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# Dislocation Analyses of Semi-Brittle Fracture (I)

## Soon-Kil Chung* and Byung-Ho Lee**

## 반취성 파괴에 관한 Dislocation 해석 ( I )

정 순 길 · 이 병 호

초    록

균일 인장하중하에 있는 고체 내부에 고립된 제 1 형 탄소성 크랙의 반취성 파괴를 경사슬립밴드 모델(inclined slip band model)로써 연속크랙전위(continuum crack dislocation) 및 연속격자 전위(continuum lattice dislocation)을 이용하여 이론적으로 연구하였다.

크랙전위및 격자전위에 관한 힘평형을 나타내는 연립특이적분방정식의 해는 크랙전위및 격자전위 에 관한 적정밀도함수를 가지고 특이항을 해소하는 조건을 부과하여 얻는다. 이 특이항 해소조건의 타당성은 처음으로 소성영역의 크기를 그 판단기준으로 검토되었으며, 그 결과 합당한 것으로 확인 되었다. 또한 상기방법으로부터 산출된 COD는 소규모소성역을 넘어서도 선형적으로 $K^2/E\sigma_Y$에 따 라 변화함을 알게된다.

상기모델에서 위축적분경로(Shrunk path)상의 $J$ 적분치를

$$J = \frac{\delta \sigma_Y}{\sin 2\theta}$$

의 형태로 유도하였는데, 이것은 $J$적분에 관한 Eshelby의 힘개념을 구체적으로 표현한다 : $J$ 는 크 랙전파방향으로 탄소성크랙정점에 작용하는 가상적인 힘이며, $\frac{1}{2}$ $J$의 한 슬립평면상에의 분력은 그 슬립평면상의 모든 격자전위에 작용하는 전단력의 총화와 같다.

## 1. Introduction

Dugdale[1] determined coplanar strip yield zone by cancelling the singularity. Recently Atkinson and Kanninen[2] also used this condition in their superdislocation model where there was, however, no information for the plastic zone size. Many other investigators[3~8] studied an inclined strip yield

* Member, Hongneung Machine Industry Co., Ltd.
** Member, Korea Advanced Institute of Science and Technology.

model since the plastic deformation at the region of the crack tip is usually confined to two slip systems inclined to the crack plane. Rice[3] analyzed approximately this model by applying the BCS model[9]. Vitak[4] and Riedel[5] improved the approximate solution by Bilby and Swinden[6], but obtained different results because of their different approach. Cherepanov[7] solved a boundary value problem of the inclined strip yield model with the Wiener-Hoph equation for small scale yielding. Lo[8] also studied this model using a complex potential method.

Unfortunately, however, there is little agreement between these investigators due to the difference in their mathematical models.

The inclined strip yield model is reconsidered in this paper with continuum crack and lattice dislocations. Moreover, the validity of the singularity cancelling condition for the plastic relaxation of the crack tip is examined by regarding the plastic zone size as a reference.

The J-integral and the crack opening displacement(COD, $\delta$) are widely accepted parameters in fracture mechanics for detecting the initial stage of crack growth. Rice[10] first applied the concept of the force on an elastic singularity introduced by Eshelby[11] to the fracture mechanics. Vitak[4] first evaluated the J-integral on the inclined strip yield model with computer. Many other investigators[12~16] studied the relationship between COD and J, experimentally. However, in simple models such as BCS model and Dugdale model, the relationship between J and COD is given by

$$J = \delta \cdot \sigma_Y.$$

Thus we first try to evaluate J by choosing a shrunk path for obtaining a direct relationship between J and COD in the inclined strip yield model which is a reasonable model of the semi-brittle plastic tensile crack.

