〈Research Paper〉

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(S-1, S) Inventory Policy Including Non-stocking Alternative as an Optimal Policy for Low Demand Items

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Abstract

When the number of stockable item types is too large in certain large scale inventory operations, it is important to *classify and screen out the items that need not be stocked*; and for the low demand or high cost items, it may be preferable to use one-for-one-ordering policy. In this paper, the problem is formulated in somewhat easier terms, and a criterion is developed that can be used in deciding what items not to stock.

1. Introduction

When the number of stockable item types is too large in certain large scale inventory operations (such as in army logistics; in this case the number is in the order of several hundred thousands), it is impractical to apply EOQ formula (or any variation thereof) to all the items. Particularly, for the low demand or high cost items, it is preferable (1) to have only a few (S) units of each item on stock, and (2) to order units one at a time as demanded.

This policy is called a continuous review (S–1, S) inventory (i.e., one-for-one-ordering) policy and fills requests on a first-come-first-served basis. This policy means that a reorder is placed whenever a demand occurs and the stock level, sometimes called "inventory position" (i.e., stock on-hand plus on-order minus unfilled demands) always remains constant thereof. Incoming request is filled immediately and passes through the station with a spare if one is available from the on-hand inventory; if not, it waits until one becomes available (i.e., complete back-ordering prevails). This particular policy is commonly used and considered optimal in low demand or high cost item inventory such as aircraft spare parts inventory [1]. Furthermore, in certain inventory situations, it may be preferable not to have any waiting stock on hand.

We assume that the demand occurs one at a time and follows a Poisson process with rate λ (monthly demand, say). The replenishment time distribution is exponential with mean τ (in months, say; $\lambda \cdot \tau < 1$) and that the replenishment times are independent random variables. This implies that orders can cross and need not be received in the same sequence in which they were placed.

The (S–1, S) inventory policy has been considered by many researchers and various sophisticated models and solutions have been put forward. However, the central theme of this paper is (1) to formulate the problem in somewhat easier (and practical) terms, and (2) to *develop a criterion that can be used in deciding what items not to stock.*

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In developing cost models, let us assume that the ratio of the following two cost factors is known (and represent the real cost situation close enough).

$$\left(\frac{₩_s}{₩_h}\right)=\frac{\text{backorder cost per item per year (e.g., due to interruption of related processes)}}{\text{inventory carrying cost per item per year (e.g., item cost}\times\text{interest rate}+\text{stocking space cost)}}$$

2. General Model for Stocking

To simplify the notation, let

$$\rho=\lambda\cdot\tau=\lambda/\mu$$

$$p(k)=\frac{\rho^k}{k!}e^{-\rho}$$

$$\overline{P}(n)=\sum_{k=n}^{\infty}\frac{\rho^k}{k!}e^{-\rho}$$

If $\pi(x)$ is the steady state probability that the net inventory (on-hand minus unfilled demands) is x, the balance equations are (equating the transition rates into and out of state x) [2],

$$\lambda\pi(x+1)+(S-x+1)\mu\pi(x-1)=[\lambda+(S-x)\mu]\pi(x);\ x=S-1,\ S-2,\ldots$$

and

$$\lambda\pi(S)=\mu\pi(S-1).$$

On successive substitution beginning with $\pi(S-1)=\rho\pi(S)$,

$$\pi(S-k)=\frac{\rho^k}{k!}\pi(S);\ k=1,2,\ldots \qquad \text{(Eq. 1)}$$

Since $\sum_{k=0}^{\infty}\pi(S-k)=\pi(S)\sum_{k=0}^{\infty}\rho^k/k!=e^{\rho}\pi(S)=1$, it follows that $\pi(S)=e^{-\rho}$ and letting $S-\mathrm{k}=x$ in (Eq. 1),

$$\pi(x)=\frac{\rho^{(S-x)}}{(S-x)!}\ e^{-\rho};\ x=S,\ S-1,\ldots \qquad \text{(Eq. 2)}$$

Noting that $x=-1$, say, implies there is one unfilled demand, the expected number of unfilled demands is

$$\sum_{x=-\infty}^{0}-x\pi(x)=\sum_{x=-\infty}^{0}-x\frac{\rho^{(S-x)}}{(S-x)!}e^{-\rho}=\sum_{k=s}^{\infty}(k-S)p(k)=\rho\overline{P}(S-1)-S\overline{P}(S) \qquad \text{(Eq.3)}$$

