manner as to minimize a given performance index.

Comparing the actual and estimated state values with reference values, the digital controller detects the error signal,

$$\delta x(k)=\begin{bmatrix} x_1(k) \\ \hat{x}_2(k) \end{bmatrix}-\begin{bmatrix} x_1^*(k) \\ x_2^*(k) \end{bmatrix}$$

and calculates the control correction term by the use of the optimal feedback gain precalculated off-line and stored in the digital computer as follows;

$$\delta u(k)=-H\ \delta x(k)$$

The actuating signal $u(k)$ is obtained as the sum of $u^*(k)$ and $\delta u(k)$ and serves as the control input to the reactor system.

3. Results and Discussions

3.1 Feedback Gain Matrix

The feedback gain matrix is given as a solution to Eq. (17) for the sampling period T=1sec. With P and Q determined, the numerical values of $H(k)$ in the interval $k=N\sim O$ are calculated by the use of recurrence Eqs. (17) and (18), letting $G(N)=O$ for given values of X and r. It is shown that if P, Q, X, r are time invariant, $G(N)=O$ and the system is controllable then $G(k)$ becomes constant as $k\rightarrow\infty$ so that the gain matrix H becomes constant matrix.[11]

Appropriate values of weighting factors were found by taking into account the transient responses of reactor until satisfactory performance is obtained for a variety of conditions at several power levels. In this paper, it was found that satisfactory performance was obtained for $a=0.01$ and $b=0.01$. Thus, feedback gain matrix used here is

$$H=[1.4395E-02\ \ 1.2966E\ 00\ \ 1.1843E\ 00]$$

These values were then used in all of the simulations to be discussed in the following.

3.2 Observer dynamics

The eigenvalues of the reactor system expressed by Eq. (22) are 1.0, 1.0 and 0.0014, respectively. Since the convergence rate of observer dynamics should be larger than that of reactor system, the eigenvalues of the observer are selected such that observer has that property. Therefore, two eigenvalues of the observer are arbitrarily selected as 0.8 and 0.001, respectively. Consequently, matrix D is obtained as;

$$D=\begin{bmatrix} -1.5811 \\ 3.0733 \end{bmatrix}$$

Therefore, the observer dynamics is obtained by the use of Eq. (24),

$$\delta\hat{x}_3(k+1)=\begin{bmatrix} 2.548 & 1.469 \\ -3.073 & -1.747 \end{bmatrix}\delta\hat{x}_3(k)+\begin{bmatrix} 1.888 \\ -1.264 \end{bmatrix}\delta u(k)+\begin{bmatrix} 0.475 \\ -0.535 \end{bmatrix}\delta x_2(k)$$

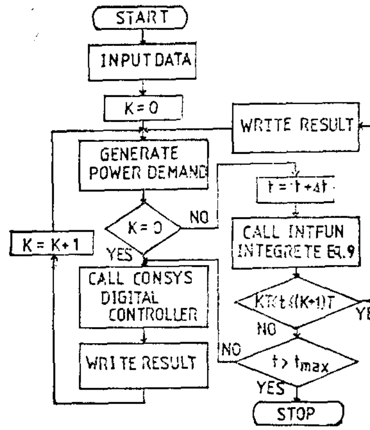

Fig. 2. Flow chart of digital simulation

3.3 Performance of Digital Control System

To examine the performance of the digital control system described in Sec. 2.5, transient responses of reactor for a variety of conditions are simulated for the case of all the state variables accessible to measurement and for the practical case where some state variables cannot be measured and should be estimated. Most of transients are obtained only on the initial 30 seconds, which in most cases is adequate for illustrations.

Fig. 2. shows a flow diagram of the program used to simulate reactor transients. In the digital simulations the computing delay and control delay are neglected to simplify the simulation study.

Fig. 3. shows the reactor transient responses with a disturbance of reactivity $\rho=0.1$ ($) when the reactor is operating at the constant power level of 85% of rated power. A step change of

reactivity at any instants gives rise to the prompt jump in nuclear power. In practical sense, disturbance can be inserted at any time. Three cases are shown in Fig. 3.