## 2. Formulation of the Model

The inclined-strip-yield continuum-dislocation model of an isolated, elastic-plastic crack in uniform tension is shown in Fig. 1 Let $f(x_0)$, $-c < x_0 < c$, be the crack-dislocation-density function and $g(x_1)$, $0 < x_1 < p$, the lattice-dislocation-density function. Then, $f(x_0)$ and $g(x_1)$ must satisfy the following

simultaneous, singular integral equations derived by:[17]

$$\sigma_a/A - \int_{-1}^{1} \frac{f(\zeta)d\zeta}{\zeta - x_0} + \sum_{i=1}^{4} \int_0^p B_i^0 (x_0, \xi_i) g(\xi_i) d\xi_i = 0,$$

(1a)

$$(\sigma_S - \sigma_R)/A - \int_0^p \frac{g(\xi)d\xi}{\xi - \xi_1} + \int_{-1}^{1} B_1^1(\xi_1, \zeta) f(\zeta) d\zeta$$

$$+ \sum_{i=2}^{4} \int_0^p B_0^i (\xi_1, \xi_i) g(\xi_i) d\xi_i = 0,$$     (1b)



**Fig. 1** Inclined-strip-yield continuum-dislocation model.

where eqn. (1a) is a force equilibrium equation for the crack dislocations and eqn. (1b) that for the lattice dislocations on the slip band of branch 1 in Fig. 1 Each kernel $B_j^i$ represents corresponding interaction stresses on the dislocations in the branch $j$ due to the dislocations in the branch $i$ normalized with respect to $A = Db(D = \mu/2\pi(1-\nu))$, where $\mu$, $\nu$ and b mean, respectively, shear modulus, Poisson's ratio and Burgers vector. And $\sigma_a$, $\sigma_Y$, $\sigma_R$, $\sigma_S$ and $p$ are applied tensile stress, flow stress, friction stress, resolved shear stress on the slip plane and length of strip yield zone (plastic zone size), respectively. Lengths are all normalized to half the crack, c.

Performing inversions[18] of Eqns.(1) with unbounded end conditions for $f$, and one end $(x_1 = 0)$ unbounded and the other $(x_1 = p)$ finite for $g$ reduces the singular integral equations into Fredholm integral equations. Changing the order of the double integrals in the Fredholm integral equations and

taking Cauchy principal values for the singular integrals give simultaneous linear equations for given points of $x_0$ and $x_1$ by replacing the integrals by sums with weight factors. Then we obtain numerical solutions of $f$ and $g$ for given numbers of base points of gaussian quadrature and for an assigned value of p. At such points $x_0 = x_{0i}$ $(i = 1, 2, ..., n)$ and $x_1 = x_{1j}$ $(j = 1, 2, ..., m)$, $f(x_{0i})$ and $g(x_{1j})$ can be obtained. Various detail expressions and derivations of the equations and the kernels $B_j{}^i$ are given in the dissertation[17].

### 3. Proper Dislocation Density Functions

In the process of the plastic relaxation by emitting lattice dislocations at the crack tips[19], the reduction in magnitude of the singularity increases the non-singular terms of the density function of the crack dislocations as in the stress fields of the crack tip region. Since, however, a satisfactory forms of the density functions of the crack dislocations is not known yet, we have investigated four possible forms of the crack-dislocation-density function.

As a first candidate, Taylor series expansion is attempted[20] for the non-singular terms with the assumption that non-singular terms are purely regular:

$$f(x) = \frac{\sigma_a}{\pi A} \frac{x}{\sqrt{(1-x^2)}} - \frac{\sigma_Y}{\pi A} \left\{ \frac{a_1 x}{\sqrt{(1-x^2)}} + \sum_{k=1}^{n-1} a_{k+1} x^{2k-1} \right\}, \tag{2}$$

where we define $a_1$ as singularity cancelling coefficient. This power series may not converge so well that the value of the singularity cancelling coefficient fluctuate severely, or may not converge at all (see Fig. 3).

If the stress field near the plastically

relaxed crack tip is assumed to be not so significantly distorted as to change the essential form of the functions which constitutes the Williams eigenfunction expansion, the second candidate can be proposed as the following:

$$f(x) = \frac{\sigma_a}{\pi A} \frac{x}{\sqrt{(1-x^2)}} - \frac{\sigma_Y}{\pi A} \left\{ \frac{a_1 x}{\sqrt{(1-x^2)}} + \mathrm{sgn}(x) \sum_{k=1}^{n-1} a_{k+1}(1-|x|)^{k/2} \right\}, \tag{3}$$

The dominant term of the stress field which is obtained with this density function are almost the same form as the Williams eigenfunction expansion. And $\mathrm{sgn}(x)$ and $(1-|x|)$ in eqn. (3) are taken for the antisymmetry of the function.