The expected value of the on-hand inventory at time t is

$$\sum_{x=0}^{s}x\pi(x)=\sum_{x=1}^{s}x\ \frac{\rho^{(S-x)}}{(S-x)!}e^{-\rho}=\sum_{k=s-1}^{0}(S-k)p(k)=S\sum_{k=0}^{s-1}p(k)-\rho\sum_{k=0}^{s-2}p(k)$$

$$=S[1-\overline{P}(S)]-\rho[1-\overline{P}(S-1)]=S-\rho+\rho\overline{P}(S-1)-S\overline{P}(S) \qquad \text{(Eq.4)}$$

The average annual cost of backorders and holding inventory is therefore

$$C(S)=₩_h(S-\rho)+(₩_h+₩_s)[\rho\overline{P}(S-1)-S\overline{P}(S)] \qquad \text{(Eq.5)}$$

Note that the average annual ordering costs are independent of S and hence need not be included in C.

If S^* is the smallest S which minimizes $C(S)$, then it is necessary that $C(S+1)-C(S)\geq 0$. That is,

$$₩_h(S+1-\rho)+(₩_h+₩_s)[\rho\overline{P}(S)-(S+1)\overline{P}(S+1)]$$
$$-₩_h(S-\rho)\ -(₩_h+₩_s)[\rho\overline{P}(S-1)-S\overline{P}(S)]$$

$$=₩_h+(₩_h+₩_s)[Sp(S)-\rho p(S-1)-\overline{P}(S+1)]$$
$$=₩_h-(₩_h+₩_s)\overline{P}(S+1)\geq 0$$

or S^* is the smallest S for which

$$\sum_{k=0}^{S}\frac{\rho^k}{k!}e^{-\rho}\geq\frac{₩_s}{₩_h+₩_s} \qquad \text{(Eq.5)}$$

Since the right hand side of (Eq.5) can be rewritten in terms of the cost ratio alone as $1/(\frac{₩_h}{₩_s}+1)$, in general, S^* is an increasing function of ρ (therefore demand λ and the mean replenishment time τ) and the backorder cost ratio $₩_s/₩_h$.

3. Non-stocking Alternative as an Optimal Policy

At first glance, (Eq.5) seems valid to identify the region of non-stocking ($S^*=0$) in terms of ρ and $₩_s/₩_h$. However, this is not certain because in the intermediate derivation steps (Eqs. 3-5), summation indices become negative if S is allowed to assume the value 0 or 1.

In order to find the conditions that $S^*=0$, let us examine when $C(1)\geq C(0)$. When $S=1$, the expected number of unfilled demands is, from (Eq.3), $\rho\overline{P}(0)-\overline{P}(1)=\rho-(1-e^{-\rho})$. The expected value of the on-hand inventory at time t is, $\sum_{x=0}^{1}x\pi(x)=[\frac{\rho^{(1-x)}}{(1-x)!}e^{-\rho}]_{x=1}=e^{-\rho}$. Therefore,

$$C(1)=₩_h e^{-\rho}+₩_s[\rho-(1-e^{-\rho})]$$

In retrospect, this value is the same as the one obtained directly from (Eq.5).

When $S=0$, the expected number of unfilled demands is, $\sum_{x=-\infty}^{0}x\pi(x)=\sum_{x=0}^{\infty}x\frac{\rho^x}{x!}e^{-\rho}=\rho$. Therefore,

$$C(0)=₩_s\cdot\rho$$

If $S^*=0$, $C(1)-C(0)\geq 0$. That is, it is optimal not to stock if

$$₩_h e^{-\rho}+₩_s[\rho-(1-e^{-\rho})]\geq ₩_s\cdot\rho$$

or

$$\left(\frac{₩_s}{₩_h}\right)\leq\frac{1}{e^{\rho}-1} \qquad \text{(Eq.6)}$$

The non-stocking region is depicted in Figure 1 along with other values of S^* in terms of ρ $(=\lambda\cdot\tau)$ and $₩_s/₩_h$. In passing, we note that (Eq.6) is equivalent to (Eq.5) when we set $S=0$ directly.

