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A Study on Output Feedback Control

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Abstract

The problem of incomplet state feedback began to appear late in 60's and early in 70's. This was motivated by inability to measure all the states of the system in practice. This survey paper traces the early developments in the subject through to the most recent achievement of gain-determining, pole-assignment, stabilization, and low sensitivity system design with output feedback.

I. Introduction

Frequently the designer of controllers for linear system does not have a complete set of state variables directly available for feedback purposes. Moreover, he may wish to generate the control variables directly by taking linear combinations of the available output variables instead of first reconstructing the state via a Kalman Filter or a Luenberger Observer[1]. At first the Optimal gain, like in optimal control theory with complete state-feedback, was obtained by minimizing the appropriately chosen cost functional. Because the cost functional depends upon the initial state, a lot of method was proposed to eliminate the dependence on the initial state[1],[2],[6],[7],[8]. Later other methods were presented to choose the gain, e.g. by error minimization[15], by maximizing an impulse response correlation matrix[16] and by approximation of the closed-loop system thransfer function to the ideal model transfer function[17], and the extension was made to the sampled data system[15].

With the problem of gain-choosing, the problem of pole-assignment was studied by many workers. The early work of Davison[25] about the number of assignable poles was developed and extended and the necessary and sufficient condition for arbitrary assignment of all of the system poles was sought[26]-[44]. Since many theorems of the pole-assignment told about the number of assignable poles and their properties a lot, but nothing about the other poles, they might be impractical when the problem of stabilization was considered. So along with the enthusiastic studies about the pole-assignment the problem of stabilization by output feedback attracted many people's attention[51]-[58]. As in recent years considerable interest grew in reducing the sensitivity of the optimal linear regulator to parameter variations, the problem of deriving low-sensitivity feedback controller for linear systems with incomplete state feedback is considered also.

II. Determination of the Optimal Constant Output Feedback Gains

In optimal control theory, all the state variables of the plant must be aviailable for feedback as required by the optimal control law. The linear

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regulator problem with complete state feedback can be stated as follows. The plant equation is given in state variable form

$$\dot{x}=Ax+Bu \quad y=Cx \quad x(0)=x_0 \qquad (2-1)$$

The optimal feedback control law minimizing the quadratic performance index

$$J=\int_0^\infty (x^TQx+u^TRu)dt \qquad (2-2)$$

is

$$u=-kx \qquad (2-3)$$

However it is usually difficult to measure every state in practice. This problem has traditionally been a serious restriction on the practical applicability of optimal control theory.

Knowing this drawback in optimal control theory, a lot of effort has been made to control the system with only available states, or incomplete state feedback.

Levine and Athans[1], and Levine, Johnson and Athans[2] derived a necessary condition for the optimal constant output feedback gains. Instead of the optimal control strategy of, (2—3), they used the optimal control law

$$u=-Fy=-Fcx \qquad (2-4)$$

with the same performance index (2—2). But simple calculations show the performance index (2—2) with (2—4) depends upon the initial states. So to eliminate the dependence upon the initial state the performance index was averaged treating the initial state x_0 as a uniformly distributed random vector. And the new cost functional was obtained.

$$J(K)=\frac{1}{2}\left\{\mathrm{tr}\int_0^\infty e^{\tilde{A}'t}\,[A+C'F'RFC]e^{\tilde{A}t}\,dt\ x_0\right\} \qquad (2-5)$$

where

$$\tilde{A}=A-BFC \qquad (2-6)$$

Then the problem of gaining the optimal control law becomes that of parameter optimization, so the following theorem.

