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## Multiobjective Decision Model with Consideration of Flexibility in Sequential Capital Budgeting

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### ABSTRACT

This paper explores a rational investment decision model in sequential capital allocation process under capital rationing. A method is proposed for measuring the new investment decision factor which is the flexibility that describes the future availability of invested funds. This flexibility is important in sequential decision process. Also presented is a multiobjective (MO) decision model into which flexibility is incorporated with the profit and risk factors. The effectiveness of this criterion is compared with the expected present value and the mean-semivariance criteria through a simulation model.

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## I . Introduction

The concept of uncertainty resolution and the methods to measure uncertainty resolution in quantitative fashion have been discussed by several authors [3, 12, 13, 15] . However, they did not express effectively the meaning of the uncertainty faced by an investor. That is, the investor's uncertainty primarily lies in the fact that how uncertainty associated with recovering the initial investment cost by the payback period is expected to be resolved through time under uncertain future. After the payback period, he returns to his original position. The investment worth of a project after that period can be evaluated through the profit measure. Therefore, the consideration of uncertainty over the entire project life does not give any significant insight into the project evaluation in sequential decision process. In this paper, the investor's uncertainty associated with recovering the initial outlay is measured in term of the concept of flexibility which describes the future availability of invested funds.

Relatively little attention has been paid on the concept of flexibility in the field of sequential capital allocation process. In particular, few models except the project balance criterion [12] in the literature have regarded the concept of flexibility as a decision factor distinct from the profit and the risk factors for evaluating a proposal.

Therefore, in addition to the profit and the risk measures accepted well in the traditional investment decision models, the idea of

flexibility is included as the major investment decision factor which affects investment decision in sequential capital budgeting process. The resulting investment model is referred to as multi-objective (MO) decision model.

## II. Factors to be Considered in Sequential Capital Rationing Decisions

### A. Expected Present Value as a Measure of Profitability

The expected return of a project is measured by the expected present value ( $E \{ PV \}$ ) of the project.

$$(1) \quad E \{ PV \} = \sum_{n=1}^N \int_{-\infty}^{\infty} x f(n, x) dx / (1+i)^n$$

where

$x$  = Random cash flow

$f(n, x)$  = Probability density function (or probability mass function) of  $x$  at the end of period  $n$

$i$  = Risk free discount rate

$N$  = Life of a project

To eliminate the effect of the size of the initial cost ( $C$ ) from  $E \{ PV \}$ , the profitability index (PI) of a project is defined as

$$(2) \quad PI = E \{ PV \} / C$$

### B. Semivariance as a Measure of Risk (Variability)

The variability associated with the cash flows of a project is measured in terms of the semivariance (SV) from  $h$  and expressed by

$$(3) \quad SV_n = \sum_{k=1}^M (x_{kn} - h_n)^2 p(x_{kn})$$

where

$h_n$  = A target cash flow at period  $n$  (normally  $h_n = E \{X\}_n$  or 0)

$M$  = Number of observations to the left of the  $h_n$

$P(x_{kn})$  = Probability occurring  $x_k$  at period  $n$

the other terms are as defined above

That is, semivariance is the variance of the probability distribution to the left of a target value and may be thought to represent a measure of downside risk. The conceptual superiority of using semivariance as a measure of risk was discussed in [8, 9] .

If the relationship between any two cash flows in the cash flow streams is assumed to be completely independent, the semivariance about the  $E \{PV\}$  for  $n$  periods is

$$(4) \quad SV = \sum_{n=0}^N SV_n / (1+i)^{2n}$$

To normalize the semivariance, risk index (RI) of a project is defined as

$$(5) \quad RI = SV / E \{PV\}$$

### C. Flexibility as a Measure of Availability of Invested Funds

When selecting a set of proposals with uncertain returns over sequential investment horizon, in each investment decision we should consider the information which reflects the variability effect of the probabilistic cash flows of proposals for the ultimate accumulation

of wealth. In particular, when investment decisions are made on sequential basis with funds generated internally under capital rationing, it is necessary to recognize that the manner in which funds become available prior to the payback period of each proposal has an effect on the future investment activities of a firm. Obtaining the information on the availability of invested funds, flexibility, is extremely valuable to a decision maker for making subsequent investment decisions, because the decision maker can consider and evaluate the chances of reinvesting funds recovered from current investment in more profitable alternatives which may arise in the near future.

In this paper, the flexibility measure for a project is proposed by utilizing the concept of negative project balance (NPB) developed recently by Park and Thuesen [12], but by not following their approach. Their approach is expressed by

$$(6) \quad FI = AUR / C = \left( \sum_{t=0}^N EGCL [ANB]_t \right) / C$$

$$(7) \quad EGCL [ANB]_t = E [ANB]_t + \delta \sigma [ANB]_t$$

where

FI = flexibility index

AUR = Area of uncertainty resolution

EGCL = The expected gain confidence limit criterion [1]

ANB = Area of NPB

C = Initial cost of a project

$\sigma [ANB]_t$  = Conditional standard deviation about the expected NPB at time t

$E \{ANB\}_t$  = Conditional expected NPB at time  $t$

$\delta$  = Coefficient of risk aversion

$EGCL\{ANB\}_t$  = Resolution index for NPB at time  $t$ .

