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TWO COMPONENT MODEL OF INITIAL MASS FUNCTION*

S. S. HONG
Department of Astronomy, Seoul National University
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ABSTRACT

Weibull analyses given to the initial mass function (IMF) deduced by Miller and Scalo(1979) have shown that the mass dependence of IMF is an $\exp[-\alpha m]$- form in low mass range while in the high mass range it assumes an $\exp[-\alpha\sqrt{m}]/\sqrt{m}$-form with the break-up being at about the solar mass. Various astrophysical reasonings are given for identifying the $\exp[-\alpha m]$ and $\exp[-\alpha\sqrt{m}]/\sqrt{m}$ with halo and disk star characteristics, respectively. The physical conditions during the halo formation were such that low mass stars were preferentially formed and those in the disk high mass stars favoured. The two component nature of IMF is in general accord with the dichotomies in various stellar properties.

I. INTRODUCTION

It is of fundamental importance to know the frequency distribution of stellar masses at birth (initial mass function or IMF) for understanding the Hubble sequence(Lynden-Bell, 1977) and the star formation processes as well. In particular, the IMF plays an important role in the chemical evolution of a galaxy by regulating yields of heavy elements. In this paper, we shall analyze the IMF deduced by Miller and Scalo(1979), and retrieve some of the imprints left from the early period of our Galactic evolution.

Numerous attempts to construct IMF's, in one form or another, from the present-day luminosity function have been plagued by observational, theoretical and conceptual uncertainties. Recently, Miller and Scalo(1979) examined very thoroughly all these difficulties and derived an IMF assuming its time-independency over the history of the disk. They consider their IMF characterizes the disk only, as will be clear shortly, it, however, should contain halo information, too. The time independency assumption may well be near the reality after the disk formation, but it can not be extended to the halo formation period. Hence, for our analysis, we will adopt their IMF as an "illdefined" nevertheless meaningful average.

Some of the present-day disk stars must have been originated from the halo, particularly those low mass stars whose main-sequence life times are longer than the disk age. Furthermore, one usually integrate the luminosity function perpendicular to the Galactic plane by multiplying two times the scale height $H(M_v)$ under the assumption of exponential distribution of stars for a given luminosity M_v. Thus, not only disk stars of halo origin but also stars currently located at the halo are included in the derivation of IMF. (See Miller and Scalo(1979) for details.)

Now, physical conditions at the time of halo star formation are likely to be quite different from those of disk star formation. Hence, we do expect to have two different IMF's depending on the mass range. The purpose of this paper is to show that the IMF derived from field stars in the solar neighborhood is indeed of such a two component nature. In section II we shall perform a Weibull analysis on the IMF to show that the IMF is at least a two-process product. In section III we shall examine implications of the two-component nature. In section IV we shall give some discussions and summarize our results.

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II. WEIBULL ANALYSIS OF THE INITIAL MASS FUNCTION

a) Weibull Statistics

Following Miller and Scalo, we denote by $m\psi(m)dm$ the stellar mass formed in unit time within the mass range between m and $m+dm$. Then, $W(m\rightarrow m_\infty)$ defined by

$$W(m\rightarrow m_\infty)=\int_m^{m_\infty} m\psi(m)\,dm \Big/ \int_{m_0}^{m_\infty} m\psi(m)\,dm \tag{1}$$

is the accumulated fractional mass distribution. Here, m_0 and m_∞ are minimum and maximum stellar masses, respectively. If the IMF follows a Weibull distribution, $W(m\rightarrow m_\infty)$ is given by a Weibull function

$$W(m\rightarrow m_\infty)=\exp[-\alpha(m-m_0)^\gamma], \tag{2}$$

where α, γ and m_0 are called scale parameter, shape parameter and location parameter (Johnson and Kotz, 1970). Weibull representation is a three-parameter fitting to the data. Taking logarithm twice to equation (2), we have

$$\ln\,\ln W^{-1}(m\rightarrow m_\infty)=\gamma\,\ln(m-m_0)+\ln\,\alpha. \tag{3}$$

Hence, the data plot as a straight line on the (ln ln−ln) plane, and one can easily determine the Weibull parameters from the Weibull plot. Then, the IMF $m\psi(m)$ is a simple differentiation of $W(m\rightarrow m_\infty)$ with respect to m.