Theorem II-I

Any matrix which minimizes (2—5) also satisfies the matrix integral equations

$$F^*=R^{-1}B'K^*L^*C'(CL^*C')^{-1} \qquad (2-7)$$

where

$$K^*=\int_0^\infty e^{A^{*\prime}\tau}\,[Q+C'F^{*\prime}RF^*C]e^{A^*\tau}\,dT \qquad (2-8)$$

$$L^*=\int_0^\infty e^{A^*\sigma}x_0e^{A^*\sigma}\,d\sigma \qquad (2-9)$$

$$A^*=A-BF^*C \qquad (2-10)$$

Alternatively assuming that K^*, L^* and F^* are solutions of (2-7)-(2-10) the K^* is also a positive-definite solution of

$$O=K^*A^*+A^{*\prime}K^*+Q+C'F^{*\prime}RF^*C \qquad (2-11)$$

and L^* is a positive definite solution of

$$O=L^*A^{*\prime}+A^*L^*+x_0 \qquad (2-12)$$

They also gave a computational algorithm for solving the above equations by iteration, but not satisfiable.

Knapp and Basuthakur[3] derived the above necessary condition using an alternate and mechanically simpler approach which has greatly simplified developments in a related problem involving output feedback with parameter uncertainty. Also another derivation of theorem II-I was made by Mendel[4], which is applicable to other limited state feedback controller design problem as well. And Hutcheson[5] made the same try with a simple direct method.

Solving the above output feedback problem (2—1), (2—2) and (2—4) gives an expression of the performance index as

$$J=x'_0K^*x_0=\mathrm{tr}(K^*x_0x'_0) \qquad (2-13)$$

where K^* is the symmetric positive definite solution of the lyapunov matrix equation (2—11).

The dependence of J on the initial state is undesirable since inital conditions are not usually known a priori. In order to alleviate this difficulty, several approaches have been taken. Levine and Athans[1],[2] treated x_0 as a uniformly distributed random vector, whereas Man[16] used the maximum eigenvalue of K^* in place of J and Dabke[7] used the Euclidean norm of K^* as an upper bound on the maximum eigenvalue. A set of performance measures that are independent of initial conditions may be defined by observing that the performance index in (2—13) is a quadratic form in the space of initial states. So Zohdy and Aplevich[8] showed a generalized measure defined on the eigenvalue of K^* can be employed in place of J

$$V_s(K^*)=\left(\sum_{i=1}^{n}\lambda_i^s\right)^{\frac{1}{s}}=\mathrm{tr}(K^{*s})^{\frac{1}{s}} \quad (2\text{—}14)$$

where λ_i is an eigenvalue of K^*. Itis readily seen that

$$V_i(K^*)=\mathrm{tr}(K^*) \quad (2\text{—}15)$$

$$\lim_{s\to\infty} V_s(K^*)=\max\ (\lambda_i)=\lambda_m \quad (2\text{—}16)$$

Along with the effort of deriving the necessary condition, many workers tried to solve the given necessary condition. Horisberger and Belanger[9] solved the problem by developing simple formulae for the gradient matrix and using the Fletcher-Powell-Davidon algorithm on the basis of the existence theorem of the optimal gain given by Horisberger[10]. It was found that the first computational algorithms for the solution suggested in (1) and (2) cannot guarantee satisfactory results, particularly in the cases when the number of outputs is much smaller than the order of the system. It is caused by the inability to find an adequate initial guess when solving the associated system of algebraic matrix equations. So Bingulac, Cuk aad Calocic[11] proposed a new computational algorithm providing an initial guess, by solving a sequence of output-constrained regulator problems, starting from the solution of a full state-feedback regulator. The solution of each preceeding control problem is used as an initial guess for the next one, where the number of output variables is reduced by 1. The last control problem in the sequence represents the specified output-constrained regulator. But knowing the rather supplementary calculations involved in the above method, Petkovski and Rakic[12] gave another computational algorithm for finding an initial guess with the use of minimum error excitation criteria presented by Kosut[13]. Several iterative techniques proposed for finding the optimal F requires an initial guess of F that stabilizes the closed-loop system. However, the case when the open-loop system is unstable makes the initial guess difficult and forces one to resort to an auxiliary stabilization algorithm. Fully understanding this situation Choi and Sirisena[14] developed an alternative algorithm which requires only a stabilizing state feedback law to initialize the search.