However, in view of the fact that flexibility measure represents the degree of funds available at each point in time before the payback period, the use of calculation procedures to measure flexibility would reveal the following problems [11] :

- 1) In the methodology, the NPB is counted twice. In other words, the NPB at each node in probability tree is added to make ANB of each brance tip and the value of  $\{ANB\}_t$  is added to make AUR. Therefore, their computation procedures would tend to obscure the concept of NPB.
- 2) To compute the resolution index ( $EGCL \{ANB\}_t$ ) for the area of NPB as seen in equation (7), they added some standard deviations of  $\{ANB\}_t$  to  $E \{ANB\}_t$  to penalize the variability in  $\{ANB\}_t$ , as though they were measuring risk. In addition, they counted twice, in evaluating a project, the effect of the variability of random cash flow in measuring the flexibility and the risk of the project.
- 3) The value of  $EGCL \{ANB\}_t$  never decrease to zero by the end of a project's life as well as the payback period and do not directly express the funds available at each point in time. Generally, the initial cost is fully recovered by the payback period of a project and a firm can use at least the initial funds at that period.  $EGCL \{ANB\}_t$  as a measure of flexibility can not explain

this point.

- 4) By their method, the magnitude of  $EGCL (ANB)_0$  is always greater than the initial cost of a project. In other words, their approach fails to recognize that the magnitude of uncertainty about recovering the initial cost at the time, when the project is proposed, should be the same as the initial outlay.

In summary, their computation procedures do not provide the means to express exactly the investor's uncertainty in quantitative terms.

However, except the problems mentioned above, the concept of NPB is more appealing as a basis of flexibility measure than any other concept, i.e., the concept of payback period and unrecovered project balance, since the NPB at each point in time provides the information on the shape of the time-dependent cash flow pattern and also provides the information regarding the rate at which the uncertainty about the recovery of the initial cost is resolved through time. This information is directly used to estimate the availability of invested funds prior to payback period. Therefore, as a basis of flexibility measure, the magnitude of the NPB at each period would be a prime concern to a decision maker.

Flexibility measure of a project is computed as the following procedures :

First, the expected NPB ( $E [NPB]$ ) at each point in time is calculated by the following recursive equation (8) :

$$NPB_0 = F_0$$

$$(8) \quad E [NPB]_n = (1+i) E [NPB]_{n-1} + \int_{-\infty}^{\infty} x f(n, x) dx, \quad n=1,2,\dots,Q-1$$

where

$F_0$  = Initial cost of a project

$Q$  = Discounted payback period which is computed by the expected cash flows

Next, to reduce the multiperiod concept of  $E [NPB]_n$  to a single value, the area under the time profile of  $E [NPB]_n$ , i.e., the total uncertainty resolution for NPB by the payback period is calculated.

This value is measured by the area of  $E [NPB]_n$  (ANPB).

$$(9) \quad ANPB = \sum_{n=0}^{Q-1} E [NPB]_n$$

Finally, to eliminate the effect of the size of the initial cost from the value of ANPB, it is necessary to normalize ANPB by dividing through the initial outlay (C). This gives

$$ANPB / C$$

the flexibility per dollar invested. In this paper, this statistic is defined as the flexibility index (FI).

Thus, if FI value of a project is relatively small, the future availability of invested funds prior to payback period is more flexible. Consequently, the firm's future investment activities before the payback period will be more active. Conversely, if this value is relatively large, they are less flexible and less active.



### III. Multiobjective Decision Model

In paper [12], the single decision criterion which was incorporated all the investment decision factors discussed in the previous section was developed for comparing proposals. However, the difficulty in establishing this index usually stems from the fact that obscure trade-offs are required. In fact, a decision maker is able to tolerate greater variability about the proposal's worth if the expected return is great enough. Or the decision maker might wish to select a current set of proposals with a allowable expected return, but a greater flexibility in order to take advantages of catching profitable investment opportunities which would arise in the future. Also, since three major decision factors are expressed in different measure of effectiveness, a goal programming formulation appears to be more realistic model of the capital budgeting problem involving multiple and conflicting objectives. Such objectives might include:

Goal 1 : Maximize the flexibility (Maintain a given flexibility level)

Goal 2 : Maximize the profitability (Maintain a given profit level)

Goal 3 : Minimize the variability (Maintain a given risk posture)

Now, if one can establish preemptive priority for all or some of these goals, then this problem is of a class known as goal programs.

The resulting model is referred to as the multiobjective (MO) decision model that a decision maker wishes to optimize.

### A . Model Formulation

In this study, we consider a budget constraint at each decision period and three objectives which a decision maker wish to optimize in a capital allocation decision process. If a decision maker has the above preemptive priority for these objectives, the problem of selecting the combination of proposals which satisfy the multiple objectives can be formulated as the following zero-one goal programming problem at each decision period :

$$(10) \text{ Minimize } \bar{a} = \{g_1(p_1), g_2(n_2, p_2), g_3(n_3, p_3), g_4(n_4, p_4)\}$$

Subject to

$$(11) \sum_{j=1}^N (BC)_j x_j + n_1 - p_1 = RBC$$

$$(12) \sum_{j=1}^N (FI)_j x_j + n_2 - p_2 = PIL$$

$$(13) \sum_{j=1}^N (PI)_j x_j + n_3 - p_3 = PIL$$

$$(14) \sum_{j=1}^N (RI)_j x_j + n_4 - p_4 = RIL$$

$$n_i, p_i \geq 0 \quad , \quad n_i \cdot p_i = 0 \quad , \quad i = 0, 1, 2, 3, 4$$

$$x_j = 1 \text{ or } 0, \text{ for all } j$$

where

$\bar{a}$  = Achievement function : a row vector measure of the attainment of the objectives or constraints at each priority level

$n_i$  = Negative deviation from goal  $i$

$p_i$  = Positive deviation from goal  $i$

$g_i(n_i, p_i)$  = Function (normally linear) of the deviation variables associated with the objectives or constraints at each priority level  $i$