Weibull statistics has a number of nice points (Johnson and Kotz, 1970): (1) It is a skewed distribution, ideal for the shape of IMF. (2) It is much more flexible than simple exponential or normal distribution. (3) It has a wide applicability in natural phenomena, particularly, for those systems whose components are interacting each other. (4) It is ideal for detecting mixed distributions. It is the fourth point that really counts in our analysis. If the data are products of two different processes, their Weibull plot becomes two straight lines (Berrettoni, 1964). Thus, the Weibull analysis is very effective in showing heterogeneous distribution. This is the reason why we employ the Weibull test in our analysis.

b) Stellar Mass Limits

As long as the normalization integral in the denominator of equation (1) covers the observed maximum mass, $\sim 65\,m_\odot$, in the IMF, detail value of m_∞ doesn't matter at all. This results from the fact that the IMF in the high mass end is a steeply declining function of mass and m_∞ enters in the analysis only as a upper limit of the normalization integral.

However, we face an acute difficulty for uncertainties in the minimum stellar mass m_0, since it enters in the analysis at two places; as a location parameter as given in equation (2) and as a lower integral limit. The observed value of $0.1\,m_\odot$ in the IMF (Miller and Scalo, 1979) is only a upper bound to the real minimum stellar mass, as we know two stars of $0.06\,m_\odot$ (Lippincott and Hershey, 1972; Heintz, 1972). Although we don't have a clear consensus on m_0, as discussed by Scalo (1978), various theoretical considerations put the minimum mass at about $0.01\,m_\odot$. Thus a non-negligible amount of unobserved mass is expected to be in the range below $0.1\,m_\odot$. We will take into account for this undetected stellar mass by introducing r defined by

$$r\equiv\int_{m_0}^{m_\infty} m\psi(m)\,dm \Big/ \int_{0.1}^{m_\infty} m\psi(m)\,dm, \tag{4}$$

which should be larger than unity.

c) Weibull Test of Two-Component Idea

With a chosen value of r and Miller and Scalo's IMF, we can evaluate, from equation (1), the accumulated fractional mass distribution $W(m\rightarrow m_\infty)$ for $m>0.1\,m_\odot$. Now, we find the location parameter m_0 which corresponds to the chosen r in such a way that the derived accumulated mass distribution plots as two straight lines on the plane formed by

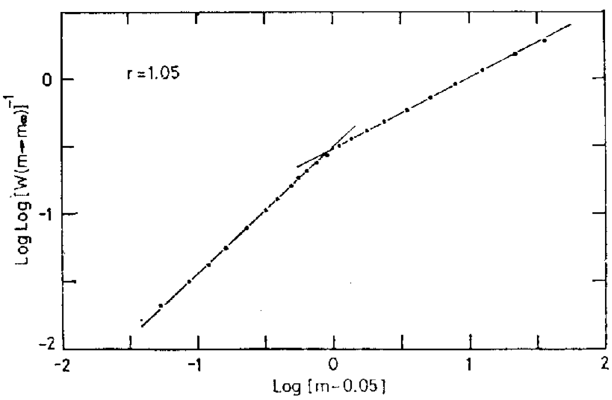

Fig. 1. One example of the Weibull plot with 5% mass correction for undetected stars between 0.1 and 0.05 $m_\odot$ mass range. For the location parameter 0.05 $m_\odot$ is chosen. Note the two straight line fit to the data and the slopes of the two lines.

