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0-DIMENSIONAL COMPACT ORDERED SPACES

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0. Introduction

It is well known [G. J.] that for a completely regular space X, the Stone-Čech compactification βX of X is characterized by the homomorphisms on the ring $C^*(X)$ onto the ring R of real numbers, and the realcompactification vX of X by the homomorphisms on the ring $C(X)$ onto the ring R. Hence a compact (realcompact, resp.) space X can be completely determined by $C^*(X)$ ($C(X)$, resp.).

In this paper, we are concerned with the analogous problem in ordered topological spaces and continuous isotones, in particular 0-dimensional ordered spaces and their 0-dimensional ordered compactifications.

Choe and Park have [C. P.] introduced the concept of bifilters to get ordered compactifications for ordered topological spaces. Here using bifilters with bases consisting of clopen decreasing sets and clopen increasing sets, respectively, we distinguish compact objects among 0-dimensional ordered spaces, and then by the analogous way with maximal clopen bifilters as in [C. P.], we construct the 0-dimensional ordered compactification of a 0-dimensional ordered space X, which gives rise to the reflection $\zeta_0: X \longrightarrow \zeta_0 X$. And for any ordered topological space X, we consider the lattice $C(X)$ of continuous isotones on X to the two point discrete chain 2. Establishing the one-one correspondence between maximal clopen bifilters on a 0-dimensional ordered space X and lattice homomorphisms on $C(X)$ onto 2, it is shown that $\zeta_0 X$ is precisely the topological ordered space of lattice homomorphisms on $C(X)$ onto 2. In consequence, we have corresponding results for $\zeta_0 X$ to those for βX and vX.

1. 0-dimensional ordered spaces

DEFINITION 1.1. An *ordered topological space* is a triple $(X, \mathscr{T}, \leqslant)$ such that $(X, \mathscr{T})$ is a topological space and $\leqslant$ is a partial order on X.

The class of ordered topological spaces and continuous isotones between them obviously forms a category, which will be denoted by **OTOP**. Then it is clear that the underlying set functor U: **OTOP**$\longrightarrow$**SET** is (onto, mono-sources)-topological [H], while **SET** is the category of sets and maps. Let 2 denote the two point discrete chain {0, 1}, and let **ZO** denote the epireflective hull of {2} in **OTOP**.

DEFINITION 1.2. An ordered topological space is called 0-*dimensional* if it is an object of **ZO**.

Since the category **OTOP** is an (onto, U-initial mono-sources) category, the following is immediate by Proposition 7.4 in [H].

PROPOSITION 1.3. *An ordered topological space X is 0-dimensional if and only if the set $C(X)$ of all continuous isotones on X to 2 forms a U-initial mono-source.*

COROLLARY 1.4. *Let X be an ordered topological space. Then X is 0-dimensional if and only if it satisfies the following conditions*:

1) *if $x \not\leq y$, there is $u \in C(X)$ such that $u(x)=1$ and $u(y)=0$,*

2) *the family of clopen decreasing sets and clopen increasing sets forms a subbase for the topology on X.*

PROOF. Noting that for any $u \in C(X)$, $u^{-1}(0)$ ($u^{-1}(1)$, resp.) is clopen decreasing (clopen increasing, resp.), and for any clopen increasing (clopen decreasing resp.) set A, the characteristic map of A ($X-A$, resp.) is a continuous isotone on X to 2, the corollary follows immediately from Proposition 1.3.

REMARK 1.5. 1) The order on a 0-dimensional ordered space is closed [N], and its topology has a base consisting of convex sets.

2) Using the three point discrete chain, Choe and Y. H. Hong have intro duced [C. H.] the concept of 0-dimensional ordered spaces, which is duly equivalent with that in this paper.