4. Conclusions

For the low demand or high cost items, (S-1, S) inventory policy is frequently very appropriate. Optimal values of the stock level S^* can be determined easily from (Eq.5), which turns out to be valid for all values of $S\geq 0$.

When the number of stockable item types is too large in certain large scale inventory operations, (Eq.6) or Figure 1 can be used to classify and screen out the items that need not be stocked.

In actual application of the (S-1, S) inventory policy, the values of S^* should be fairly small.

If this is not so, some form of bulk ordering might be more economical. At the present moment, this boundary is not understood clearly (except that when $\lambda \cdot \tau \geq 1$, one-for-one-ordering is insufficient), and left as a future research area.

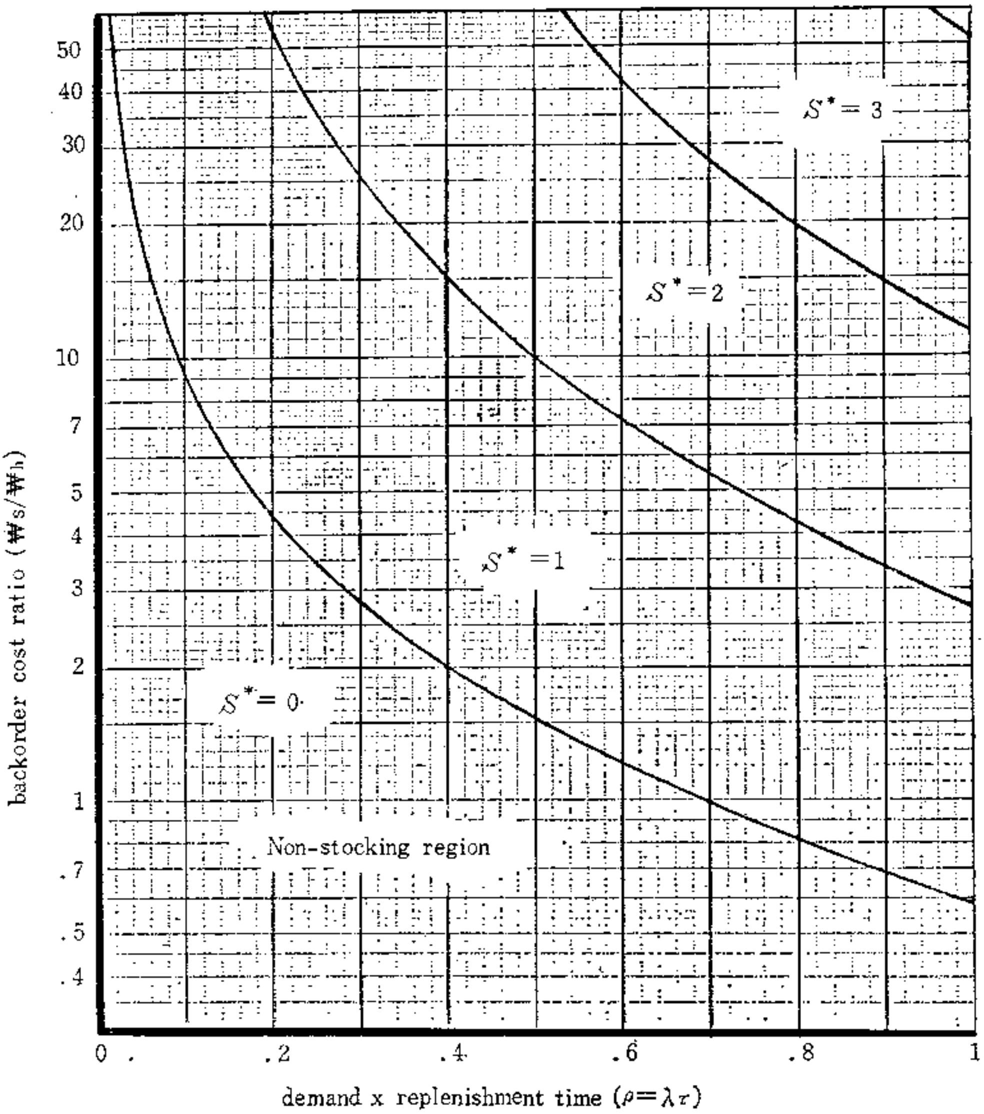

Fig. 1. Optimal Values of Stock Level S^*

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