The Nonstationary Vibration of Asymmetry Shaft carrying two Discs Passing through Critical Speeds

(Analysis by Perturbation Theory)

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The nonstationary vibration of a rotor carrying two discs with a limited driving torque is studied theoretically by using the method of the perturbation theory.

The influence of the asymmetry, torque, damping and phase difference in passing through a critical speed is studied in detail, considering the interaction between the driving source and the vibration system.

Introduction

In many installations of modern high speed machineries, the running speed of the machine is in excess of the resonant of the system, and so starting up or stopping the machine could result in the vibration with large amplitude.

The approaches for this problem are devided into two categories. One is the approach assuming that an energy source is one which acts on the vibration system, but does not experience any interaction from the system. The other is the case that the characteristics of the energy source must be considered because of an appreciable interaction between the energy source and the vibration system.

The former has been tried by many authors since Lewis(1943), Dimentberg(1961) and Fernlund(1963). The latter has been tried with a linear and nonlinear system by Kononenko(1964). W. Hübner(1965) studied analytically a simple vibration model. Kawai, Iwastsubo(1974) reported on the case of a vibration of asymmetry shaft carrying a disc. Nonami, Miyashita(1978) reported the problem of rotor passing through critical speed with gyroscopic effect.

These investigations have not been treated the nonstationary vibration of asymmetry shaft carrying two discs systematically.

In this paper, the author describes vibration characteristics of asymmetry shaft carrying two discs during passing through its critical speed.

The analysis is made using the method of Bogolyubov's perturbation theory. The numerical calculations are done with digital computer(FANAFACOM U-300).

Nomenclature

$0\text{-}xyz$: fixed rectangular coordinate system

x_1, y_1, x_2, y_2 : coordinate of geometrical center of the rotor

$x_{1G}, y_{1G}, x_{2G}, y_{2G}$: coordinate of gravitational center of the rotor

m_1, m_2 : mass of the discs

I : moment of inertia of the rotor

c_{11}, c_{12}, c_{22} : stiffness of the shaft

$2\Delta c_{11}, 2\Delta c_{12}, 2\Delta c_{22}$: differences between maximum and minimum values of c_{11}, c_{12}, c_{22}

k_1, k_2 : coefficient of the external damping

d_1, d_2 : coefficient of the internal damping

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$\mathcal{E}_1, \mathcal{E}_2$: eccentricity of the discs
ϕ : revolution angle of the rotor
$\omega_{1,2}$: the first and second critical speeds
$M-S\dot{\phi}$: driving torque
$R\dot{\phi}$: resisting torque of driving source
β : angle between $\mathcal{E}_1$ and $\mathcal{E}_2$

Equations of motion

The derivation of the equations of motion is shown in the Appendix.

$$m_1\ddot{z}_1+c_{11}z_1+c_{12}z_2=m_1\mathcal{E}_1\dot{\phi}^2e^{i\phi}-(k_1+d_1)\dot{z}_1-id_1\dot{\phi}z_1 -\Delta c_{11}\bar{z}_1e^{2i\phi}-\Delta c_{12}\bar{z}_2e^{2i\phi}$$

$$m_2\ddot{z}_2+c_{22}z_2+c_{12}z_1=m_2\mathcal{E}_2\dot{\phi}^2e^{i(\phi+\beta)}-(k_2+d_2)\dot{z}_2 -id_2\dot{\phi}z_2-\Delta c_{22}\bar{z}_2e^{2i\phi}-\Delta c_{12}\bar{z}_1e^{2i\phi}$$

$$I\ddot{\phi}=M-S\dot{\phi}-R\dot{\phi}-\frac{i}{2}\{c_{11}\mathcal{E}_1(z_1e^{-i\phi}-\bar{z}_1e^{i\phi}) +c_{12}\mathcal{E}_1(z_2e^{-i\phi}-\bar{z}_2e^{i\phi})-c_{22}\mathcal{E}_2(z_2e^{-i(\phi+\beta)}-\bar{z}_2e^{i(\phi+\beta)}) +c_{12}\mathcal{E}_2(z_1e^{-i(\phi+\beta)}-\bar{z}_1e^{i(\phi+\beta)})-d_1(\dot{\bar{z}}_1z_1-\dot{z}_1\bar{z}_1) -d_2(\dot{\bar{z}}_2z_2-\dot{z}_2\bar{z}_2)\}-d_1\dot{\phi}z_1\bar{z}_1-d_2\dot{\phi}z_2\bar{z}_2+1/2\Delta c_{11}\{(z_1^2 +\bar{z}_1^2)\sin 2\phi+i(z_1^2-\bar{z}_1^2)\cos 2\phi\}+1/2\Delta c_{22}\{(z_2^2 +\bar{z}_2^2)\sin 2\phi+i(z_2^2-\bar{z}_2^2)\cos 2\phi\}+1/2\Delta c_{12}\{(z_1z_2 +\bar{z}_1\bar{z}_2)\sin 2\phi-(\bar{z}_1z_2+z_1\bar{z}_2)\cos 2\phi\} \quad \cdots\cdots(1)$$