The first is the case which a reactivity disturbance is occurred at $T_d=0.0$ sec. (T_d represents the time when disturbance is inserted). In accessible case where all state variables are measurable, the digital control system detects the reactivity deviation at $t=0.0$ sec because allstate variables are measured and modifies instantaneously the actuating signal to the control rod to reduce the reactivity disturbance to zero quickly. But, in inaccessible case where reactivity is unmeasurable, the control system detects the nuclear power deviation at the next sampling instant. Therefore, control rod is actuated after exactly one sampling period. Thus the overshoot of nuclear power is larger in latter case than in former case.

The second and third cases are $T_d=0.5$ sec and $T_d=0.9$ sec, respectively. In this cases, the transients of these two control schemes are nearly same because the control system actuates the control rod at the same time, $t=1.0$ sec.

Fig. 4 shows the simulated responses to a 10% step increase in power demand frrom 80% to 90% of rated power at $t=0.0$ sec. This is a considerable change in demand. In transient responses of a ccessible and inaccessible cases, some degree of discrepancies exist. But, since these discrepancies decrease rapidly to zero in about 6 seconds, these can be neglected for control purposes.

Fig. 5 shows the response to a ramp increase in load demand. The power demand is determined

$$L=(0.8+0.005t)\times 100\%, \quad 0\leq t\leq 20 \text{ sec}$$
$$L=90\% \qquad t>20 \text{ sec}$$

i.e., the demanded load change is 0.5%/sec, which corresponds to 30%/min. This is an adequate rate of change. One can note that severe overshoots in the nuclear power are avoided when a ramp load change is considered since an abrupt change in reactivity is not occurring. The actual nuclear power is delayed for about one sampling period, because the digital controller begins to sense the power deviation after one sampling period.

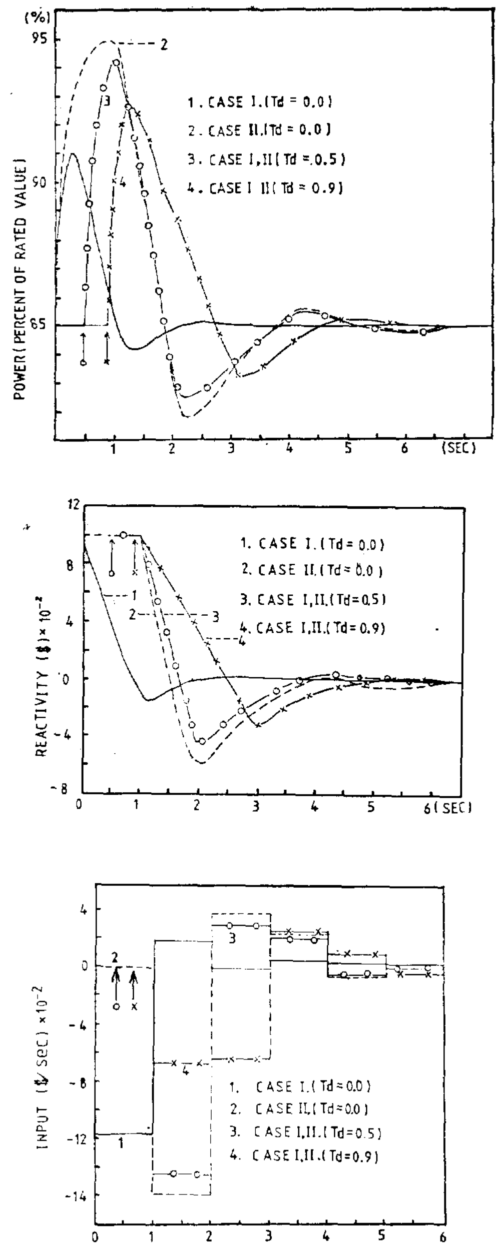

Fig. 3. Transients following a step reactivity disturbance. (CASE Ⅰ; accessible case, CASE Ⅱ; inaccessible case and Td is the time when disturbance is inserted)