Table 1. Examples of Graphical Tests

unseen mass correction	r	1.038	1.050	1.065
locationparameter	m_0	$0.06 m_\odot$	$0.05 m_\odot$	$0.03 m_\odot$

$\ln \ln W^{-1}(m \to m_\infty)$ and $\ln(m-m_0)$.
We have performed such a graphical test with a number of different trial values of r, and found corresponding loation parameter which gives two straight line fit to the data. Table 1 summarizes the results, and one example of the Weibull plot is given in Fig. 1.
From the table, we note that a decrease in the minimum mass m_0 naturally accompanies with an increase in the undetected mass fraction. It should be pointed out, however, that the undetected fraction doesn't seem to increase indefinitely as m_0 becomes small. Our experience based on many graphical tests places a limit on r at about 1.10; the undetected mass can not exceed 10% of the observed mass. As shown in Fig. 2, an over-correction for the unseen mass by some 15% makes the curve curving up at the low mass end. At $r=1.10$, m_0 becomes negligibly small in comparison to the observed minimum value of $0.1\, m_\odot$ in the IMF. Because there are no data available for $m<0.1\, m_\odot$ and the real minimum stellar mas is negligible to $0.1\, m_\odot$, we approximate the location parameter m_0 to zero in our further analysis.

In Fig. 2 we have shown three cases of unseen mass correction: $r=1.06$, 1.085 and 1.15. The correction for the undetected mass doesn't change the accumulated mass distribution at all at the high mass end. Under-correction makes the low-mass points curving down; while over-correction makes them curving up. Further more, improper correction for the unseen mass gives poor fit to the two straight line, particularly, at their intersecting region. Thus, the Weibull test as shown in Fig. 2 gives us confidence on the two-component idea of the initial mass function. We may conclude that the IMF of the field stars in the solar neighborhood is a two-process product.

d) Weibull Parameters of the IMF

In Table 2 we have listed Weibull parameters obtained by the least square method for three cases of the unseen mass correction. The location parameter m_0 is kept at zero. As stated before, $r>1.10$ cases do not give a good straight-line Weibull plot with $m_0=0$. So do the cases of $r<1.07$ with $m_0=0$. We think $r=1.08$ is a reasonable choice for the IMF. Considering many uncertainties involved in the derivation of the IMF, we adopt 1 and 1/2 for the shape parameter in the low and high mass range, respectively.

Noting $\alpha^{-1} \simeq 1.4$ is a natural mass unit in the low mass range and $\alpha^{-2} \simeq 2$ in the high mass range, we have following Weibull representations of the IMF:

$$W_l(m \to m_\infty) = \exp[-m/1.4], \tag{5a}$$

and

$$W_h(m \to m_\infty) = \exp[-\sqrt{m/2}], \tag{5b}$$

where the mass is in units of the solar mass. It is interesting to note that the natural unit mass in the low mass range is smaller than the one in the high range. Furthermore, both natural unit masses are sufficiently larger than the expected minimum stellar mass $\sim 0.01\, m_\odot$ that $m-m_0$ can be safely equated to simply m. This justifies our choice of $m_0 \simeq 0$. Although the boundary between the two mass ranges is likely to be at about one solar mass, it is admittedly uncertain. From the Weibull analysis alone, it is not possible to locate the mass boundary accurately. We can say only that the low mass IMF has an upper mass cut-off and the high mass IMF does a lower mass cut-off. Thus, in the mass range between ~ 1 and $\sim 2\, m_\odot$ both IMF are comparable each other. (Further discussions are given in the last section.)

From the Weibull representation of mass distribution, one can construct corresponding IMF's by a simple differentiation. Ignoring the normalization constants we have

Table 2. Weibull Parameters of the IMF

	$r=1.07$		$r=1.08$		$r=1.09$	
	γ	α	γ	α	γ	α
low mass range	1.052	0.720	1.004	0.718	0.961	0.718
high mass range	0.526	0.703	0.523	0.712	0.520	0.720

$$m\psi_H(m) \propto \exp[-\alpha m], \quad (6a)$$

and

$$m\psi_D(m) \propto \exp[-\alpha\sqrt{m}]/\sqrt{m};\ m > \sim 1 \quad (6b)$$

Here we give the same scale parameter α, as we are interested in the mass-dependency only. The subscripts H and D denote the halo and disk, respectively. The reason for H and D notation will be clear shortly.