3) An ordered topological space X is 0-dimensional if and only if it is 2-regular ordered space [C. H.], [P3]. Hence for any $X \in OTOP$, let $h: X \longrightarrow 2^{C(X)}$ be the map defined by $h(x)=(u(x))_{u \in C(X)}$, and let zX be the subspace of $2^{C(X)}$ with $h(X)$ as its underlying set. Then the map $z: X \longrightarrow zX$ ($z(x)=h(x)$) is the ZO-reflection of X.

The following definition is due to Y. S. Park [P2].

DEFINITION 1.6. *A* pair $(\mathscr{F}, \mathscr{G})$ of filters on a partially ordered set X is said to be a *bifilter* on X if $\mathscr{F}$ ($\mathscr{G}$, resp.) has a base consisting of decreasing (increasing, resp.) sets and $F\cap G\neq\phi$ for any $F\in\mathscr{F}$ and $G\in\mathscr{G}$. If $\mathscr{B}$ ($\mathfrak{C}$, resp.) is a base for $\mathscr{F}$($\mathscr{G}$, resp.), then $(\mathscr{B},\mathfrak{C})$ is called a base for the bifilter $(\mathscr{F},\mathscr{G})$.

REMARK. Let X be a partially ordered set.

1) If $(\mathscr{F}, \mathscr{G})$ is a bifilter on X, then the join filter of $\mathscr{F}$ and $\mathscr{G}$, denoted by $\mathscr{F}\vee\mathscr{G}$, exists.

2) For bifilters $(\mathscr{F}, \mathscr{G}), (\mathscr{H}, \mathscr{K})$ on X, we define a relation $(\mathscr{F}, \mathscr{G})\subseteq(\mathscr{H}, \mathscr{K})$ if and only if $\mathscr{F}\subseteq\mathscr{H}$ and $\mathscr{G}\subseteq\mathscr{K}$. In case, $(\mathscr{F},\mathscr{G})$ is said to be *contained in* $(\mathscr{H}, \mathscr{K})$. Then every bifilter is contained in a maximal bifilter, i.e. maximal element with respect to the relation $\subseteq$.

DEFINITION 1.7. Let X be an ordered topological space. Then a bifilter $(\mathscr{F}, \mathscr{G})$ on X is called *clopen* if $\mathscr{F}$ ($\mathscr{G}$, resp.) has a base consisting of clopen decreasing (clopen increasing, resp.) sets. By a *maximal clopen bifilter* on X we mean a clopen bifilter not contained in any other clopen bifilfter.

PROPOSITION 1.8. *Let X be an ordered topological space. A clopen bifilter $(\mathscr{F}, \mathscr{G})$ on X is a maximal clopen bifilter if and only if for any continuous isotone $u: X\longrightarrow 2$, i.e. $u\in C(X)$, $u(\mathscr{F}\vee\mathscr{G})$ is convergent.*

PROOF. Let $(\mathscr{F}, \mathscr{G})$ be a maximal clopen bifilter and $u\in C(X)$. Suppose $u^{-1}(0)\notin\mathscr{F}$ and $u^{-1}(1)\notin\mathscr{G}$. Then by the maximality, there are $F, F'\in\mathscr{F}$ and $G, G'\in\mathscr{G}$ such that $u^{-1}(0)\cap F\cap G=\phi$ and $u^{-1}(1)\cap F'\cap G'=\phi$. Since $X=u^{-1}(0)\cup u^{-1}(1)$, $(F\cap G)\cap(F'\cap G')=(F\cap F')\cap(G\cap G')=\phi$, which is a contradiction. It is now clear that if $u^{-1}(0)\in\mathscr{F}$, then $u(\mathscr{F}\vee\mathscr{G})\longrightarrow 0$, and if $u^{-1}(1)\in\mathscr{G}$, then $u(\mathscr{F}\vee\mathscr{G})\longrightarrow 1$. For the converse, let $(\mathscr{H}, \mathscr{K})$ be a clopen bifilter with $(\mathscr{F}, \mathscr{G})\subseteq(\mathscr{H}, \mathscr{K})$. Take any clopen increasing set $K\in\mathscr{K}$. Then the characteristic map u of K is a continuous isotone. If $u(\mathscr{F}\vee\mathscr{G})\longrightarrow 0$, then there is $F\in\mathscr{F}$ and $G\in\mathscr{G}$ with $u(F\cap G)=0$. Since F is decreasing, $u(F)=0$, so $F\cap K=\phi$, and we have a contradiction. Thus $u(\mathscr{F}\vee\mathscr{G})\longrightarrow 1$, i.e. there is $F\in\mathscr{F}$ and $G\in\mathscr{G}$ such that $u(F\cap G)=1$. Since G is increasing, $u(G)=1$, i.e. $G\subseteq K$ and hence $K\in\mathscr{G}$. Thus $\mathscr{G}=\mathscr{K}$. Dually, one can conclude $\mathscr{F}=\mathscr{H}$. Therefore $(\mathscr{F}, \mathscr{G})$ is a maximal clopen bifilter.