We are interested in the vibration of the system in the neighbourhood of the resonance, therefore it is worthwhile to transform the first and second terms of eq. (1) into principal coordinates and thus represent the system as simple oscillators, interacting one with another and with the driving source. By substituting the variables $z_1=u_1+u_2$, $z_2=s_1u_1+s_2u_2$ into eq. (1), the transformed equations are obtained.

where $s_1=\dfrac{\omega_1{}^2m_1-c_{11}}{c_{12}}$,

$$s^2=\frac{\omega_2{}^2m_1-c_{11}}{c_{12}}$$

and ω_1, ω_2 are the natural frequencies of the system.

The transformed equations are expressed as follows:

$$\ddot{u}_1+\omega_1{}^2u_1=\frac{1}{\mu_1}(D_1+s_1D_2)$$

$$\ddot{u}_2+\omega_2{}^2u_2=\frac{1}{\mu_2}(D_1+s_2D_2) \quad \cdots\cdots(2)$$

where

$$\mu_1=m_1+s_1{}^2m_2$$

$$\mu_2=m_1+s_2{}^2m_2$$

$$D_1=m_1\mathcal{E}_1\dot{\phi}^2e^{i\phi}-(d_1+k_1)(\dot{u}_1+\dot{u}_2)-id_1\dot{\phi}(u_1+u_2) -\Delta c_{11}(u_1+u_2)e^{2i\phi}-\Delta c_{12}(s_1u_1+s_2u_2)e^{2i\phi}$$

$$D_2=m_2\mathcal{E}_2\dot{\phi}^2e^{i(\phi+\beta)}-(d_2+k_2)(s_1\dot{u}_1+s_2\dot{u}_2) -id_2\dot{\phi}(s_1u_1+s_2u_2)-\Delta c_{22}(s_1\bar{u}_1+s_2\bar{u}_2)e^{2i\phi} -\Delta c_{12}(\bar{u}_1+\bar{u}_2)e^{2i\phi}$$

Approximate equations

In this paper, we will apply the method of Bogolyubov's perturbation theory to obtain an approximate solution of eq. (2).

It is convenient to introduce the substitution

$$u_1=A_1\cos\phi+B_1\sin\phi$$

$$u_2=A_2\cos\phi+B_2\sin\phi$$

$$\dot{u}_1=-A_1\omega_1\sin\phi+B_1\omega_1\cos\phi$$

$$\dot{u}_2=-A_2\omega_2\sin\phi+B_2\omega_2\cos\phi \quad \cdots\cdots(3)$$

Where u_1, u_2 is the from for the solution of eq. (2) for the x, y coordinates, the complex quantities A_1, B_1, A_2 and B_2 are determined as unknown functions of time so that eq. (3) satifies eq. (2).

Eq. (2) is transformed to

$$\frac{dA_1}{dt}=-(\dot{\phi}-\omega_1)B_1-\frac{\phi_1}{\mu_1\omega_1} (A_1,B_1,A_2,B_2,A_1,B_1,A_2,B_2,\phi,\dot{\phi})\sin\phi$$

$$\frac{dB_1}{dt}=(\dot{\phi}-\omega_1)A_1+\frac{\phi_1}{\mu_1\omega_1} (A_1,B_1,A_2,B_2,\bar{A}_1,\bar{B}_1,\bar{A}_2,\bar{B}_2,\phi,\dot{\phi})\cos\phi$$

$$\frac{dA_2}{dt}=-(\dot{\phi}-\omega_2)B_2-\frac{\phi_2}{\mu_2\omega_2} (A_1,B_1,A_2,B_2,\bar{A}_1,\bar{B}_1,\bar{A}_2,\bar{B}_2,\phi,\dot{\phi})\sin\phi$$

$$\frac{dB_2}{dt}=(\dot{\phi}-\omega_2)A_2+\frac{\phi_2}{\mu_2\omega_2} (A_1,B_1,A_2,B_2,\bar{A}_1,\bar{B}_1,\bar{A}_2,\bar{B}_2,\phi,\dot{\phi})\cos\phi \quad \cdots\cdots(4)$$