III. IMPLICATIONS OF THE TWO COMPONENT IMF

a) Halo IMF and Disk IMF

Now, we ask ourselves what causes this dichotomy in the IMF of field stars in the solar neighborhood. We can have the following two different points of view: First, during the whole history of the disk two distinctly different mechanisms have operated for star formation and the field stars are all of disk origin. Second, some of the field stars sampled in the derivation of the IMF are of halo origin, and the physical conditions in the halo were different from those in the disk in such a way that less massive stars were preferentially formed in the halo while massive ones in the disk. In the second view, the star formation mechanism itself may have been the same for both the halo and disk; only environmental conditions control the frequency distribution of stellar masses at birth.

At the present stage, it is difficult to critically assess these two alternatives. Instead we go with the second point of view by simpy giving a few supporting evidences for it: (1) From considerations of main sequence life times of low mass stars we know that the field stars gone into the analysis of the IMF contain stars of halo origin. Furthermore, the method employed in the derivation of the IMF implicitly brings halo stars in the sample. (2) Known stellar birth sites, open clusters, OB, T and R associations, are not sufficient enough to account for all the gas consumption rate, if associations are deficient in low mass stars (Miller and Scalo, 1978; von Hoerner, 1968). An additional birth place is required especially for the majority of low mass stars. (3) There are many clusters whose mass spectra are deficient in the mass range below $\sim 1\ m_{\odot}$ (Yu and Yun, 1981; Taff, 1974; van den Bergh, 1961), which is incompatible with the derived flat IMF below $\sim 1\ m_{\odot}$. (4) A single coagulation mechanism can produce the two different mass spectra depending on the mass-dependency of coalescence rate (Silk and Takahashi, 1979; Nakano, 1966). According to Silk and Takahashi (1979), $\exp[-\alpha m]$-type results from mass-independent coalescence rate ($\lambda=0$ in their notation), while $\exp[-\alpha\sqrt{m}]/\sqrt{m}$-type from linear-mass dependent rate ($\lambda=1$). Thus, it is natural to associate the low mass IMF with the halo characteristics and the high mass IMF with the disk characteristics. From now on, we shall call $\exp[-\alpha m]$ the halo IMF and $\exp[-\alpha\sqrt{m}]/\sqrt{m}$ the disk IMF.

b) Dichotomies in Stellar Properties

Comparison between halo IMF and disk IMF given in equation (6) shows that the contribution from the halo is no longer negligibe in number wise for stars later than mid F-type whose masses are less than about $1/\alpha \simeq 1.4\ m_{\odot}$. Thus, late type field stars are composed of "genetically" two different subgroups. In this point of view, the following dichotomies in stellar properties find natural causes: (1) a sudden decrease in rotational velocity for stars later than $\sim$F4, (2) a substantial increase in velocity dispersion for stars later than $\sim$F4, and (3) a marked increase in scale height of stellar distribution above the Galactic plane for stars later than $\sim$F6. This type of explanation for the dichotomy suggests that detail studies on the distributions of rotation velocity and velocity dispersion within a narrow range of late spectral types will reveal some sort of bimodal nature in the distribution. This will help us fix the normalization constants in halo and disk IMF's or total stellar birth rates in halo and disk.

IV. DISCUSSIONS AND CONCLUSIONS

At the high mass limit, both the halo and disk IMF's have natural cut-offs. On the other hand at the low mass limit, the disk IMF rapidly grows due to its $1/\sqrt{m}$-term. Although the disk IMF yields still a finite total mass, we may require a modification to equation (6b) in order to facilitate a cut-off of the disk component at the mass limit. It is unfortunate that in cases of mixed distributions the Weibull analysis alone doesn't tell us whether such a cut-off is necessary or not for the given case. However, observations often show that mass spectra of open clusters are deficient in low mass stars. Hence, we probably need some sort of low-mass cut-off in the disk IMF. This is the reason why we introduced a limit to the mass range as in equation (6b).