Since the early work of Levine and Athans[1], [2] most of the papers were confined to continuous time case. Yahagi[15] extended this problem to sampled-data systems. First he derived the corresponding state equations and performance index as follows.

$$x[(k+1)T]=\alpha(T)x(kT)+\alpha(T)u(kt) \quad (2\text{—}17)$$

$$y(kT)=Cx(kT),\ u(kT)=-Fy(kT) \quad (2\text{—}18)$$

$$J_1=T\sum_{k=0}^{\infty}x'(kT)Lx(kT)+u'(kT)Mx(kT) + x'(kT)M'u(kT)+u'(kT)Wu(kT) \quad (2\text{—}19)$$

where

$$\alpha(\lambda T)=\exp(\lambda TA) \quad (2\text{—}20)$$

$$\beta(\lambda T)=\int_0^{\lambda T}\alpha(T)\ \mathrm{B}dT \quad (2\text{—}21)$$

$$L=\int_0^1\alpha'(\lambda T)\ Q\alpha(\lambda T)\ d\lambda \quad (2\text{—}22)$$

$$M=\int_0^1\beta'(\lambda T)Q\beta(\lambda T)d \quad (2\text{—}23)$$

$$W=\int_0^1[\beta(\lambda T)Q\beta(\lambda T)+R]\ d\lambda \quad (2\text{—}24)$$

And in this case he derived the necessary condition for the optimal output feedback gain in three equations which are quite similar to continuous version. Also in the same paper he obtained the optimal gain in a different way. Subtracting the state x which is obtained from complete state-feedback from the state x which is obtained from incomplete state-feedback he made a new equation

$$e=\dot{x}-\dot{\bar{x}}=[A-BK]\ e+B[FC-K]x$$
$$e(0)=0 \quad (2\text{—}25)$$

Letting

$$v=(-FC+K)x \quad (2\text{—}26)$$

then

$$\dot{e}=[A-BK]e+Bv \quad (2\text{—}27)$$

with an intention to minimize the trajectory error driving function with the cost function

$$J_2=\int_0^{\infty}v'(t)Pv(t)dt \quad (2\text{—}28)$$

he derived a necessary condition for the optimal feedback gain in terms of three equations. Here also, this technique was extended to sampled data system.

Apart from the many works mentioned previously the optimal gain was obtained by using impulse response or transfer function. Subbayyan and Vaithilingan[16] approached the problem with an approximation of the impulse response of the closed-loop system with incomplete state feedback to the optimal closed-loop system with complete state feedback. So they tried to solve the problem by maximizing an impulse response correlation index given below

$$\text{I.R.C.}=\frac{|g^*,\hat{g}|}{\sqrt{(g^*,g^*)(\hat{g},\hat{g})}} \qquad (2-29)$$

where g^* is the impulse response of the optimal closed-loop and $\hat{g}$ is the impulse response with incomplee state feedback. T.C. Hsia[17] tackled the problem in the frequency domain. His approach is based on the approximation of the closed-loop system transfer function to the ideal model transfer function. The closed loop transfer function can be written as

$$G^*(s)=\frac{Y^*(s)}{R^*(s)}=C[sI-A+BK]^{-1}B$$
$$=f\frac{1+\alpha_1 s+\alpha_2 s^2+\cdots+\alpha_m s^m}{1+\beta_1 s+\beta_2 s^2+\cdots+\beta_n s^n}\quad n>m \qquad (2-30)$$

and

$$\hat{R}(s)=\frac{Y^*(s)}{\hat{R}(s)}=C[sI-A+BK]^{-1}B$$
$$=g\frac{1+\alpha_1 s+\alpha_2 s^2+\cdots+\alpha_m s^m}{1+y_1 s+y_2 s^2+\cdots+y_n a^n} \qquad (2-31)$$

where $G^*(s)$ is the transfer function with complete state-feedback, and $G(s)$ is the transfer function with incomplete state feedback. It is noted that the two transfer functions have the same numerator polynomials. The feedback gain K is selected such that