where

$$\phi_1=D_1+s_1D_2$$

$$\phi_2=D_1+s_2D_2$$

It is worthwhile to consider it in the form

$$A_1=a_1+\mathcal{E}E_{a1}(t,\ a_1,\ b_1,\ a_2,\ b_2,\ \Omega)$$

$$B_1=b_1+\mathcal{E}E_{b1}(t,\ a_1,\ b_1,\ a_2,\ b_2,\ \Omega)$$

$$A_2=a_2+\mathcal{E}E_{a2}(t,\ a_1,\ b_1,\ a_2,\ b_2,\ \Omega)$$

$$B_2=b_2+\mathcal{E}E_{b2}(t,\ a_1,\ b_1,\ a_2,\ b_2,\ \Omega)$$

$$\dot{\phi}=\Omega+\mathcal{E}E_\Omega(t,\ a_1,\ b_1,\ a_2,\ b_2,\ \Omega) \quad \cdots\cdots(5)$$

a_1, a_2, b_1, b_2 and Ω will be slowly varying function of time, while $\mathcal{E}E_{a1}$, $\mathcal{E}E_{b1}$, $\mathcal{E}E_{a2}$, $\mathcal{E}E_{b2}$ and $\mathcal{E}E_\Omega$ will be small periodic function.

c_1, a_2, b_1, b_2 and Ω, are determined from the equations of the first approximation.

In the region of the first resonance, resonant vibration close to the frequency ω_1 are characterized by a fundamental variation of the coordinate

z_1 in the this region.

The averaged equations are expressed as follows:

$$\frac{da_1}{dt}=-(\Omega-\omega_1)b_1-\frac{1}{2\mu_1\omega_1}\Big\{(ip_{11}-p_{12})\Omega^2+q_{11}\omega_1a_1-iq_{12}\,\Omega b_1-\frac{i}{2}(k_1+2k_2)\bar{a}_1+\frac{1}{2}(k_1+2k_2)\bar{b}_1\Big\}$$

$$\frac{db_1}{dt}=(\Omega-\omega_1)a_1+\frac{1}{2\mu_1\omega_1}\Big\{(p_{11}+ip_{12})\Omega^2-q_{11}\omega_1b_1-iq_{12}\Omega a_1-\frac{i}{2}(k_1+2k_2)\bar{b}_1-\frac{1}{2}(k_1+2k_2)\bar{a}_1\Big\}$$

$$\frac{d\Omega}{dt}=\frac{1}{I}\Big[M-S\phi-R\phi+\frac{i}{2}\omega_1q_{12}(a_1\bar{b}_1-\bar{a}_1b_1)-\frac{1}{2}\Omega q_{12}(a_1\bar{a}_1+b_1\bar{b}_1)+\frac{i}{4}\{d_{11}(\bar{a}_1-a_1)+d_{12}(\bar{a}_1e^{i\beta}-a_1e^{-i\beta})\}-\frac{1}{4}\{d_{11}(\bar{b}_1+b_1)+d_{12}(\bar{b}_1e^{i\beta}+b_1e^{-i\beta})\}+\frac{1}{8}\Delta c_{11}\{2(a_1b_1+\bar{a}_1\bar{b}_1)+i(a_1^2-b_1^2)-i(\bar{a}_1^2-\bar{b}_1^2)\}+\frac{1}{8}\Delta c_{22}\,s_1^2\{2(a_1b_1+\bar{a}_1\bar{b}_1)+i(\bar{a}_1^2-\bar{b}_1^2)-i(a_1^2-b_1^2)\}+\frac{1}{4}\Delta c_{12}\,s_1\{a_1b_1+\bar{a}_1\bar{b}_1-\bar{a}_1a_1+b_1\bar{b}_1)\}\Big]\quad\cdots\cdots(6)$$

where

$p_{11}=m_1\mathcal{E}_1+s_1m_2\mathcal{E}_2\cos\beta$

$p_{12}=s_1m_2\mathcal{E}_2\sin\beta$

$q_{11}=(k_1+d_1)+s_1^2(k_2+d_2)$

$q_{12}=d_1+s_1^2d_2$

$d_{11}=\mathcal{E}_1(c_{11}+s_1c_{12})+\mathcal{E}_2(c_{12}+s_1c_{22})\cos\beta$