REMARK 1.9. By the proof of the above proposition, for any maximal clopen bifilter $(\mathscr{F}, \mathscr{G})$ and any $u\in C(X)$, $u^{-1}(0)\in\mathscr{F}$ if and ony if $u(\mathscr{F}\vee\mathscr{G})\longrightarrow 0$, and $u^{-1}(1)\in\mathscr{G}$ if and only if $u(\mathscr{F}^{\vee}\mathscr{G})\longrightarrow 1$.

DEFINITION 1.10. Let $(\mathscr{F}, \mathscr{G})$ be a bifilter on an ordered topological space X, then $(\mathscr{F}, \mathscr{G})$ is said to *converge to* x if $\mathscr{F}^{\vee}\mathscr{G}$ converges to x in X, and a point y is said to be an *adherence point* of $(\mathscr{F}, \mathscr{G})$ if y is an adherence point of $\mathscr{F}^{\vee}\mathscr{G}$.

REMARK 1.11. For any ordered topological space X and $x\in X$, $\mathscr{D}(x)$ ($\mathscr{I}(x)$, resp.) denotes the family of clopen decreasing (clopen increasing, resp.) neighborhoods of x. It is clear that $(\mathscr{D}(x), \mathscr{I}(x))$ is a base for a clopen bifilter. Moreover, if X is 0-dimensional, then by Corollary 1.4, $\mathscr{D}(x)^{\vee}\mathscr{I}(x)$ is a local base at x and hence by proposition 1.8 $(\mathscr{D}(x), \mathscr{I}(x))$ is a base for a maximal clopen bifilter. In the following, the bifilter generated by $(\mathscr{D}(x), \mathscr{I}(x))$ will be again denoted by $(\mathscr{D}(x), \mathscr{I}(x))$.

THEOREM 1.12. *Let X be a 0-dimensional ordered space. Then the following are equivalent*:

1) *X is compact.*
2) *Every clopen bifilter has an adherence point.*
3) *Every maximal clopen bifilter is convergent.*

PROOF. 1)$\Longrightarrow$2) Clear.

2)$\Longrightarrow$3) Let x be an adherence point of any maximal clopen bifilter $(\mathscr{F}, \mathscr{G})$. Then by the maximality of $(\mathscr{F}, \mathscr{G})$, $(\mathscr{D}(x), \mathscr{I}(x))$ is contained in $(\mathscr{F}, \mathscr{G})$, so that by remark 1.11, $(\mathscr{F}, \mathscr{G})$ converges to x.

3)$\Longrightarrow$1) By the Alexander subbase theorem, it is enough to show that every filter $\mathscr{W}$ consisting of clopen increasing sets and clopen decreasing sets has an adherence point. Let $\mathscr{F}$ ($\mathscr{G}$, resp.) be the family of all decreasing (increasing, resp.) members of $\mathscr{W}$, then $(\mathscr{F}, \mathscr{G})$ is clearly a base for a clopen bifilter. Thus there is a maximal clopen bifilter $(\mathscr{H}, \mathscr{K})$ with $(\mathscr{F}, \mathscr{G})\subseteq(\mathscr{H}, \mathscr{K})$. Since $\mathscr{H}^{\vee}\mathscr{K}\longrightarrow x$ for some $x\in X$, x is obviously an adherence point of $\mathscr{W}$.