$$\frac{G^*(j\omega)}{f}\simeq\frac{G(j\omega)}{g} \qquad (2-32)$$

the approximation in (2—32) can be best carried out by applying the magnitude criterion[18]

$$\left|\frac{G^*(j\omega)/f}{G(j\omega)/g}\right|^2=1 \qquad (2-33)$$

Substituting (2—30) and (2—31), (2—33) becomes

$$\left|\frac{1+y_1 s+y_2 s^2+\cdots+y_n s^n}{1+\beta_1 s+\beta_2 s^2+\cdots+\beta_n s^n}\right|^2_{s=j\omega}=1 \qquad (2-34)$$

But his work is limited to single variable case so it needs to be extended to the multivariable case.

Using only the outputs of the system has its outstanding merit that no observers or no derivative actions are needed, but because we use only a part of all the informations about the system, the control possibility is limited. So extending the output feedback problem to the optimal PI or PID controller design in continuous and discrete systems, we can have a better control over the system.

Most of the chemical processes have time delays. And this fact makes the control more difficult. In these time delay systems the controller design by output feedback, PI or PID action remains challenging.

III. On Pole-Assignment

Since the fundamental result was presented by Wonham[21], the problem of pole assignment has received much attention and has been expected to bridge the gap between classical and modern control theory. The result of (21) states that, if the system is controllable it is pole-assignable, that is, the eigenvalues (the poles) of the closed-loop system can be assigned arbitrarily by selecting an appopriate state feedback. Since the complete state observation which was assumed in (21) is unlikely to most practical situations. It has been desirable to find the condition under which the system is pole-assignable with incomplete state observation. If some dynamic elements are allowed in the feedback loop, the elegant result of Brasch and Pearson[22] gives an answer to this question. Another approach has been the one using the Luenberger observer[23] or the Kalman filter[24].

In 1970 Davison[25] appeared with his early work on pole-assignment with output feedback with the following system

$$\dot{x}=Ax+Bu$$
$$y=Cx \qquad (3-1)$$

where y is an p-dimensional output vector, x is an n-dimensional state vector, u is an m-dimensional input vector, and A, B, and C are constant matrices of appropriate size and he proved theorem which said

Theorem III-1

Consider the system given by (3—1). If (A,B) are controllable, if C has rank $p\leq n$ and if A has eigenvalues that are distinct or are repeated such that the eigenvalues of each Jordan block of the Jordan cononical form of A are distinct, then a linear feedback of the output variables

$$u=-Fy \qquad (3-2)$$

where F is a constant gain matrix, can always be found so that p eigenvalue of the closed-loop matrix $A-BFC$ are arbitrarily close (but not necessarily equal) to p preassigned values, where the preassigned values are chosen so that any complex numbers appearing do so in complex conjugate pairs.

He also gave an algorithm for finding output feedback gain F. The above theorem makes certain assumptions regarding the Jordan cannonical structure of A, which holds iff A is a cyclic matrix. This assumption can be removed by using the results of (22). By making a further assumption on the observability, Davison and Chatterjee[26] obtained the following theorem.

Theorem III-2.

Consider the system given by (3—1). If the triple (C,A,B) is complete (i.e., (C,A) observable, (A,B) contralloble) and if C has rank $p\leq n$ and B has rank $m\leq n$, then a linear feedback of the output variables

$$u=-Fy \qquad (3-3)$$

where F is a constant gain matrix can always be found so that max (p,m) eigenvalues of the closed-loop matrix-loop matrix $A-BFC$ are arbitrarily close (buy not necessarily equal) to max (p,m) preassigned values, where the preassigned values are chosen as in Theorem III-1.

Sridhar and Lindorff[27] gave an alternate proof of Davison's theorem on pole placement and showed that in some cases, more than max (p,m) poles can be assigned arbitrarily. In (28) they obtained the same result as in (27) and assuming the system is output stabilizable, gave a least square design technique to approximate the desired pole locations when it is not possible to place all the poles.