In eqn. (3), the nonsingular terms are regular functions except the first square root term. This study tests the significance of the square root term effectively by taking the following density function as the third candidate,

$$f(x) = \frac{'\sigma_a}{\pi A} \frac{x}{\sqrt{(1-x^2)}} - \frac{\sigma_Y}{\pi A} \left\{ \frac{a_1 x}{\sqrt{(1-x^2)}} + a_2 \cdot \mathrm{sgn}(x) \sqrt{(1-|x|)} + \sum_{k=1}^{n-2} a_{k+2} x^{2k-1}, \right\} \tag{4}$$

by replacing the regular part of the non-singular terms in eqn. (3) with the Taylor series expansion. eqn. (4) is called a "mixed function".

Finally, this study tests the [N,N] or [N, N-1] Padé approximant for the non-singular terms which provides efficient rational function[23].

$$f(x) = \frac{\sigma_a}{\pi A} \frac{x}{\sqrt{(1-x^2)}} - \frac{\sigma_Y}{\pi A} \left\{ \frac{a_1 x}{\sqrt{(1-x^2)}} + \frac{\sum_{i=0}^{M} c_i x^i}{\sum_{i=0}^{N} b_i x^i} \right\}, \tag{5}$$

where the second term in the bracket is the [N,M] Padé approximant.

## 4. Solution with Singularity Cancelling Condition

An additional condition is needed for the length of the slip band, p, which depends on the applied stress, $\sigma_a$, and is nonlinearly involved in eqns. (1). The boundary condition have already used in the inversion process of eqns. (1), so we takes the singularity cancelling condition as a necessary additional condition. Since the true stress intensity factor is given by

$$K_t = \sqrt{2}\,\pi^{3/2} A \lim_{x \to c} \{(c-x)^{\frac{1}{2}} f(x)\}$$

$$= \sigma_Y \sqrt{\pi c}\left(\frac{\sigma_a}{\sigma_Y} - a_1\right),$$

the singularity cancelling condition will be satisfied when the true stress intensity factor vanishes, that is, when

$$a_1 = \frac{\sigma_a}{\sigma_Y} \tag{6}$$

Now, we can obtain sets of numerical values on $a_i$ $(i=1,2,...,n)$ with respect to each of the crack-dislocation-density functions given by eqns. (2-4), respectively, for an assigned value of p under a fixed applied stress. Proper crack-dislocation-density functions must give smoothly convergent value of $a_1$, not being necessary the value of eqn. (6), with increasing values of n. Under the fixed applied stress, p is then adjusted so that $a_i$ may vary. The plastic zone size, p, can not be fixed until the adjusted value of p yields $a_i$ which satisfies the singularity cancelling condition, eqn. (6), within reasonable accuracy.

For the crack-dislocation-density function, eqn. (5), we have to solve simultaneous nonlinear equations. The least square me-

thod is used for this purpose with a minimization computer program. The process for the determination of $a_i$ and p is essentially the same as the previous cases.

A widely accepted parameter of the crack opening displacement is given by[4~5].

$$\delta = 2b \sin\theta \int_0^p g(\xi)\,d\xi, \tag{7}$$

where eqn. (7) is numerically integrated by using the values of $g(x_{1j})$ and its gaussian quadrature in the above numerical solution of eqns.(1).

## 5. The J-integral on the Shrunk Path

For the crack on the plane $y_0=0$, the J-integral is given by

$$J = \int_\Gamma (W dS_1 - \sigma_{jk}\partial_k/\partial x_1 dS_j), \quad (x_1 \equiv x_0, x_2 \equiv y_0) \tag{8}$$

where $dS_j$ becomes a line element of an integration contour, $\Gamma$, which goes in an anti-clockwise direction from the bottom to the top of the crack surface, as shown in Fig. 2.
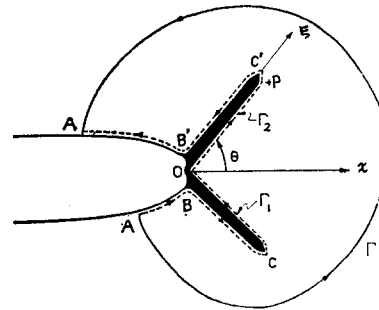


Fig. 2 Shrunk path of J-integral in the inclined strip yield model.