$d_{12}=\mathcal{E}_2(c_{12}+s_1c_{22})\sin\beta$

$r_1=\Delta c_{11}+s_1^2\,\Delta c_{22}$

$r_2=\Delta c_{12}\,s_1$

By substituting the variables $a_1=a_{11}+ia_{12}$, $b_1=b_{11}+ib_{12}$, $\bar{a}_1=a_{11}-ia_{12}$ and $\bar{b}_1=b_{11}-ib_{12}$ into eq. (6), then the equations are expressed as follows:

$$\frac{da_{11}}{dt}=-(\Omega-\omega_1)b_{11}-\frac{1}{2\mu_1\omega_1}\Big\{-p_{12}\Omega^2+q_{11}\omega_1a_{11}+q_{12}\Omega b_{12}+\frac{1}{2}(r_1+2r_2)(b_{11}-a_{12})\Big\}$$

$$\frac{da_{12}}{dt}=-(\Omega-\omega_1)\,b_{12}-\frac{1}{2\mu_1\omega_1}\Big\{p_{11}\Omega^2+q_{11}\omega_1a_{12}-q_{12}\Omega b_{11}-\frac{1}{2}(r_1+2r_2)(a_{11}+b_{12})\Big\}$$

$$\frac{db_{11}}{dt}=(\Omega-\omega_1)a_{11}+\frac{1}{2\mu_1\omega_1}\Big\{p_{11}\Omega^2-q_{11}\omega_1b_{11}-q_{12}\Omega a_{12}-\frac{1}{2}(r_1+2r_2)(a_{11}+b_{12})\Big\}$$

$$\frac{db_{12}}{dt}=(\Omega-\omega_1)a_{12}+\frac{1}{2\mu_1\omega_1}\Big\{p_{12}\Omega^2-q_{11}\omega_1b_{12}+q_{12}a_{11}\Omega-\frac{1}{2}(r_1+2r_2)(b_{11}-a_{12})\Big\}$$

$$\frac{d\Omega}{dt}=\frac{1}{I}\Big[M-S\Omega-R\Omega+(a_{11}b_{12}-b_{11}a_{12})\omega_1q_{12}-\frac{1}{2}q_{12}\Omega(a_{11}^2+a_{12}^2+b_{11}^2+b_{12}^2)+\frac{1}{2}d_{11}a_{12}-\frac{1}{2}d_{11}b_{11}-\frac{1}{2}d_{12}\{(b_{11}-a_{12})\cos\beta+(a_{11}+b_{12})\sin\beta\}+\frac{1}{2}\Delta c_{11}(a_{11}b_{11}-a_{12}b_{12}-a_{11}a_{12}+b_{11}b_{12})+\frac{1}{2}\Delta c_{22}s_1^2(a_{11}b_{11}-a_{12}b_{12}-a_{11}a_{12}+b_{11}b_{12})+\frac{1}{2}\Delta c_{12}s_1\{a_{11}b_{11}-a_{12}b_{12}-\frac{1}{2}(a_{11}^2+a_{12}^2-b_{11}^2-b_{12}^2)\}\Big]\quad\cdots\cdots(7)$$

Moreover, to simplify these equations, the variables $v_{11}=a_{11}-b_{12}$, $v_{12}=b_{11}+a_{12}$, $v_{13}=a_{11}+b_{12}$ and $v_{14}=b_{11}-a_{12}$ are introduced. eq. (11) become

$$\frac{dv_{11}}{dt}=-(\Omega-\omega_1)\,v_{12}-\frac{1}{2\mu_1\omega_1}(q_{11}\omega_1v_{11}+q_{12}\Omega v_{11})$$

$$\frac{dv_{12}}{dt}=(\Omega-\omega_1)\,v_{11}-\frac{1}{2\mu_1\omega_1}(q_{11}\omega_1v_{12}-q_{12}\Omega v_{12})$$

$$\frac{dv_{13}}{dt}=-(\Omega-\omega_1)\,v_{14}+\frac{1}{2\mu_1\omega_1}\{2p_{12}\Omega^2-q_{11}\omega_1v_{13}+q_{12}\Omega v_{13}-(r_1+2r_2)v_{14}\}$$

$$\frac{dv_{14}}{dt}=(\Omega-\omega_1)v_{13}-\frac{1}{2\mu_1\omega_1}\{-2p_{11}\Omega^2+q_{11}\omega_1v_{14}-q_{12}\Omega v_{14}+(r_1+2r_2)v_{13}\}$$

$$\frac{d\Omega}{dt}=\frac{1}{I}\Big[M-S\Omega-R\Omega+\frac{1}{4}q_{12}(\omega_1-\Omega)(v_{13}^2+v_{14}^2)-(\omega_1+\Omega)(v_{11}^2+v_{12}^2)\}-\frac{1}{2}d_{11}v_{14}-\frac{1}{2}d_{12}v_{13}+\frac{1}{2}r_1v_{13}v_{14}+\frac{1}{4}r_2v_{11}(2v_{12}-v_{13}+v_{14})\Big]\quad\cdots\cdots(8)$$