When the order of the system n is large, however, the algorithm in(25) may run into numerical difficulties due to the illconditioning of a matrix which must be inverted. So Davison and Chow[31] presented a modification of the algorithm which avoids this problem, and so allowing large systems ($n>10$) to be considered. Their algorithm consists of two parts. In the first part, a feedback gain matrix F^* is found so that the closed-loop system matrix $(A-BF^*C)$ has distinct eigenvalues (this implies that the closed-loop system matrix becomes cyclic). In the second part, a feedback gain matrix F_0 is found so that the closed-loop system matrix $(A-BF^*C-BF_0C)$ has eigenvalues arbitrarily close to some preassigned values. The required gain matrix becomes, therefore, $F=F^*+F_0$ Davison and Wang[32] considered the poleplaceme nt problem and extended the early works having the following theorem

Theorem III-3

Given (C,A,B) controllable and observable with $A=R^{n\times n}$, rank $B=m$, rank $C=p$, then for almost all (B,C) pairs, there exists an output gain matrix F so that $A-BFC$ has min $(n, m+p-1)$ eigenvalues assigned arbitrarily close to min $(n, m+p-1)$ specified symmetric eigenvalues.

The above theorem implies in particular, that almost every linear time-invariant, multivariable system can be stabilized by using only output feedback provided $m+p\geq n+1$

Antsaklis and Wolovich[33] showed that the rank of an appropriately defined real matrix Ω represents an upper bound on the number of closedloop poles which can be completely and arbitrarily assigned via constant gain output feedback and the observability index of an appropriately defined single-input system represents a measure of the order of dynamic compensation which is required for complete and arbitrary pole placement

Different from the conventional approachusing the characteristic equation, with an approach based on the properties of the eigenspaces of the closed-loop dynamics, Kimura[34] showed if the system is controllable and observable and if $n\leq m+p-2$, an almost arbitrary set of distinct closed-loop poles is assignable by constant gain output feedback and moreover the minimum order of the dynamic compensator required for almost arbitrary pole assignment.

Kimura, again[35] faced the problem with a purely geometrical approach. He showed that pole assignability is strongly connected to the structural properties of systems in state space with the basic idea of connecting the concept of controllability subspace with the subspace which represents the set of all candidates for closed-loop eigenvectors. So he gained a result which says arbitrary pole assignment is possible for almost all systems if $n<m+p+\nu-1$, $p>\mu$, $m>\nu$ where ν and μ are the so-called controllability index and the observability index of the system, respectively. And in (37) Vardulakis, also, derived a sufficient condition for n-specified eigenvalues to be assigned under constant output feedback.

Munro and Vardulakis[37] presented a simple test given in analytic matrix terms which provides both necessary and sufficient condition for arbitrary assignment of all of the system poles using only constant output feedback. With state feedback gain K the closed-loop dynamic equation with complete state feedback is

$$\dot{x}=(A-BK)x \tag{3—4}$$

and the following theorem is given

Theorem III-4

A necessary and sufficient condition for all of the poles of a complete system described by (3-1) to be arbitrarily assigned using constant ouput feedback is that at least one of the set of state-feedback matrices K, which achieves the same pole assignment, and one of the g_1-inverses of C satisfy the consistency relationship

$$KC^{g_1}C=K \tag{3—5}$$

Using co-ordinate transform, Patel[38] derived the necessary and sufficient condition condition of arbitrary placement of the n poles and gave a method to find the corresponding output feedback gain.

The above problem was considered and extended by Munro[39]. He showed the necessary and sufficient condition for an output feedback solution to exist is equivalent to conditions imposed by the controllability and observability indices of the given system. He revealed that the desired closed-loop eigenvalue but also be chosen such that the pair (A_C,B) has the same controllability indices as the pair (A,B). It was also shown that for an output feedback solution to exist, A_C must in addition be chosen such that the pair (A_C,C) has the same observability indices as the pair (A,C)

Seraji[40], also gave a necessary and sufficient condition and algorithm for the same problem.