The path-independence of the J-integral permits us to choose the shrunk path ABC OC'B'A' shown with dashed line in Fig. 2 as our integral contour. The integration on the path AB and B'A' do not contribute to the value of J. Thus, on the net shrunk

path $\Gamma_1+\Gamma_2$ ($\Gamma_1=$BCO, $\Gamma_2=$OC$'$B$'$), the J-integral of eqn. (8) becomes, on account of the symmetry of the model,

$$J=2\int_{\Gamma_2}\left\{W\sin\theta-\sigma_{\theta\xi}\frac{\partial u_\xi}{\partial\xi}\frac{1}{\cos\theta}\right\}d\xi, \qquad (9)$$

where $\sigma_{\theta\xi}$ is the shear stress acting on the lattice dislocations at the points $\xi$ on the strip yield zone, and $\theta$ is the inclined angle of the slip plane. The shear stress $\sigma_{\theta\xi}$ is composed of all the interactive shear stresses due to crack and lattice dislocations and the resolved shear stress of the applied tensile stress. This is written as

$$\sigma_{\theta\xi}=\sigma_s+A\int_0^p\frac{g(\xi)d\xi}{\xi_1-\xi}+A\int_{-c}^c B_0^1(\xi_1,\zeta)f(\zeta)d\zeta$$

$$+A\sum_{i=2}^4\int_0^p B_i^1(\xi_1,\xi_i)g(\xi_i)d\xi_i. \qquad (10)$$

Since all the lattice dislocation are in equilibrium, eqn. (1b) gives

$$\sigma_{\theta\xi}=\sigma_R. \qquad (11)$$

Substituting eqn. (11) into eqn. (9) and considering the positive definiteness of the elastic strain energy density $W$, we obtain

$$J=-2\int_{\Gamma_2}\sigma_{\theta\xi}\frac{\partial u_\xi}{\partial\xi}\frac{1}{\cos\theta}d\xi$$

$$=-\frac{2}{\cos\theta}\left\{\int_0^p\sigma_R\frac{\partial u_\xi^-}{\partial\xi}d\xi+\int_p^0(-\sigma_R)\frac{\partial u_\xi^+}{\partial\xi}d\xi\right\}$$

$$=\frac{2}{\cos\theta}\int_0^p\sigma_R\frac{-\partial(2u_\xi^+)}{d\xi}$$

$$=\frac{2\sigma_R}{\cos\theta}\int_0^p bg(\xi)d\xi \qquad (12a)$$

$$=\frac{\delta\sigma_Y}{\sin 2\theta}, \qquad (12b)$$

where $u_\xi^+$ and $u_\xi^-$ represent the upper and lower displacement field of the strip yield zone, respectively, and Eqn. (7) and the following relation have been used:

$\sigma_R=\sigma_Y/2$ (Tresca material).

Since the total force, $F_1$, acting on all the lattice dislocations on one branch of the strip yield zones, is given by

$$F_1=\int_0^p b\sigma_{\theta\xi}\cdot g(\xi)d\xi$$

$$=\int_0^p b\cdot\sigma_R\cdot g(\xi)d\xi,$$

the $J$ given by eqns. (12) means the hypothetical force acting at the plastic crack tip in the direction of the crack plane and the resolved force of a half the $J$ on the slip plane is equal to the total force acting on all the lattice dislocations on one branch of the strip yield zones. This may be a more concrete or specific statement of the Eshelby force concept of the J-integral in the inclined strip yield model of the plastically relaxed crack.

## 6. Results and Discussion

The convergency of $a_1$ is shown in Fig. 3 for the four kinds of the forms of the crack-dislocation-density function. We can see that the minimum value of n may be fixed 12 for satisfactory convergence of $a_1$. The function using a Taylor series expansion gives poor convergence of $a_1$ for the lower applied loads ($\sigma_a/\sigma_Y=0.1$). The convergence of $a_1$ is poorer with an increase of the applied load
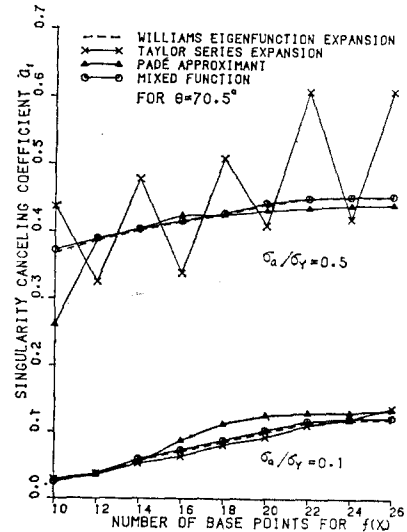