We have based our analysis solely on the IMF that Miller and Scalo (1979) derived under the assumption of constant total stellar birth rate and the disk age of 1.2×10^{10} years. They derived IMF's for the cases of maximum decreasing and maximum increasing birth rate, too. The low-mass cut-off in the disk IMF is definitely needed for the case of maximum decreasing birth rate; while it is not for maximum increasing case. Thus, answer as to the low-mass cut-off should wait until further observations become available, especially, for $m<0.1\ m_{\odot}$.

We do not expect that the time-dependency of stellar birth rate and the disk age would change the mass-dependency in the halo IMF, $\exp[-\alpha m]$, and in the disk IMF, $\exp[-\alpha\sqrt{m}]/\sqrt{m}$. The scale parameter will be somewhat changed depending on the birth rate and the disk age, so will the location of intersecting point of the two straight lines of the Weibull plot seen in Fig. 2. However, detail studies on the effects of stellar birth rate and the disk age deserve further considerations.

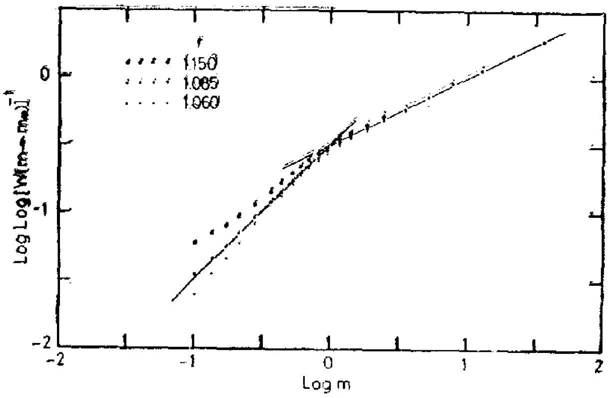

Fig. 2. Weibull tests with varying amount of undetected mass correction. The location parameter is assumed to be small in comparison to $0.1\ m_{\odot}$. Please note the effect of over- and under-correction for the unseen stellar mass. The two lines have slopes 1 and 1/2, respectively.

We now summarize our results from this work. Two-component nature of the initial mass function has been demonstrated by Weibull analysis. Identification of the two components with halo and disk characteristics offers a natural reason for the dichotomies in various stellar properties. The difference in mass-dependency between the halo IMF and disk IMF is interpreted as a consequence of the differences in physical conditions during the period of halo star formation and that of disk star formation.

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REFERENCES

Berretoni, J. N. 1964, *Industrial Quality Control*, Aug., 71.

Heintz, W. D. 1972, A. J. **77**, 160.

Johnson, N. L. and Kotz, S. 1970, *Distributions in Statistics, Continuous Univariate Distributions*-I, p. 250.

Lippincott, S. L. and Hershey, J. L. 1972, A. J., **77**, 679.

Lynden-Bell, D. 1977, in *Star Formation*, ed by T. de Jong and A. Maeder (Reidel: Holland), p. 291.

Miller, G. E. and Scalo, J. M. 1978, P.A.S.P., **90**, 508.

Miller, G. E. and Scalo, J. M. 1979, Ap. J. Suppl., **41**, 513.

Nakano, T. 1966, Progr. Theoret. Phys., **36**, 515.

Scalo, J. M. 1978, in *Protostars and Planets*, ed by T. Gehrels (Univ. Arizona Press: Tucson), p. 265.

Silk, J. and Takahashi, T. 1979, Ap. J., **229**, 242.

Taff, L. G. 1974, A. J., **79**, 1280.

van den Bergh, S. 1961, Ap. J., **134**, 554.

von Hoerner, S. 1968, in *Interstellar Ionized Hydrogen*, ed by Y. Terzian (Benjamin: New York), p. 101.

Yu, K. L. and Yun, H. S. 1981, Presprint.