Seraji[41] dealt with the same problem of the unattainability of certain closed-loop pole positions in single-input multi-output systemes with constant feedback. He showed that there are two classes of unattainable pole positions which are unattainable individually and those which are unattainable simultaneously. The individually unattainable pole positions are identified as the locations of common zeros of the open-loop system, and a set of simultaneously unattainable pole positions results in linearly dependent rows in a certain matrix.

Vardulakis[42] investigated the problem associated with the allowable s-plane regions for the closed-loop system poles under constant output feedback. It was shown that under certain conditions result which can be seen as a natural extension of the classical root-locus ideas can be obtained.

$$\begin{aligned}\dot{x}&=Ax+Bu\\ y&=Cx+Du\end{aligned} \tag{3—6}$$

In the pole-assignment problem mentioned previously, the matrix D was eliminated. Seraji[43] considered the problem of pole-placement in the general case where there is direct transmission from input to output with a control law

$$u=V-Fy \tag{3—7}$$

We have closed-loop dynamic equation as follow

$$\begin{aligned}\dot{x}&=(A-BF(I+DF)^{-1}C)\ x\\ &\quad+(B-BF(I+DF)^{-1}D)v\\ y&=(I+DF)^{-1}Cx+(I+DF)^{-1}Dv\end{aligned} \tag{3—8}$$

Let's define the constant mxp matrix P as

$$P=F(I+DF)^{-1} \tag{3—10}$$

Then eqn (3—8) becomes

$$\dot{x}=(A-BPC)x+(B-VPD)v \qquad (3-11)$$

Using the existing methods, a matrix P can be found which assigns the eigenvalues of $A-BPC$. Once P is found the required output feedback matrix F is obtained by solving eqn (3—10) as

$$F=P(I+DF) \qquad (3-12)$$

or

$$F=(I-PD)^{-1}P \qquad (3-13)$$

where the inverse matix exists in general.

Furthermore Chammas and Leondes[44] considered the problem of pole assignment by piecewise constant output feedback for linear time-invariant systems with infrequent observation. An algorithm for computing the output feedback gain was presented, together with necessary and sufficient conditions for pole assignment. The resultant output feedback gain is a periodic piecewise constant function of time, that takes at most $(n-m)$ different values, which leads to an output feedback control law that can be easily implemented on line.

In cases when proportional ouput feedback alone fails to satisfy the poleassignment problem, controller with proportional-plus-derivative output feedback is found to be useful. Seraji and Tarokh[45] and Paraskevopoulous[46] investigated this problem, but a very little is known. So the same questions, e.g., the number gnd position of assignable poles, necessary and sufficient condition for arbitrary pole-placement and frequency domain implication as in the constant output feedbackcase still remains to be answered.

If even with the above type controller, arbitrary pole-assignment cannot be achieved, the controller of higher order can be considered. And in this case the controller of least order will be desirable. So designing the compensator of least order can be an interesting problem.

IV. On Output Feedback Stabilization

Stabilization of high-order system by constant state feedback of dynamic output feedback canbe impractical from the viewpoint of measurement of the compensator device. So the method which stabilizes the system with the output that can be measured easily by constant gain has been studied. In the pole-placement problem of section III, the subject of arbitrarily assigning the number of poles which are determined by the characteristics of the system was considered, but we had no control over the other poles. So these results may be of no use in the problem of stabilization. Therefore output feedback stabilization which locates all the system poles in the left half plane of complex frequency plane needs to be considered.

The fact that the uncontrollable mode and unobservable mode cannot be changed by output feedback was known by Jameson[49] and Nandi and Herzog[50], so the condition that the uncontrollable mode and unobservable mode are stable is presupposed in the output stabilization problem.