2. 0-dimensional ordered compactification

Let *ZCO* denote the full subcategory of *ZO* formed by all compact objects. Since every 0-dimensional ordered space is Hausdorff, a 0-dimensional ordered space is compact if and only if it is a 2-compact ordered space. Hence *ZCO* is an epireflective subcategory of *ZO* [C. H.], [P3].

In what follows, X will always denote a 0-dimensional ordered space. Now we follow the way of Wallman type order compactification introduced by Choe and Park [C.P.] to get the *ZCO*-reflection of X, employing maximal clopen bifilters instead of maximal closed bifilters.

Let $\zeta_0 X$ be the collection of all maximal clopen bifilters on X. We define a relation $\leqslant$ on $\zeta_0 X$ as follows: $(\mathscr{F}, \mathscr{G}) \leqslant (\mathscr{H}, \mathscr{K})$ if and only if $\mathscr{H} \subseteq \mathscr{F}$ and $\mathscr{G} \subseteq \mathscr{K}$. Then the relation $\leqslant$ is clearly a partial order on $\zeta_0 X$.

REMARK 2.1. Considering the characteristic map of a clopen increasing set (the complement of a clopen decreasing set, resp.), one can easily conclude by remark 1.9 that $(\mathscr{F}, \mathscr{G}) \leqslant (\mathscr{H}, \mathscr{K})$ if and only if for any $u \in C(X)$, $\lim u(\mathscr{F}^{\vee}\mathscr{G}) \leqslant \lim u(\mathscr{H}^{\vee}\mathscr{K})$.

For a clopen decreasing set A and a clopen increasing set B in X, define
$A^d = \{(\mathscr{F}, \mathscr{G}) \in \zeta_0 X : A \in \mathscr{F}\}$, $B^i = \{(\mathscr{F}, \mathscr{G}) \in \zeta_0 X : B \in \mathscr{G}\}$.

Then it is clear that A^d (B^i, resp.) is decreasing (increasing, resp.) and that for clopen decreasing sets A, A', $(A \cap A')^d = A^d \cap A'^d$, and for clopen increasing sets B, B', $(B \cap B')^i = B^i \cap B'^i$. Hence $\{A^d \cap B^i : A$ is a clopen decreasing set and B is a clopen increasing set$\}$ forms a base for a topology on $\zeta_0 X$, which will be denoted by $\mathscr{T}^*$. Since $\zeta_0 X - A^d = (X-A)^i$ and $\zeta_0 X - B^i = (X-B)^d$, A^d is clopen decreasing and B^i is clopen increasing in $(\zeta_0 X, \mathscr{T}^*, \leqslant)$. For the brevity, $(\zeta_0 X, \mathscr{T}^*, \leqslant)$ will be as before denoted by $\zeta_0 X$. Furthermore, let $\zeta_0 : X \longrightarrow \zeta_0 X$ be the map defined by $\zeta_0(x) = (\mathscr{D}(x), \mathscr{I}(x))$.

We recall that a continuous isotone $f : X \longrightarrow Y$ is called 2-extendable if for any $u \in C(X)$, there is $v \in C(Y)$ with $vf = u$.