$x_j$  =  $j$ -th decision variable  $\begin{cases} 0, & \text{if proposal } j \text{ is not selected} \\ 1, & \text{if proposal } j \text{ is selected} \end{cases}$

$(BC)_j$  = Initial cost of proposal  $j$

RBC = Amounts of funds available at each decision period

$(FI)_j$  = Flexibility index of proposal  $j$

FIL = Allowed level of flexibility indicated by a decision maker as expressed in FI

$(PI)_j$  = Profitability index of proposal  $j$

PIL = Desired level of profit indicated by a decision maker as expressed in PI

$(RI)_j$  = Risk index of proposal  $j$

RIL = Risk posture indicated by a decision maker as expressed in RI

$N$  = Total number of proposals

The above linear zero-one goal programming problem can be solved by way of applying a general, sequential goal programming algorithm developed by Ignizio [5, 6].

Depending upon the perceived importance, the total number of the different classes of preemptive priority levels will be six combinations for three objectives. According to the decision environment at each period in sequential capital budgeting process, the preemptive priorities for these goals would be determined adaptably by a decision maker to make optimal decision.

**B . Decision Models to be Compared with MO Decision Model**

To compare with the MO criterion in sequential capital budgeting process under capital rationing, two of the most widely suggested decision criteria in the literature are selected [2] . They are the expected PV criterion and the mean-semivariance (M-SV) criterion.

Under the expected PV criterion, a decision maker is assumed to be risk-indifferent such that his problem is to select the feasible solution vector  $(x_1, x_2, \dots, x_n)$  having the largest expected PV without violating the budget constraint. The vector is obtainable by solving the zero-one integer programming problem.

(15) Maximize  $z = \sum_j E [PV]_j X_j$

(16) Subject to  $\sum_j BC_j x_j \leq RBC$

$x_j = 1$  or  $0$ , for all  $j$

M-SV criterion [8, 9] utilized in this study, if all the investment proposals are mutually independent (co-semivariance=0), can be written

(17) Maximize :  $\mu - \lambda (SV) = \sum_j E_j x_j - \lambda \sum SV_j x_j$

(18) Subject to  $\sum_j BC_j x_j \leq RBC$

$x_j = 1$  or  $0$ , for all  $j$

where

$\mu$  = Expected PV of the accepted set of proposals

SV = Semivariance of PV of the accepted set of proposals

$\lambda$  = Coefficient of risk aversion

$E_j$  = Expected PV of proposal  $j$

$SV_j$  = Semivariance of proposal  $j$

which can be solved as a zero-one integer programming.

#### IV. Simulation Model

To test the effectiveness of the MO criterion with the expected PV and the M-SV criteria, simulation model is developed. The basis for comparing the relative effectiveness of the different criteria is their ability to maximize the total accumulation of a firm's capital with limitation on funds available for investment over periodic decision process. In this study, it is assumed that a decision maker is aware of all proposals to be considered at the current decision period, but no precise knowledge of the investment proposals that will be submitted for consideration in the future period in sequential decision process. The funds available at each decision period are all optimally allocated within the current proposals according to the decision criteria tested. Therefore, the postponement of investment is not allowed.

In the simulation, in order to limit the numerical calculation without affecting the simulation purpose, the study period is set  $H=20$ , and the number of decisions are at least four times with the average life of proposals of 15 year. That is, the regular periodic decisions occur at 0, 5, 10, and 15 year in sequential decision process. Through this process, we can evaluate how many of the profitable future investment opportunities can be realized with funds

recovered from the investments made in the previous decision period. In addition, it can be appraised that how much more flexible proposals do contribute to the ultimate accumulation of the capital. The simulation model is comprised of the following two phases.

### **A . Phase I : Generation of a Periodic Schedule of Investment Proposals**

A schedule of investment proposals is nothing but a set of investment opportunities which may be submitted to reflect various investment situations during a decision period. To characterize each investment proposal, it is necessary to generate the PI, the first investment cost, the life, the RI, and the FI of each proposal. These parameters are controlled by assumed probability distributions. Those functions discussed below to generate parameters are chosen as examples because they are relatively simple to handle and might be reasonable in certain situations. It should be noted that the wealth accumulation of the firm is influenced not by the schedule of investment proposals generated in this section but by the decision models tested, because the decision criteria are applied to the same schedule of investment proposals. In this analysis, not only the proposals generated but also the cash flows of each proposal are considered to be mutually independent.

#### **1 . The Profitability Index (PI )**

At each decision period, the PI is assumed to be a random variable which has a uniform distribution, and it is assumed that the PI

distribution grows or remains constant as time passes. Therefore, the effectiveness of the models will be tested in view of the two cases, growing and constant PI distributions over the horizon period.