**Fig. 3** Convergency of the singularity cancelling coefficient.

and severe fluctuation of the value of $a_1$ occurrs for $\sigma_a/\sigma_Y = 0.5$. The three functions $f$ of eqns. (3-5) behave well. A fast convergence is achieved by the Padé approximant but has a defect that simultaneous nonlinear equations must be solved. Fig. 3 proves that the square root depenendence is the essential property in the density function. Fig. 4 shows that the dependence of $p$ on the applied
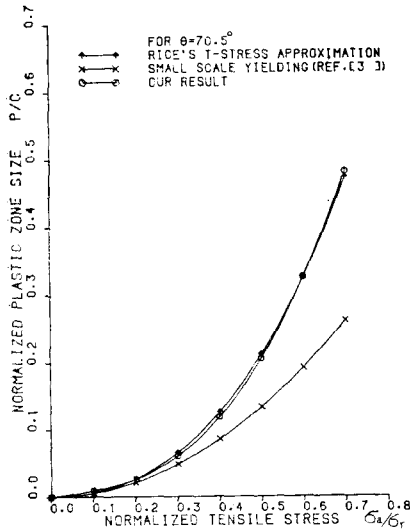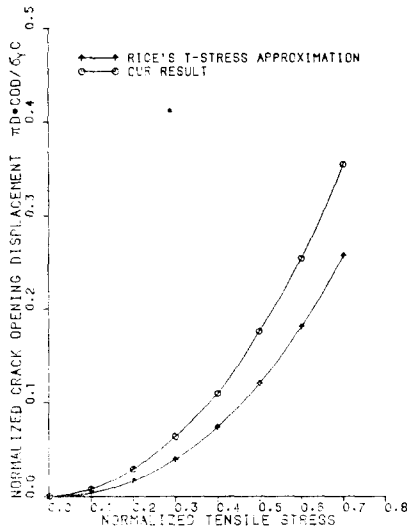


**Fig. 4** Plastic zone size versus applied tensile stress.



**Fig. 5** Crack tip opening displacement versus applied stress.
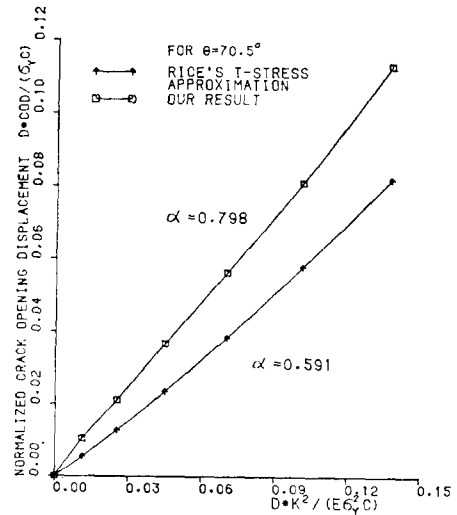


**Fig. 6** Normalized crack opening displacement to obtain the linearizing constant.

stress. Here it is noted that the plastic zone size of this method coincides well with the Rice's T-stress approximation[3]. As far as the plastic zone size is concerned, results of other investigators [4-5],[7-8] fit well with that of the Rice's T-stress approximation in the small scale yielding regime. Thus our result will further corroborate the generally accepted T-stress approximation. Fig. 5 shows the nondimensional COD versus the applied stress by eqn. (7) as well as Rice's T-stress approximation for a comparison. Another normalized plot for the COD shown in Fig. 6 is based on the formula

$$\delta = \alpha \frac{K^2}{E\sigma_Y} \qquad (13)$$

where $K$ is the stress intensity factor, $E$ the Young's modulus. Present method gives 0.8 and Rice's method[3] about 0.59 to the $\alpha$. Results of other investigators ranges from 1.159 to 0.425[2],[17].