Using a geometric approach M.T.Li[51] derived the sufficient condition, Theorem IV-1 for the stabilization of linear time invariant system and this was proved by Denham[52] using alternative approach.

$$\dot{x}=Ax+Bu \qquad (4-1)$$

$$y=Cx$$

$$u=-Fy \qquad (4-2)$$

$$\dot{x}=(A-BFC)x \qquad (4-3)$$

In the completely controllable and observable system given by (4—1), $R(C)$ and $N(C)$ being the range space and null space respectively, let S be the orthogonal subspace of T where T is the largest invariant space of A' contained in $R(C)$. And U is the smallest invariant space of A' containing $N(B')$

Theorem IV-1

System (4-1) is stabilizable if

$$\{\text{modes of } A' \text{ associated with } U\} \cap \{\text{modes of } A \text{ associated with } S+N(H)\}$$

are stable.

Using the fact [54] that if the constant feedback gain of unity rank is represented as the dyadic form of $F=qk$, then the closed loop characteristic polynomial $G_f(s)$ is given in a simple form as

$$G_f(s)=|sI-A|+k[\mathrm{Cadj}(sI-A)B]q \qquad (4-4)$$

Seraji[53] showed that the system is output feedback stabilizable if the missing coefficient of power of s in $|sI-A|$ is strictly positive in $k\,[\mathrm{Cadj}(sI-A)B]q$.

McBrinn and Roy[55] developed a computational algorithm to find the feedback gain which minimizes the real part of the least stable eigenvalue of (4—1) to maximize the stability of the least stable pole of the system. And the problem of maximizing the most negative Hurwitz determinant was considered by T.E. Fortmann[56].

To find the gain which makes the real part of the eigenvalues of the closed-loop system less than zero, Kreisselmeiler[57] treated the problem of minimizing the quadratic cost functional of (4—5) and used Riccati equation

$$J(uF)=E\left\{\int_0^\infty (u+Fy)'\ (u+Fy)\,dt\right\} \qquad (4-5)$$

Sirisena and Choi[58] improved previous works and dealt the problem of stabilization by placing the poles in the prescribed region of the complex frequency plane. They developed a solution technique and computer algorithm to find the constant output feedback gain which places the closed-loop system pole in the region given by the inequality $h(\sigma,\omega)<0$, where $h(\sigma,\omega)$ is a continuous and differentiable function of the complex frequency $s=\sigma+j\omega$

V. on Sensitivity with Output Feedback

In recent years there has been considerable interest in reducing the sensitivity of the optimal linear regulator to parameter variations. This is because a regulator designed to be optimal for one set of system parameter values may well cease to be optimal when these system parameters deviate from their norminal values. The sensitivity constrained controllers presuppose the system state variables and sensitivity terms to be available for feedback control purposes, a condition often not met in practice. So the problem of deriving low-sensitivity feedback controller for linear systems with incomplete state information is considered.

In 1974 Kurtaran and Sidar[59] considered the problem of sensitivity of optimal performance of linear quadratic stochastic systems with output feedback, and derived the optimal-cost sensitivity.

Sirisena and Choi[60] dealt with the problem of the design of output feedback control systems with minimum pole sensitivity to plant parameter changes. Two different design approaches were proposed. In one approach, exact pole-placement is obtained at the nominal values of the values of the pole sensitivity to small parameter variations. The second approach yields optimal, though not exact, pole placement in a minimum mean square error sense over a prescribed range of parameter values.

The problem of output feedback controller design with low trajectory sensitivity to small parameter variations was considered for both deterministic and stochastic systems by O'Reilly[61]. The performance index was of the standard linear regulator type modified to include a quadratic sensitivity term. For deterministic systems, a concise derivation of a low-sensitivity output feed back controller was presented using the methd of lagrange multipliers. For stochastic systems, use was made of the observer-estimator of O' Reilly and Newmann to present a unified approch to low-sensitivity controller design.

VI. Conclusion

In this paper the early works and later achievements of output state feedback problem were surveyed. Since the optimal control theory and the pole-placement problem treated by Wonham, impracticality to have the complete state informations induced many worker's attention to the study of output state feedback problem. With this natural trend, this paper treated the output feedback problem in four categories, obtaining the optimal gain, pole-assignment, stabilization, and low-sensitivity controller design with output feedback.

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