Case I : Growing PI Distribution of Future Investment Proposals

At period 0 : 1.32 - 2.57

At period 5 : 1.84 - 3.54

At period 10 : 3.54 - 6.62

At period 15 : 6.62 - 12.06

Case II : Constant PI distribution of Future Investment Proposals

At each decision period : 1.32 - 2.57

## 2. Initial Investment Cost

The proposal's first cost( $C$ ) is assumed to be a random variable which has a normal distribution,  $F(C)$ , with known mean ( $\bar{C} = 20$ ), and known standard deviation ( $\sigma_c = 5$ ). As the total capital of the firm being simulated grows from period to period, it is logical to expect that the funds available for investment also grow from period to period. Thus, if the average cost ( $\bar{C}$ ) of the proposals and the number of proposals per decision period are held constant, the budget after some periods of operation would be large enough to undertake all the proposals contained in the schedule of investment proposals. If it is the case, it has the same effect as when there is no financial constraint. Therefore, it is assumed that the number of proposals (ten proposals) remains constant over the study period, and the first

cost of each proposal grows at the expected growth rate of total capital, 15%, so that it would be possible to maintain a reasonable balance between the funds available for investment and the total of the first cost in each period's schedule of investment proposals. Then, the initial cost of each proposal at each decision period is determined by

$$(19) C_t = (\bar{C} + \sigma_c F(C)) (1+0.15)^t$$

### 3 . Proposal's Life

A proposal's life is assumed to be a random variable which has an exponential distribution. To avoid the lives of proposals to be extremely long, it is necessary to limit the maximum life which proposals can take. Thus, three parameters are used to define this truncated exponential distribution ( $L_{\min}=10$  year,  $\bar{N} = 15$  year,  $L_{\max}=20$ ). Then, the truncated distribution,  $F(N)$ , would be

$$(20) F(N) = (1/F(L_{\max})) \left[ 1 - \exp(-1/(\bar{N} - L_{\min})) \cdot (N - L_{\min}) \right]$$

Thus, a proposal's life ( $N$ ) is determined by

$$(21) N = L_{\min} - (\bar{N} - L_{\min}) \ln \left[ 1 - F(L_{\max}) F(N) \right]$$

This probability distribution is considered to remain constant throughout the study period.

### 4 . The Risk Index (RI )

For the proposal whose the profitability index (PI), initial cost ( $C$ ), and life ( $N$ ) are randomly determined, the RI of this proposal can not be determined directly from these pieces of information



only. Therefore, the RI is assumed to be a random variable which has a normal distribution,  $F(R)$ , with known mean ( $\bar{R} = 10$ ) and known standard deviation ( $\sigma_R = 3$ ). Then, the RI for a proposal is determined by

$$(22) \text{ RI} = \bar{R} + \sigma_R F(R)$$

## 5. The Flexibility Index (FI)

As investigated in Section II, the FI for a proposal is mainly determined by its random cash flow pattern without being affected directly its first cost, life, and expected PV. Therefore, it would be reasonable to assume that the FI for a proposal is a random variable which has a normal distribution,  $F(F)$ , with known mean ( $\bar{F} = 5$ ), and known standard deviation ( $\sigma_F = 2$ ). Then, the FI for a proposal is determined by

$$(23) \text{ FI} = \bar{F} + \sigma_F F(F)$$

## B. Phase II : Application of Decision Criteria to Schedule of Investment Opportunities and Computation of Statistics

### 1. Computation of Amounts of Funds Available in Future Decision Periods

The initial amounts of dollars available,  $B_0$ , at the first decision period ( $t=0$ ) is assumed to be \$100 in this simulation. Then, the amounts of funds available for investment,  $B_5$ , at decision period  $t=5$  can be viewed as consisting of the cash receipts  $R_5$ , to be recovered in  $t=5$  for those investments made at  $t=0$ , plus the remaining

amounts of funds which were not invested at  $t=0$ . It is assumed that the remaining funds will be invested in highly liquid investments at a minimum attractive rate of return (MARR), so that the funds will be available at the next decision period. Further, in attempt to approximate amounts of funds available at the following decision period, it is necessary to estimate the values,  $R_t$ , where  $R_t$  is the cash receipts to be recovered in  $t$  for those investments made before. A simple method to derive a meaningful estimate of  $R_t$  as a function  $t$ , which does not affect the final results, can be developed as follows: First, it is necessary to compute the payback period ( $Q$ ) by using the FI value of a proposal. For simplicity, we assume that the approximate pattern of uncertainty resolution for NPV prior to the payback period is linearly decreased to that period as in Figure 1.

Then,  $Q = 2 FI$

By using the payback period of a proposal calculated,  $R_t$  can approximately be computed through equation 24, 25, and 26.

i)  $t \leq Q$

$$(24) \quad R_t = C \left[ (1+i)^t - (Q-t)/Q \right]$$

ii)  $Q < t \leq N$

If it is assumed that the future value after  $Q$  is linearly increased by the end of proposal's life as in Figure 2, we can easily estimate  $R_t$ .

$$(25) \quad R_t = C(1+i)^Q + \left[ C PI (1+i)^N - C(1+i)^Q \right] (t-Q) / (N-Q)$$

iii)  $t > N$

$$(26) \quad R_t = C PI(1+i)^t$$

In the simulation, this methodology is directly applied to estimate  $R_t$  approximately. Therefore, the total amounts of funds available for investment at decision period  $t(B_t)$  is :