Many investigators[12-16] used to present the relationship between the $J$ and COD of the form

$$J = m\sigma_Y \delta,$$

where most of the results on the proportional constant $m$ ranges from 1 to 2 for elastic perfect plastic materials. However, $m$ attains up to 3.5 if strain hardening effect is considered. Several experimental results on the $m$ are illustrated in the following. Robinson[12] obtained $m=1.0$ for a low strain hardening material, En 24 steel, and 2.6 for a high strain hardening material, En 32 steel. Hallstein and Blauel[13] reported $m=1.48\pm0.15$ for the reactor pressure vessel steel 22NMo-Cr37. Broek[14] found 2.2 using 7075-T6 and 7079 aluminum. Date, et al.[15] obtained 2.0 in low strength steels. Tracy[16] also found almost the same value 2.0 in his finite element solution.

In our inclined strip yield model, the proportional constant $m$ is evaluated as

$$m=\frac{1}{\sin2\theta},$$

thus it depends on the angle $\theta$ of the strip yield zones. Generally, the angle has been treated as a macroscopically averaged constant so that the strip yield zone can present a simple approximation to the diffused macroscopic plastic zone. Rice[3], and Atkinson and Kanninen[2] used the angle $\theta=70.5°$ which give the maximum shear stress on the inclined strip yield zone. Cherepanov[7] obtained $\theta=72°$, the angle for the maximum plastic zone size, in his solution of the strip yield model with the boundary value problem technique. Lo[8] defined the inclined angle for what minimizes the potential energy of the crack system, and obtained $\theta=75.1°$.

Thus, we see that the proportional constant, $m$, which relates $J$ with COD can be fixed within the range

$$1.589\leq m\leq2.01 \tag{14}$$

with the macroscopic inclined angle ranging

$$70.5°\leq\theta\leq75.1°.$$

eqn. (14) shows good agreement with the most of the experimental results mentioned above and with some numerical results[16]. The agreement suggest that the inclined strip yield model is a useful and simple model for the elastic-plastic tensile crack.

Substituting eqn.[13] into eqn. (12b) gves

$$J=1.43J_e \tag{15}$$

where $J_e$ is the J-integral of the elastic crack and $\nu=1/3$ has been used. eqn. (15) agrees qualitatively with the experimental result and theoretical estimation on the J-integral of the elastic-plastic crack performed by the EPRI ductile fracture analysis group[21]. eqn (15) shows that the initiation of the growth of the plastically relaxed crack will take place more easily than the pure brittle fracture instability. The problem of the semi-brittle fracture instability in the inclined-strip-yield continuum-dislocation model have been clearly studied by Lee and Chung[22].

## 7. Conclusions

**1)** The solution of the force equilibrium equations on the inclined-strip-yield continuum-dislocation model has been obtained by the aid of a singularity cancelling condition at the tip of elastic-plastic crack.

**2)** The singularity cancelling condition may be the condition for plastic relaxation of the crack tip if all the lattice dislocations emitted from the crack tip are mobile. This conclusion is based on the coincidence of the plastic zone size by this method with that of generally accepted Rice's T-stress approximation.

**3)** The proper crack-dislocation-density function has been obtained. The square root term is indispensible for the nonsingular part of the density function. Consequently,

this means that the square-root stress field is indispensible in considering the stress field of the plastically relaxed crack-tip region.

4) The COD varies lineary with $K^2/E\delta_Y$ even beyond the limit of the small scale yielding, that is,

$$\delta = 0.8K^2/E\delta_Y.$$

5) The J-integral on the shrunk path in the inclined-strip-yield continuum-dislocation model is given by

$$J = \frac{\delta\sigma_Y}{\sin 2\theta},$$

where the inclined angle, $\theta$, is the macroscopic, constant angle ranging

$$70.5° \leq \theta \leq 75.1°$$

This $J$ shows good agreement with the most of experimental results.

6) Our result on the J-integral on the shrunk path gives a concrete statement of the Eshelby force concept of the J-integral in the inclined strip yield model: $J$ is the hypothetical force acting at the plastic crack tip in the direction of the crack plane and the resolved force of a half the $J$ on the slip plane (strip yield zone) is equal to the total force acting at the all the lattice dislocations on one branch of the slip planes.