On the other hand, the approximate solution of eq. (2) in the region of the second resonance is a similar to that used previously

$$\frac{dv_{21}}{dt}=-(\Omega-\omega_2)v_{22}-\frac{1}{2\mu_2\omega_2}(q_{21}\omega_2v_{21}+q_{22}\Omega v_{21})$$

$$\frac{dv_{22}}{dt}=(\Omega-\omega_2)v_{21}-\frac{1}{2\mu_2\omega_2}(q_{21}\omega_1v_{22}-q_{22}\Omega v_{22})$$

$$\frac{dv_{23}}{dt}=-(\Omega-\omega_2)v_{24}+\frac{1}{2\mu_2\omega_2}\{2p_{22}\Omega^2-q_{21}\omega_2v_{23}+q_{22}\Omega v_{23}-(r_1'+2r_2')v_{24}\}$$

$$\frac{dv_{24}}{dt}=(\Omega-\omega_2)v_{23}+\frac{1}{2\mu_2\omega_2}\{-2p_{21}\Omega^2+q_{21}\omega_2v_{24}-q_{22}\Omega v_{24}+(r_1'+2r_2')v_{23}\}$$

$$\frac{d\Omega}{dt}=\frac{1}{I}\Big[M-S\Omega-R\Omega+\frac{1}{4}q_{22}\{(\omega_2-\Omega)(v_{23}^2+v_{24}^2)-(\omega_2+\Omega)(v_{21}^2+v_{22}^2)\}-\frac{1}{2}d_{21}v_{24}-\frac{1}{2}d_{22}v_{23}+\frac{1}{2}r_1'v_{23}v_{24}+\frac{1}{4}r_2'v_{21}(2v_{22}-v_{23}+v_{24})\Big]\quad\cdots\cdots(9)$$

Since the approximate equations (8) and (9) cannot be solved analytically, they can be solved numerically. Considering the stationary vibration for eq. (8) and (9), the following stationary solutions are obtained.

$$a_1^2=\frac{1}{4}(v_{13}^2+v_{14}^2)$$

$$a_2^2=\frac{1}{4}(v_{23}^2+v_{24}^2)$$

$$\psi_1=\tan^{-1}\frac{v_{13}}{v_{14}}$$

$$\psi_2=\tan^{-1}\frac{v_{23}}{v_{24}} \quad \cdots\cdots(10)$$

where

$$v_{13}=\frac{p_{12}K_3-p_{11}K_4}{K_1^2+K_2} \qquad v_{23}=\frac{p_{22}K_8-p_{21}K_9}{K_6^2+K_7}$$

$$v_{14}=\frac{p_{11}K_3+p_{12}K_5}{K_1^2+K_2} \qquad v_{24}=\frac{p_{21}K_8+p_{22}K_{10}}{K_6^2+K_7}$$

$$K_1=2\mu_1\omega_1(\Omega-\omega_1)$$

$$K_2=(r_1+2r_2)^2+(q_{11}\omega_1-q_{12}\Omega)^2$$

$$K_3=2(q_{11}\omega_1-q_{12}\Omega)\Omega^2$$

$$K_4=2\Omega^2\{2\mu_1\omega_1(\Omega-\omega_1)+(r_1+2r_2)\}$$

$$K_5=2\Omega^2\{2\mu_1\omega_1(\Omega-\omega_1)-(r_1+2r_2)\}$$

$$K_6=2\mu_2\omega_2(\Omega-\omega_2)$$

$$K_7=(r_1'+2r_2')^2+(q_{21}\omega_2-q_{22}\Omega)^2$$

$$K_8=2(q_{21}\omega_2-q_{22}\Omega)\Omega^2$$

$$K_9=2\Omega^2\{2\mu_2\omega_2(\Omega-\omega_2)+(r_1'+2r_2')\}$$

$$K_{10}=2\Omega^2\{2\mu_2\omega_2(\Omega-\omega_2)-(r_1'+2r_2')\}$$

Nonstationary vibration during passing through critical speeds

The approximate eqs. (8)and(9) was calculated with digital computer (FANAFACOM U-300) by the Runge-Kutta-Gill's method for the initial conditions of eq. (10).

The steady state at 0.9 times the critical speed are chosen as the initial conditions.

The driving torque used for the calculation has a limited power.

Fig. 1 shows the comparison of the amplitude between the stationary and the nonstationary motion in the neighbourhood of the resonance.

The critical speed is higher and maximum amplitude is smaller than that of the stationary state.

The resultant angular accelaration through the critical speed is smaller than that of the ideal driving source.

The amplitude characteristics after passing through the critical speed made periodic cycle to the stationary amplitude.