LEMMA 2.2. *The map $\zeta_0 : X \longrightarrow \zeta_0 X$ is a dense embedding in OTOP such that ζ_0 is 2-extendable.*

PROOF. It is a routine verification that ζ_0 is $1-1$ isotone and that for any clopen decreasing set A, $\zeta_0(A) = A^d \cap \zeta_0(X)$ and for any clopen increasing set B, $\zeta_0(B) = B^i \cap \zeta_0(X)$. Hence ζ_0 is an embedding. Furthermore, for any $(\mathscr{F}, \mathscr{G}) \in \zeta_0 X$, $\zeta_0(\mathscr{F}^{\vee}\mathscr{G})$ converges to $(\mathscr{F}, \mathscr{G})$ in $\zeta_0 X$, so that ζ_0 is dense. For any

$u \in C(X)$, define $\bar{u} : \zeta_0 X \longrightarrow 2$ by $\bar{u}((\mathscr{F}, \mathscr{G})) = \lim u(\mathscr{F}^{\vee}\mathscr{G})$. Then one has $\bar{u}\zeta_0 = u$, and $\bar{u}$ is isotone by remark 2.1. Hence it remains to show the continuity of $\bar{u}$. Suppose $\bar{u}((\mathscr{F}, \mathscr{G})) = 1$ (the case $\bar{u}((\mathscr{F}, \mathscr{G})) = 0$ is similar). Let $B = u^{-1}(1)$, then $B \in \mathscr{G}$. Hence B^i is a neighborhood of $(\mathscr{F}, \mathscr{G})$ and $\bar{u}(B^i) = 1$, so that $\bar{u}$ is continuous at $(\mathscr{F}, \mathscr{G})$. This completes the proof.

THEOREM 2.3. *For any 0-dimensional ordered space X, the dense embedding $\zeta_0 : X \longrightarrow \zeta_0 X$ is the ZCO-reflection of X.*

PROOF. Suppose $(\mathscr{F}, \mathscr{G}) \not\leq (\mathscr{H}, \mathscr{K})$ in $\zeta_0 X$. Then by remark 2.1, there is $u \in C(X)$ such that $\lim u(\mathscr{H}^{\vee}\mathscr{K}) = 0$ and $\lim u(\mathscr{F}^{\vee}\mathscr{G}) = 1$. Let $\bar{u} : \zeta_0 X \longrightarrow 2$ be the extension of u, i.e. $\bar{u}\zeta_0 = u$, then $\bar{u}((\mathscr{F}, \mathscr{G})) = \bar{u}(\lim \zeta_0(\mathscr{F}^{\vee}\mathscr{G})) = \lim \bar{u}\zeta_0 (\mathscr{F}^{\vee}\mathscr{G}) = \lim u(\mathscr{F}^{\vee}\mathscr{G}) = 1$, and similarly one has $\bar{u}((\mathscr{H}, \mathscr{K})) = 0$. Thus by corollary 1.4 together with the definition of the topology on $\zeta_0 X$, $\zeta_0 X$ is a 0-dimensional ordered space. Since $\zeta_0 : X \longrightarrow \zeta_0 X$ is now a dense embedding into the Hausdorff space, ζ_0 is uniquely 2-extendable. Since *ZCO* is the category of 2-compact ordered spaces, it is enough to show that $\zeta_0 X$ is compact. Let $\mathscr{W}$ be a filter base on $\zeta_0 X$ consisting of the subbasic closed sets, $\mathscr{F} = \{A : A^d \in \mathscr{W}\}$, and $\mathscr{G} = \{B : B^i \in \mathscr{W}\}$. Then it is obvious that $(\mathscr{F}, \mathscr{G})$ is a base for a clopen bifilter. Let $(\mathscr{H}, \mathscr{K})$ be a maximal clopen bifilter containing $(\mathscr{F}, \mathscr{G})$, then it is again clear that $(\mathscr{H}, \mathscr{K})$ belongs to every member of $\mathscr{W}$.

3. Lattice homomorphisms on $C(X)$

Since 2 is a topological distributive lattice with 0 and 1, the set $C(X)$ of all continuous isotones on X to 2 is again a distributive lattice with 0 and 1 as a sublattice of the power lattice 2^X. In this paper, by lattice homomorphisms we mean those preserving 0 and 1. It is clear that $C : \mathbf{OTOP} \longrightarrow \mathbf{DLatt}_0^1$ is a contravariant functor.