## References

1. D.S. Dugdale; Yielding of Steel Sheets Containing Slits, Journal of the Mechanics and Physics of Solids, Vol. 8, pp. 100-104, 1960.

2. C. Atkinson and M.F. Kanninen; A Simple Representation of Crack Tip Plasticity, Acta Metallurgica, Vol. 19, pp. 679-683, 1971.

3. J. R. Rice; Limitation to the Small Scale Yielding Approximations for Crack Tip Plasticity, Journal of the Mechanics and Physics of Solids, Vol. 22, pp. 17-26, 1974.

4. V. Vitak; Yielding on Inclined Planes at the Tip of a Crack loaded in Uniform Tension,

Journal of the Mechanics and Physics of Solids, Vol. 24, pp. 263-275, 1976.

5. H. Riedel; Plastic yielding on Inclined Slip-Planes at a Crack Tip, Journal of the Mechanics and Physics of Solids, Vol. 24, pp. 277-289, 1976.

6. B.A. Bilby and K. H. Swinden; Representation of Plasticity at Notches by Linear Dislocation Arrays, Proceedings of the Royal Society of London, Vol. A285, pp. 22-33, 1965.

7. G.P. Cherepanov; Plastic Rupture Lines at Tip of a Crack, P.M.M., Vol. 40, pp. 720-728, 1976.

8. K.K. Lo; Modelling of Plastic Yielding at a Crack Tip by Inclined Slip Planes, International Journal of Fracture, Vol. 15, pp. 583-589, 1979.

9. B.A. Bilby, A.H. Cottrell and K.H. Swinden; The Spread of Plastic Yield from a Notch, Proceedings of the Royal Society of London, Vol. A272, pp. 304-314, 1963.

10. J.R. Rice; in Fracture, Vol. 2, Ed. H. Liebowitz, Academic Press, pp. 191-311, 1968.

11. J.D. Eshelby; The Continuum Theory of Lattice Defects, Solid State Physics, Vol. 3, Eds. F. Seitz and D. Turnbull, pp. 79-144, 1956.

12. J.N. Robinson; An Experimental Investigation of the Effect of Specimen Type on the Crack Tip Opening Displacement and J-integral Fracture Criteria, International Journal of Fracture, Vol. 12, pp. 723-737, 1976.

13. T. Hallstein and J.G. Blauel; On the Relation of the Crack Opening Displacement to the J-integral, International Journal of Fracture, Vol. 13, pp. R385-390, 1977.

14. D. Broek; Correction between Fracture Toughness and Stretched Zone Size, ICF3, Munich, Germany, 1973.

15. K. Date, et al.; Crack Tip Blunting, the J-integral and Cleavage Fracture Toughness in Low Strength Steels, Tohoku Univercity, x-876-77, 1977.

16. D.M. Tracy; Finite Element Solutions for Crack Tip Behavior in small Scale Yielding,

Journal of Engineering Materials and Technology, Vol. 98, pp.146-151, 1976.

17. S.K. Chung; A Dislocation Theory of Semi-Brittle Fracture, Ph. D. Dissertation, Korea Advanced Institute of Science, 1981.

18. A.K. Head and N. Louat, The Distribution in Linear Arrays, Australian Journal of Physics, Vol. 8, pp.1-7, 1955.

19. J.R. Rice and R. Thomson; Ductile versus Brittle Behavior of Crystals, Philosophical Magazine, Vol. 29, pp.73-97.

20. T. Yokobori and A. Kamei; A Criterion for Low-Stress Brittle Fracture of Materials with Crack or Sharp Notch Based on Combined Micro-And-Macro Fracture Mechanics, Engineering Fracture Mechanics, Vol. 5, pp.819-836, 1973.

21. T.U. Marston; The EPRI Fracture Research Program, unpublished work, Electric Power Research Institute, Palo Alto, Ca. U.S.A.

22. B.H. Lee and S.K. Chung; Semi-Brittle Fracture Instability in the Inclined-Strip-Yield Continuum-Dislocation Model, Material Science and Engineering, to be published, 1981.

23. G. A. Baker, Jr.; Padé Approximant in Theoretical Physics, Academic Press, London, 1970.