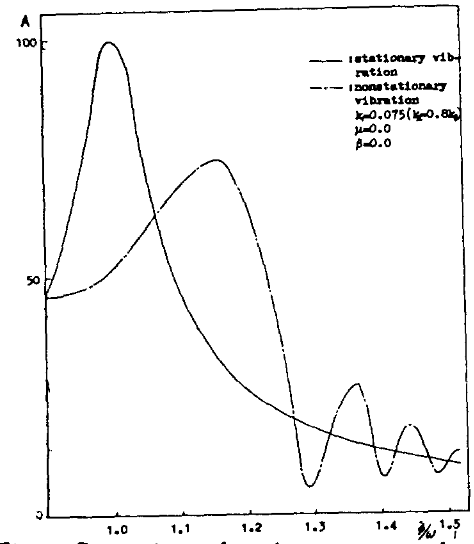

Fig. 1. Comparison of stationary state and nonstationary state.

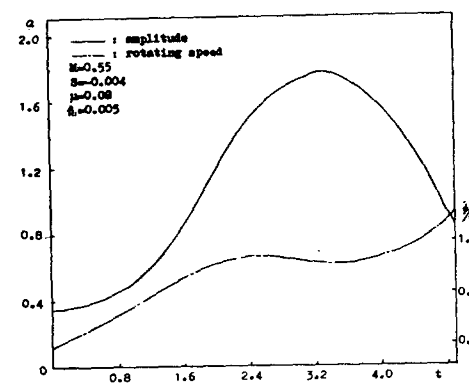

Fig. 2. Interaction for very limited power.

Fig. 2 shows the characteristics of the amplitude and the rotating speed of the rotor for very limited power.

Due to the interaction between the driving torque and the vibration system, the speed characteristics varies.

The acceleration becomes smaller in the nonstationary vibration to the maximum amplitude and becomes larger in the period of decreasing amplitude.

These relations are more remarkable in smaller torque M, larger slope S, and resisting torque R. Fig. 3 shows the influences of the torque M at the values of $S=-0.0168$, $k_1=0.005$, $k_2=0.8k_1$.

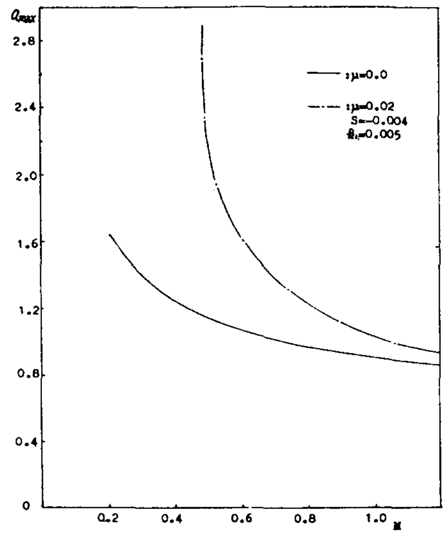

Fig. 3. Influence of driving torque.

For $M=2.14$, because of resisting torque in the neighbourhood of the resonance is greater than the driving torque, so rotor could not passing through the critical speed.

When the driving torque has very limited power, much time needs to pass through the critical speed and the expands the unstable state.

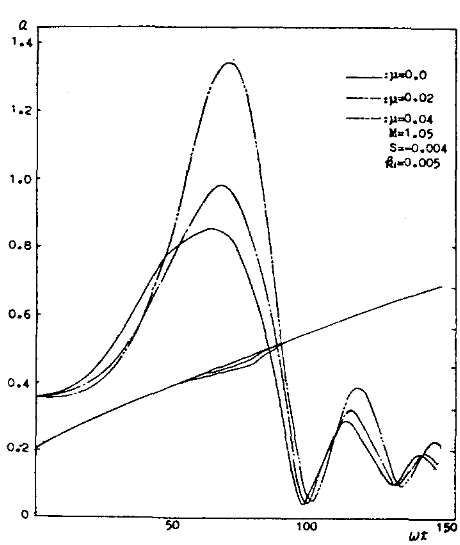

Fig. 4. Relation of driving torque and maximum amplitude.

Fig. 4 shows the characteristics of the maximum amplitude for the variable driving torque.

When the torque increases gradually, the maximum amplitude decreases and then become nearly constant.

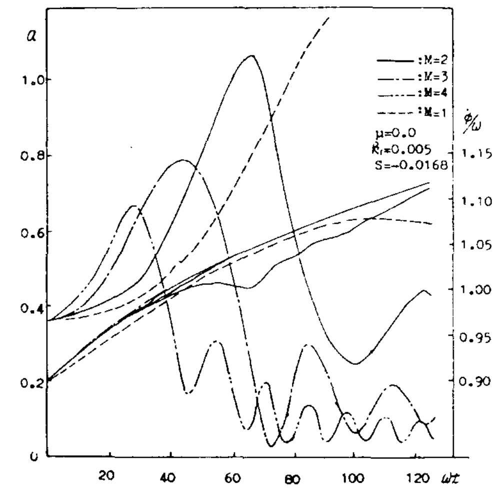

Fig. 5. Influnce of asymmetry of the shaft.