PROPOSITION 3.1. *There is one-one correspondence between the set $SC(X)$ of lattice homomorphisms on $C(X)$ to 2 and the set of all maximal clopen bifilters on X.*

PROOF. For any lattice homomorphism $h : C(X) \longrightarrow 2$, let $\mathscr{F} = \{u^{-1}(0) : h(u) = 0\}$ and $\mathscr{G} = \{v^{-1}(1) : h(v) = 1\}$. Since h preserves 0 and 1, i.e. h is onto, $\mathscr{F}$

and $\mathscr{G}$ are non-empty families of non-empty sets. Since $u^{-1}(0)\cap v^{-1}(0)=(u\vee v)^{-1}(0)$ and $u^{-1}(1)\cap v^{-1}(1)=(u\wedge v)^{-1}(1)$, $(\mathscr{F}, \mathscr{G})$ is a base for a clopen bifilter. In fact, $(\mathscr{F}, \mathscr{G})$ is a base for a maximal clopen bifilter. Indeed, if $h(u)=0$, then $u(\mathscr{F}^{\vee}\mathscr{G})$ converges to 0 and if $h(v)=1$, then $v(\mathscr{F}^{\vee}\mathscr{G})$ converges to 1. Hence by Proposition 1.8, $(\mathscr{F}, \mathscr{G})$ generates a maximal clopen bifilter which is denoted by $(\mathscr{F}h, \mathscr{G}h)$. We note that $h(u)=\lim u(\mathscr{F}^{\vee}\mathscr{G})$. Let $(\mathscr{F}, \mathscr{G})$ be a maximal clopen bifilter, then define $h_{(\mathscr{F},\mathscr{G})}: C(X)\longrightarrow 2$ by $h_{(\mathscr{F},\mathscr{G})}(u)=\lim u(\mathscr{F}^{\vee}\mathscr{G})$. Again by Proposition 1.8, $h_{(\mathscr{F},\mathscr{G})}$ is well defined, and using Remark 1.9 together with the fact that $\mathscr{F}$ and $\mathscr{G}$ are prime filters with respect to clopen decreasing sets and clopen increasing sets respectively, $h_{(\mathscr{F},\mathscr{G})}$ is a lattice homomorphism. It is now obvious that the correspondence $h\longrightarrow(\mathscr{F}_h, \mathscr{G}_h)$ is the inverse of the correspondence $(\mathscr{F}, \mathscr{G})\longrightarrow h_{(\mathscr{F},\mathscr{G})}$. This completes the proof.

COROLLARY 3.2. *A 0-dimensional ordered space X is compact if and only if every lattice homomorphism $h: C(X)\longrightarrow 2$ is fixed, i.e. there is $x\in X$ such that $h(u)=u(x)$ for all $u\in C(X)$.*

PROOF. This is an immediate consequence from Theorem 1.12 and Proposition 3.1.

The above corollary amounts to saying that a 0-dimensional compact ordered space can be recovered by its lattice $C(X)$ of continuous isotones on X to 2.

Using Theorem 2.3, Proposition 3.1, and Theorem 2.2 in [P3], one has the following:

THEOREM 3.3. *Let X be a 0-dimensional ordered space. Then the ZCO-reflection $\zeta_0: X\longrightarrow\zeta_0 X$ of X is given as follows: $\zeta_0 X$ is the subspace $SC(X)$ of $2^{C(X)}$ consisting of all lattice homomorphisms on $C(X)$ to 2 and $\zeta_0(x)(u)=u(x)$ $(x\in X, u\in C(X))$.*

COROLLARY 3.4. (Priestley [P5]) *For any 0-dimensional compact ordered space X, the map $\zeta_0: X\longrightarrow SC(X)=\zeta_0 X$ is an isomorphism in OTOP. In particular, for 0-dimensional compact ordered spaces X, Y, X is isomorphic with Y in OTOP if and only if $C(X)$ is isomorphic with $C(Y)$ as lattices.*

COROLLARY 3.5. *Let X be a 0-dimensional ordered space and Y a 0-dimensional compact ordered space. Then for any lattice homomorphism $f: C(Y)\longrightarrow C(X)$,*

..ere is a unique continuous isotone $u: X \longrightarrow Y$ *such that* $C(u)=f$.