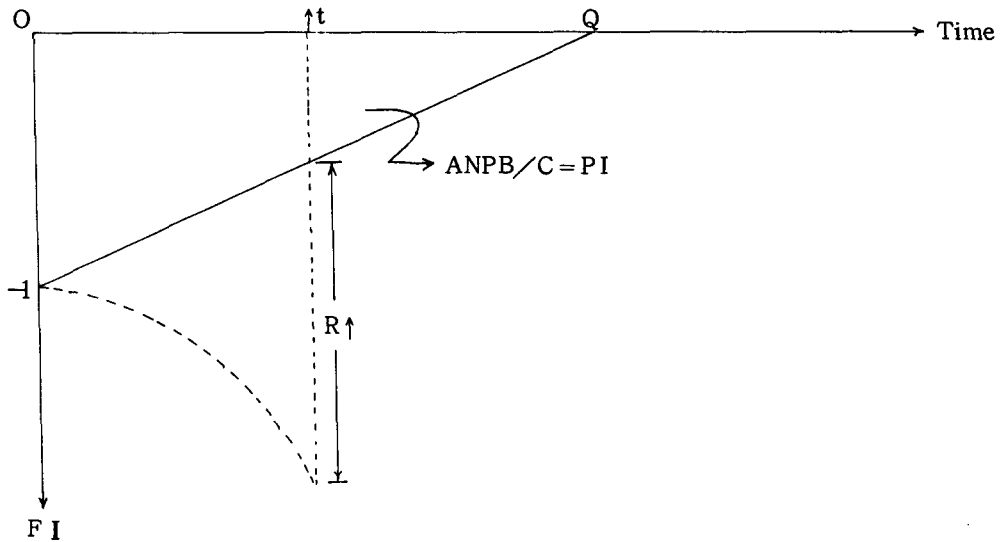


Figure 1. The Approximate Pattern of Uncertainty Resolution for NPV.

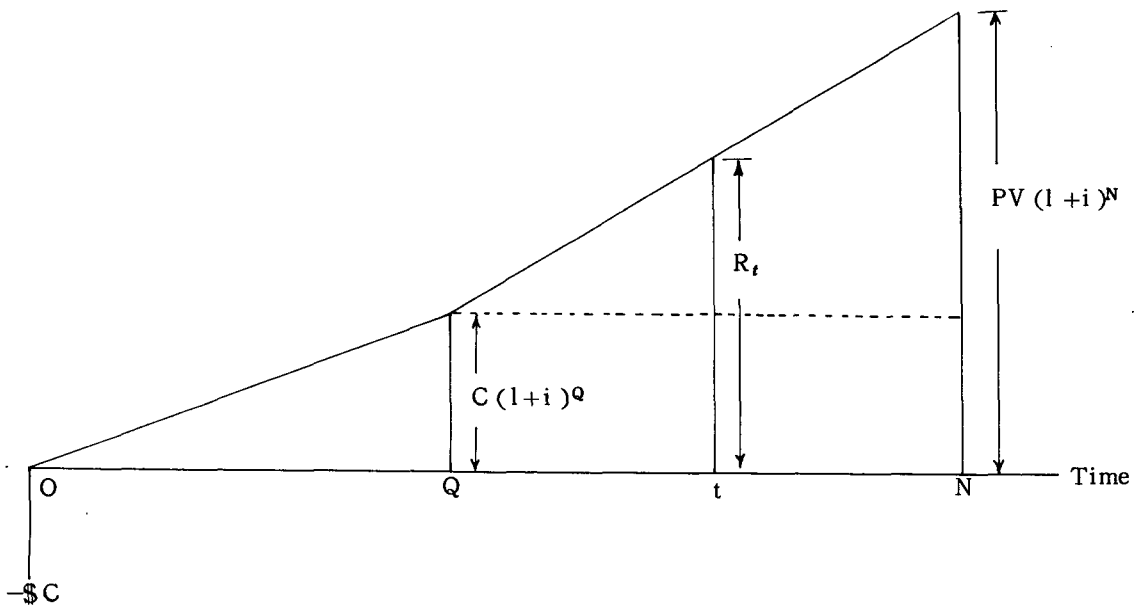


Figure 2. The Approximate Net Future Value Pattern of a Proposal

$$(27) \quad B_t = \sum_{j=1}^K R_{jit} + RF(1 + MARR)^a$$

where

$R_{jit}$  = The receipts to be recovered in period  $t$  from the proposal  $j$  made at decision period  $i$

$RF$  = The amounts of remaining funds not invested at previous decision time

$K$  = Number of proposals

$a$  = Interval of each decision time

$MARR$  = Minimum attractive rate of return

## 2 . Selection of Proposals with Budget Limitation

All decision criteria discussed in Section III require the selection of proposals that optimize their objective function without allowing for the acceptance of fractional proposal. Under the assumption that proposals considered at each decision period are independent, the optimization technique to be used in the proposal selection is zero-one integer programming. Then, the structure of MO decision model is to optimize multiobjective function with a single budget constraint. Therefore, zero-one goal programming algorithm is utilized for the optimal selection of proposals for each decision period.

## 3 . The Horizon Value as a Measure of Effectiveness

The accumulated capital or "Horizon Value" at a particular decision period  $t=T$  is defined as the present worth at period  $T$  of the

future receipts of investment that extend beyond T but were made on or before period T. The proposals undertaken at or before period T have the horizon value (HV) at period T as follows :

$$(28) \quad HV = \sum_{j=1}^N C_j PI_j (1+i)^T$$

where

$C_j$  = Initial cost of proposal j

N = Number of proposals

### C. Interpretation of the Simulation Results

The results of this simulation are concerned with the amounts of the wealth accumulation for the different decision models tested. The results are highly dependent upon the PI distribution of the future investment proposals, and depend upon how the PIL, FIL, and RIL were determined by a decision maker. These results are obtained by 10 simulation runs for the given  $\lambda$ , MARR, discount rate, PIL, FIL, and RIL.

#### 1. Case I : Growing PI Distribution of Future Investment Proposals

The expected funds available for investment at each decision period and the expected horizon values (HV) of the different decision models are represented respectively in Figures 3 and 4. FI-MO means that the first priority goal is to maintain the flexibility level, the second goal is to attain the profit level, and the third goal is to maintain the risk posture.