The influence of asymmetry of the shaft is shown in Fig. 5.

Under the condition of $M=1.05$, $k_1=0.005$, $k_2=0.8k_1$, when asymmetry of the shaft increases, the maximum amplitude in the neighbourhood of the resonance increases rapidly, and rotating speed decreases.

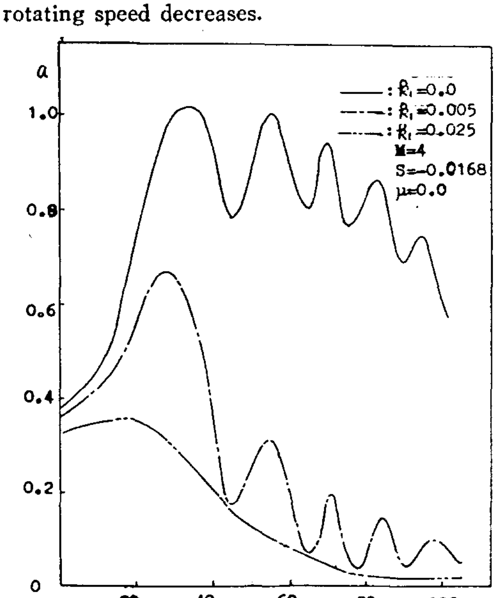

Fig. 6. Influence of damping.

Fig. 6 shows the influences of the external damping at $M=4$.

Taking a smaller values of k_1, the maximum amplitude increases rapidly, and nonstationary state will be more longer.

For $k_1=0$, the amplitude after passing through the critical speed decreases very slowly.

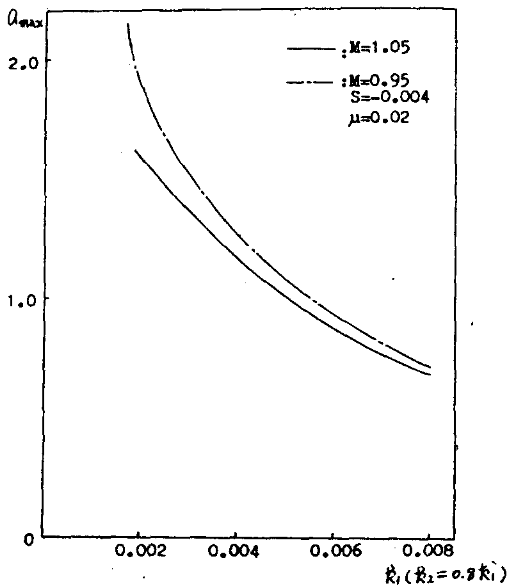

Fig. 7. Relation of damping and maximum amplitude.

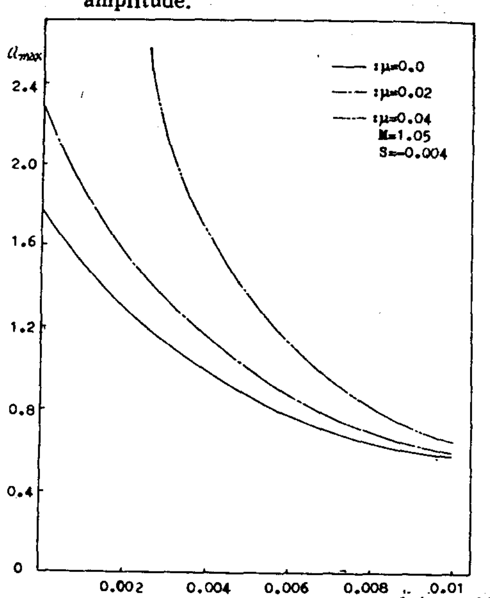

Fig. 8. Relation of damping and maximum amplitude.