PROOF. Let $u: X \longrightarrow Y$ be the composite of $\zeta_0: X \longrightarrow \zeta_0 X = SC(X)$ and $S(f)$: $SC(X) \longrightarrow SC(Y)$. Since $pr_u S(f) = pr_{f(u)}$ $(u \in C(Y))$ where pr_u denotes the uth projection, $S(f)$ is continuous isotone. Since $SC(Y) = \zeta_0 Y = Y$, $u: X \longrightarrow Y$ is the unique continuous isotone with $C(u)=f$.

4. Concluding remarks

4.1. Using maximal o-zero-dimensional filters in [C.H], Choe and Y. H. Hong have implicitly constructed the ZCO-reflection of a 0-dimensional ordered space. We note that the concept of o-zero-dimensional filters is external but that of maximal clopen bifilters is internal.

4.2. Using maximal o-completely regular filters whose concept is obviously external, Choe and Hong have constructed the Nachbin compactification of a completely regular ordered space ([C.H.]). Hence the problem whether there is an internal way to characterize the compactification, arises.

4.3. By Corollary 3.5, a 0-dimensional compact ordered space X is characterized by the lattice structure of $C(X)$. It is then a natural question that what kind of algebra structures of $C(X)$ characterizes the compact ordered space X.

4.4. Since realcompact spaces are characterized by their rings of real continuos maps, or real z-ultrafilters [G. J.], there arise the same problems as 4.2 and 4.3 for realcompact ordered spaces, R-compact ordered spaces, 0-dimensional realcompact ordered spaces, and N-compact ordered spaces (N is the discrete ordered chain of natural numbers).

4.5. In [C.P.] and [P4], using maximal closed bifilters, Choe and Park have constructed the Wallman type ordered compactification of a convex ordered topological space with a semi-closed order. Unfortunately, the order on the compactification need not be semi-closed. However the order on $\zeta_0 X$ is closed. Thus it is natural to ask whether there is a method using bifilters or else to get the compactification with a continuous order (see [P1] for a partial answer).

4.6. For a 0-dimensional space X with the discrete order, $\zeta_0: X \longrightarrow \zeta_0 X$ coincides with the Banaschewski's 0-dimensional compactification $\zeta: X \longrightarrow \zeta X$ [B].

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REFERENCES

[B] B. Banaschewski, *Über nulldimensionale Räume*, Math. Nachr., 13(1955). 129—140.

[C. H.] T. H. Choe and Y. H. Hong, *Extensions of completely regular ordered spaces*, Pacific J. Math., 66(1976), 37—48.

[C. P.] T. H. Choe and Y. S. Park, *Wallman's type order compactifications*, Pacific J. Math., 82(1979), 339—347.

[G. J] L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, New York, 1960.

[H] H. Herrlich, *Topological functors*, Gen. Top. and its Appl., 4(1974), 125—142.

[N] L. Nachbin, *Topology and order*, Van Nostrand, New York, 1965.

[P 1] B. H. Park, *On Wallman-type extensions*, Kyungpook Math. J., 19(1979), 183—200.

[P 2] Y. S. Park, *Ordered topological spaces and topological semilattices*, Doctoral Dissertation, McMaster Univ. Hamiltion, 1977.

[P 3] Y. S. Park, *On P-compact ordered spaces*, Kyungpook Math. J., 17(1977), 117—120.

[P 4] Y. S. Park, *Wallman's type order compactifications* II, Kyungpook Math. J., 19 (1979), 151—158.

[P 5] H. A. Priestley, *Representation of distributive lattices by means of ordered Stone spaces*, Bull. London Math. Soc., 2(1970), 186—190.