4. Conclusions

In this study, the design of optimal digital

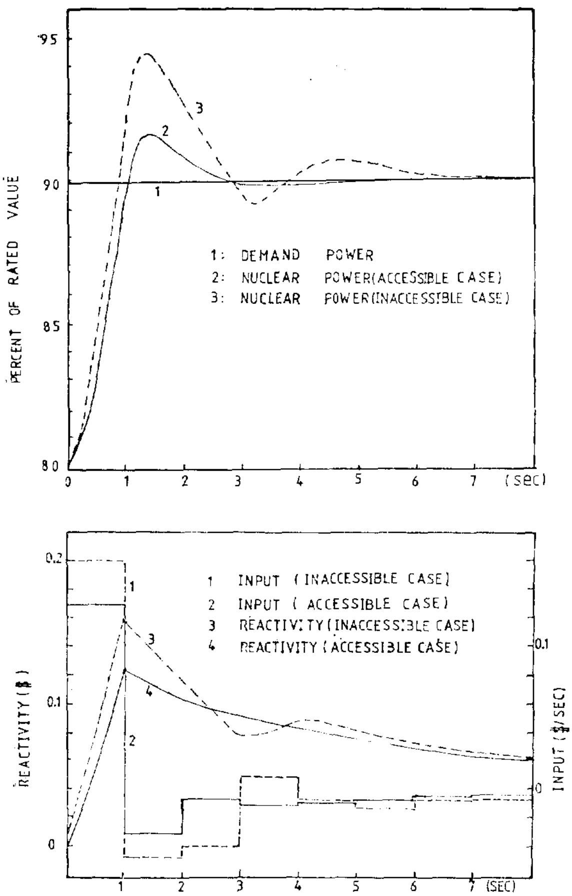

Fig. 4. Transients following a 10% step increase in power demand

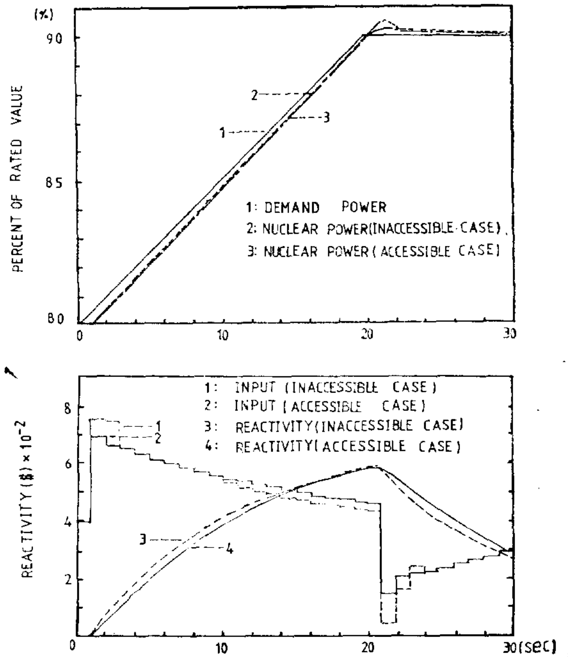

Fig. 5. Transienst following a 0.5%/sec (for 20 sec) ramp change in power demand

control system for a nuclear reactor is treated for a steady state control and load-following control. The dynamic equations of the nuclear reactor are nonlinear, which are linearized at the constant power level of 85% of rated power. The feedback controller is designed by the use of this linearized time invariant model and a square performance index.

As a state estimator we designed the minimal order observer by Gopinath's method to estimate the delayed neutron precursor density and reactivity. This observer is proved to give good estim ated values by simulation studies.

Simulation studies are performed for various operating conitions in the range of 80%~100% of rated power and for each condition, the characteristics of two control schemes, namely, accessible case and inaccessible case compared. The comparisons show that the control show that the control system which contains the observer has very good transient characteristics in all cases.

In the control of whole power range, the suggested time invariant model does not seem to be the best approach. To solve this problem, it is desirable for the digital controller to memory several linearized models obtained in advance at appropriate power levels and switch from one model to another according to corresponding power levels.

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