As seen in Figure 3, according to the determined FIL, PIL, and

RIL, the expected available funds at each decision period of FI-MO decision model are greater than those of the other decision models. Such greater funds available for investment give advantage in selection of proposals at the following decision period. This advantage makes the expected HV of FI-MO greater, about 19-57%, than those of the other models as seen in Figure 4. Consequently, it is obvious that the MO decision model into which the new decision factor, flexibility, is incorporated may maximize the wealth accumulation more than the other decision models in sequential capital budgeting process.

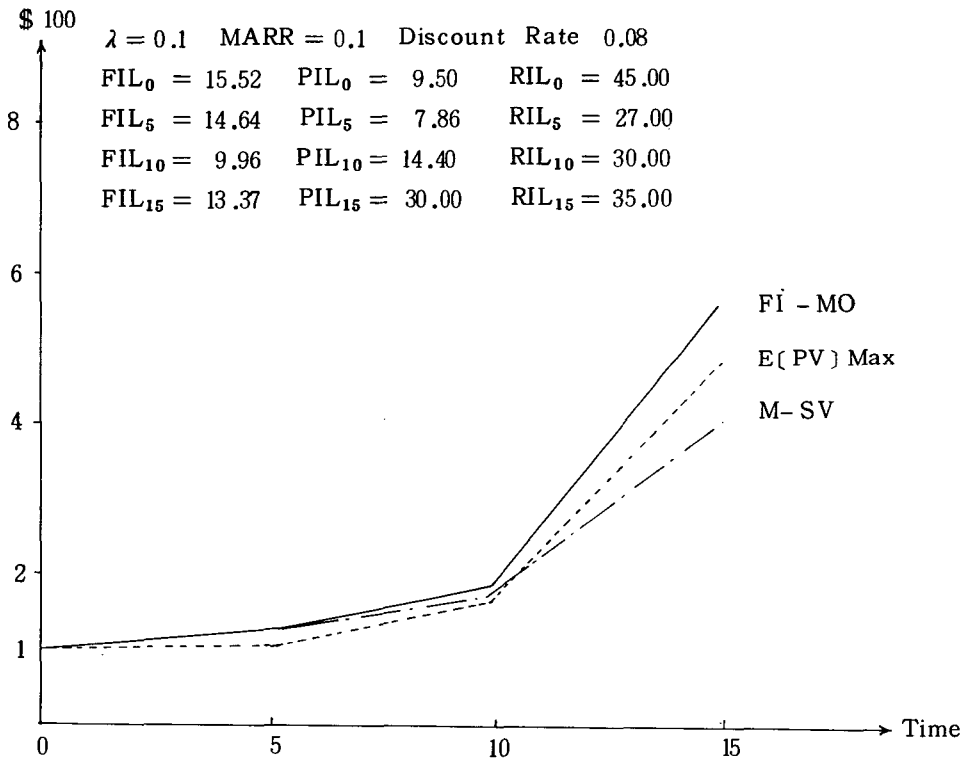


Figure 3 . Expected Available Funds in the Case of Growing PI Distribution .

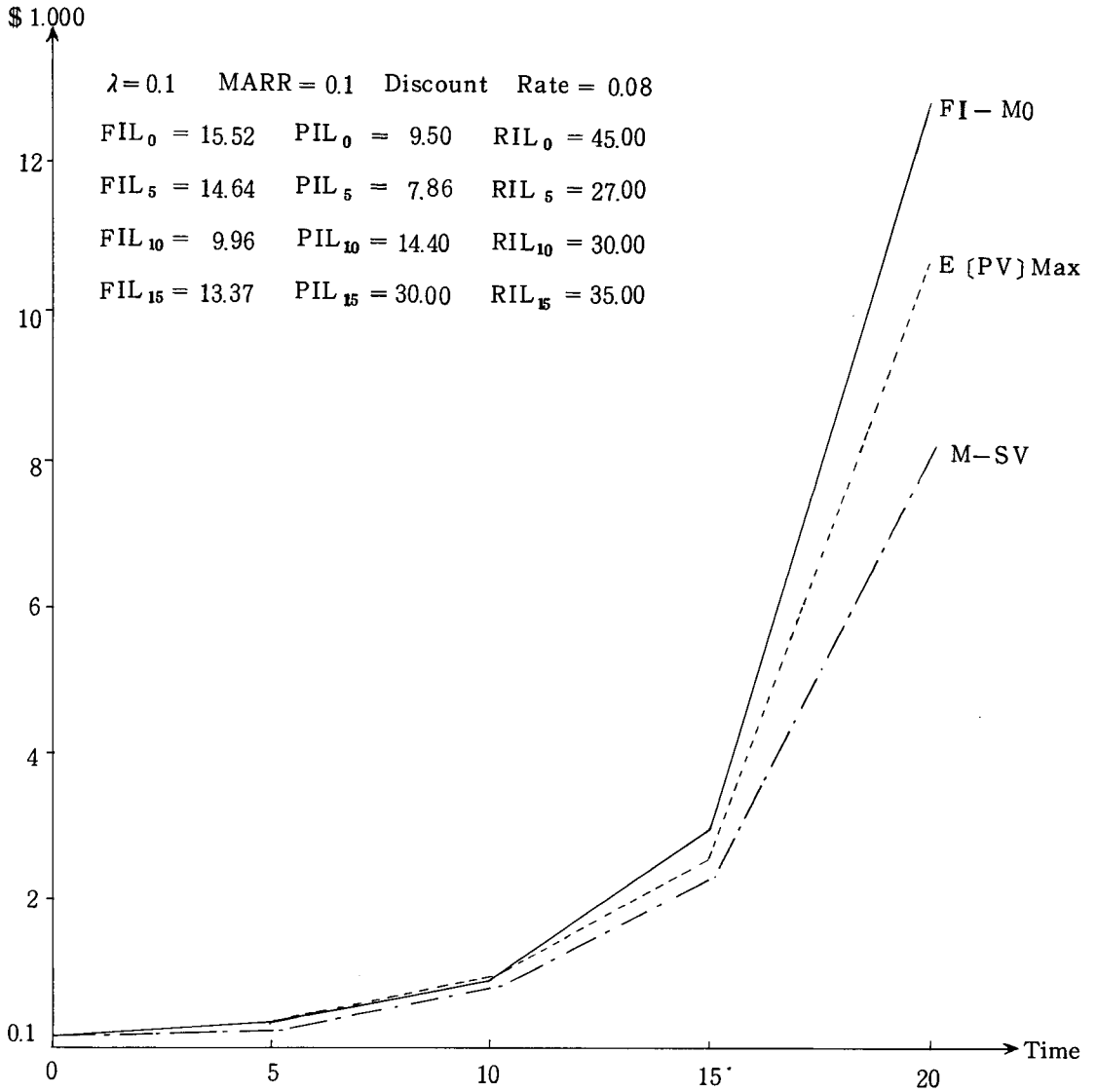
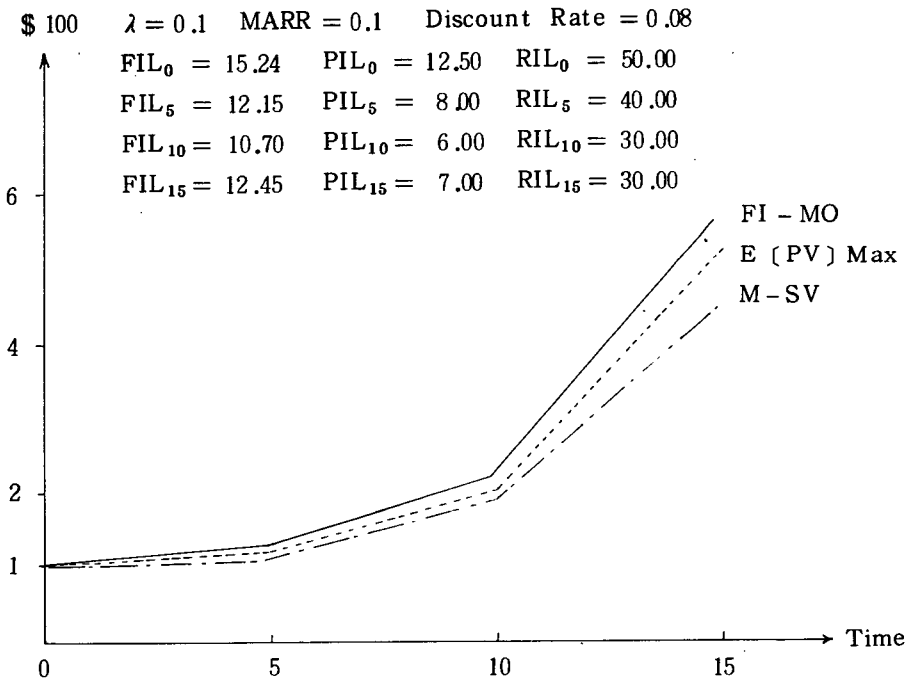


Figure 4 . Expected Horizon Values in the Case of Growing PI Distribution .

**2. Case II : Constant PI Distribution of Future Investment Proposals**

The expected funds available for investment at each decision period and expected HV of the different decision models are expressed in Figure 5 and 6. In this case, the FIL, PIL, and RIL determined by a decision maker do greatly affect the wealth accumulation of a firm. Sometimes, FI-MO model may make the HV less than the other models tested. However, in the case of the FIL, PIL, and RIL values are determined as in Figures 5 and 6, the expected for HV for FI-MO is greater, about 11%, than the expected PV maximization model, and about 34% than the M-SV model. That is, the MO model may maximize the wealth accumulation of a firm by establishing adaptably the flexibility level more than any other models tested.



**Figure 5. Expected Available Funds in the Case of Constant PI Distribution.**



### V. Conclusions

In this paper, we have proposed the new investment decision model, MO criterion by which the optimal selection of a set of proposals may be achieved in sequential capital budgeting process under capital rationing. The new investment decision factor, flexibility, in addition to the profit and the risk of a proposal should be considered to maximize the wealth accumulation of a firm under those situations.

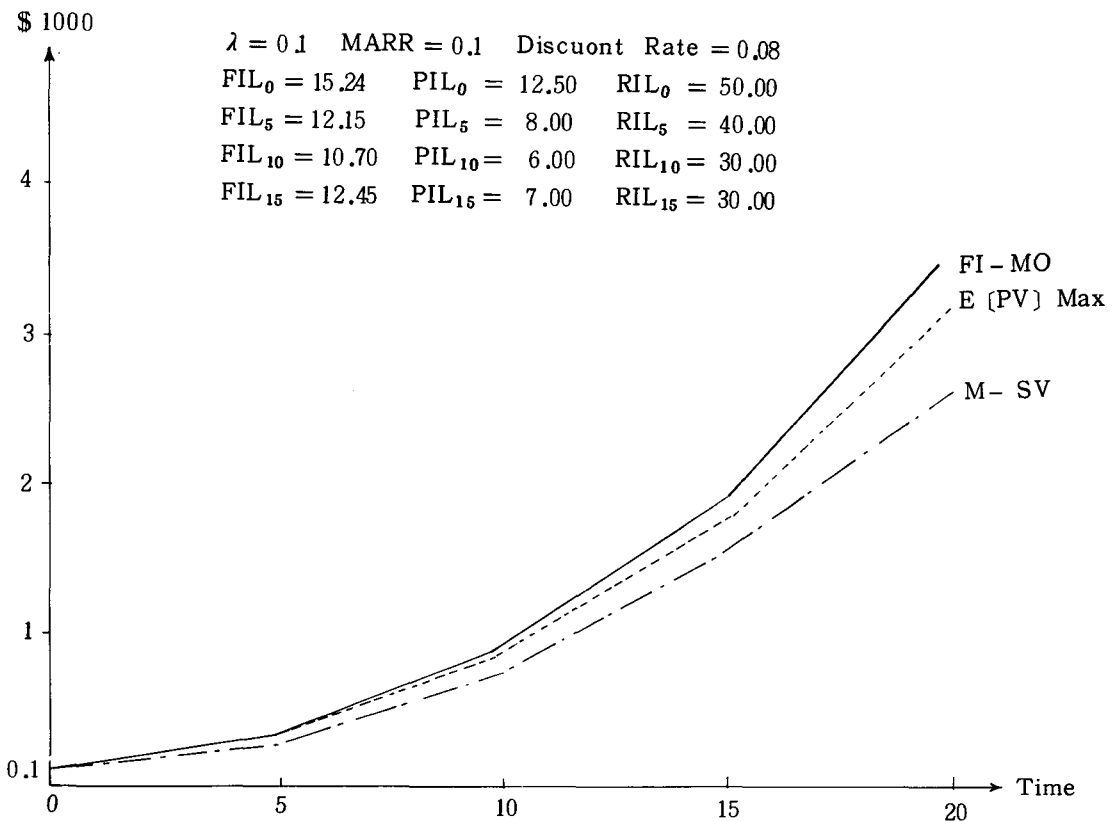


Figure 6. Expected Horizon Values in the Case of Constant PI Distribution.

As seen in Figure 4 for the growing PI distribution of schedule of future investment opportunities, in the case with consideration of flexibility as a investment decision factor, the expected horizon value of a firm is greater than that in the case without consideration of flexibility (the expected PV maximization model and mean-semivariance model), as much as 19-57%. However, in the case of constant PI distribution, the expected horizon value may be greater or less than the other models tested, according to how the FIL, PIL, and RIL are determined by a decision maker.

In addition, it is another advantage of MO criterion that this model would be used adaptably to the dynamic decision environment at each decision period. That is, a decision maker may give the flexibility the first priority in MO model when funds available for investment are in shortage during the next decision period and a current set of proposals has an allowable expected return. While the decision maker may give the profit or the risk the first priority according to the financial environment of the firm.

## REFERENCES

1. Baumol, W.J., "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," *Management Science*, vol. 10, no 1 (October 1963), pp. 174-182.
2. Bey, R.P., and Porter, R.B., "An Evaluation of Capital Budgeting Portfolio Models Using Simulated Data," *The Engineering Economist*, vol. 23, no. 1 (1977), pp. 41-65.
3. Bierman, H.J., and Hausman, W.H., "The Resolution of Investment Uncertainty Through Time," *Management Science*, vol. 18, no. 12 (August 1972), pp. B-654-662.
4. Hillier, F.S., "The Derivation of Probabilistic Information for the Evaluation of Risky Investments," *Management Science*, vol. 9, no. 3 (April 1963), pp. 443-457.
5. Ignizio, J.P., "An Approach to the Capital Budgeting Problem with Multiple Objectives," *The Engineering Economist*, vol. 21, no. 4 (1976), pp. 259-272.
6. Ignizio, J.P. "Goal Programming : A Tool for Multiobjective Analysis," Unpublished Manuscript, The Pennsylvania University, 1978.
7. Kaplan, S., and Barish, N.N., "Decision-Making Allowing for Uncertainty of Future Investment Opportunities," *Management Science*, vol. 13, no. 10 (June 1967), pp. B-569-577.
8. Mao, James C.T., "Models of Capital Budgeting, E-V VS E-S\*," *Journal of Financial and Quantitative Analysis*, vol. 5 (1970), pp. 657-675.

9. Mao, James C.T., and Brewster, J.F., "An E-S<sub>h</sub> Model of Capital Budgeting," *The Engineering Economist*, vol. 15, no. 2(1970), pp. 103-121.
10. Min, K.R., and Park, K.S., "Flexibility and Sequential Investment Analysis," *Journal of the Korean Institute of Industrial Engineers*, vol. 5, no. 2 (Dec. 1979), pp. 15-20.
11. Para-Vasquez, A.S., and Oakford, R.V., "Simulation as a Technique for Comparing Decision Procedures," *The Engineering Economist*, vol. 21, no. 4(1976), pp. 221-236.
12. Park, C.S., and Thuesen, G.J., "Combining the Concept of Uncertainty Resolution and Project Balance for Capital Allocation Decisions," *The Engineering Economist*, vol. 24, no. 2 (1979), pp. 109-127.
13. Robichek, A.A., and Myers, S., "Valuation of the Firm : Effects of Uncertainty in a Market Context," *Journal of Finance*, vol. 21 (May 1966), pp. 161-179.
14. Salazar, R.C., and Sen, S.K., "A Simulation Model of Capital Budgeting under Uncertainty," *Management Science*, vol. 15, no. 4 (December 1968), pp. B-161-179.
15. Van Horne, James C., "The Analysis of Uncertainty Resolution in Capital Budgeting for New Products," *Management Science*, vol. 15, no. 8 (April 1969), pp. B-376-386.