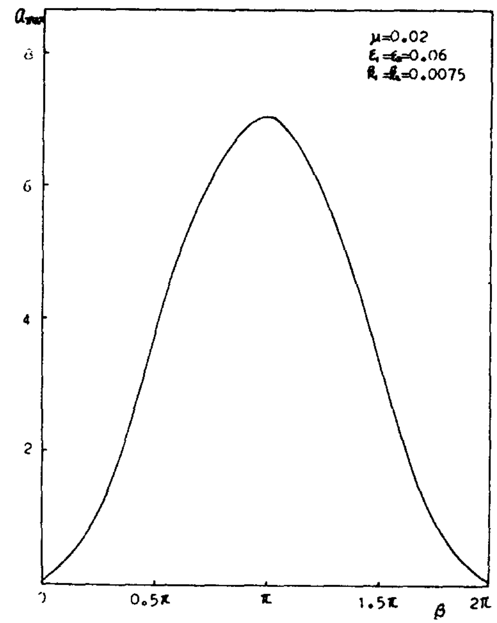

Fig. 9. Influence of angle (β) between ε_1 adn ε_2.

Fig. 7 shows the characteristics of the maximum amplitude at $M=0.95$, 1.05 when the damping varies.

Fig. 8 shows the damping influences when asymmetry of the shaft varies.

These are coincides with those of Iwatsubo's results, i. e., when damping is small, asymmetry of the shaft and the interaction between driving source and the vibration system are much influenced respectively.

In comparisons of Fig. 7 and Fig. 8, Torque and damping influences similarly in with maximum amplitude. Fig. 9 shows the influence of β. The maximum amplitude in passing through the critical speeds is the largest at $\beta=\pi$.
It is caused by the relative motion of eccentricity and coincides with the characteristics of the stationary state.

Conclusions

In this paper, the analyses in nonstationary vibration through a critical speeds are made as follows;

1) The characteristics in passing through the critical speeds of the rotating shaft system

with two discs are similar to those of the rotating shaft system with a disc.

The characteristics in the neighbourhood of the first and the second resonance are coincided with quantitive.

2) The interactions between the driving torque and vibration system are more remarkable in smaller torque, larger slope and resisting torque.

3) In the system with a very limited power, the maximum amplitude in passing through a critical speeds decreases with a decrease of asymmetry of the shaft, and decreases an increases of damping, torque, and is the largest at $\beta=\pi$.

Appendix : Equations of motion

For the purpose of simplicity, the following assumption is set in the analysis, i. e., the rotor is rigid and has a weightless shaft.

The external damping forces are assumed to be proportional to the velocity of the center of the discs, while the internal damping forces are assumed to be propotional to the velocity of the bending deformation of the shaft. It can be neglected that the torsional vibration of the shaft and the gyroscopic moments.

The kinetic energy of the rotor is

$$T=\frac{1}{2}\dot{\phi}^2(m_1r_1^2+m_2r_2^2)+\frac{1}{2}m_1\{(\dot{x}_1-\varepsilon_1\dot{\phi}\sin\phi)^2+(\dot{y}_1+\varepsilon_1\dot{\phi}\cos\phi)^2\}+\frac{1}{2}m_2[\{\dot{x}_2-\varepsilon_2\dot{\phi}\sin(\phi+\beta)\}^2+\{\dot{y}_2+\varepsilon_2\dot{\phi}\cos(\phi+\beta)\}^2]\cdots\cdots(\text{A-1})$$

The potential energy due to flexure in the shaft is

$$V=\frac{1}{2}c_{11}(x_1^2+y_1^2)+c_{12}(x_1x_2+y_1y_2)+\frac{1}{2}c_{22}(x_2^2+y_2^2)+1/2\Delta c_{11}\{(x_1^2-y_1^2)\cos 2\phi+2x_1y_1\sin 2\phi\}+\Delta c_{12}\{(x_1x_2-y_1y_2)\cos 2\phi+(x_1x_2+y_1y_2)\sin 2\phi\}+1/2\Delta c_{22}\{(x_2^2-y_2^2)\cos 2\phi+2x_2y_2\sin 2\phi\}\cdots(\text{A-2})$$

The dissipation function due to viscous forces is

$$W=\frac{1}{2}k_1(\dot{x}_1^2+\dot{y}_1^2)+\frac{1}{2}k_2(\dot{x}_2^2+\dot{y}_2^2)\frac{1}{2}d_1(\dot{x}_1^2+\dot{y}_1^2)+\frac{1}{2}d_2(\dot{x}_2^2+\dot{y}_2^2)-2d_1\dot{\phi}(x_1\dot{y}_1-\dot{x}_1y_1)-2d_2\dot{\phi}(x_2\dot{y}_2-\dot{x}_2y_2)\cdots\cdots(\text{A-3})$$

To express the equations of motion of this rotor system, we use Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_n}\right)-\frac{\partial T}{\partial q_n}+\frac{\partial W}{\partial \dot{q}_n}+\frac{\partial V}{\partial q_n}=0 \quad (n=1,\ 2,\cdots,5)\cdots(\text{A-4})$$

where $q_1=x_1,\ q_2=y_1,\ q_3=x_2,\ q_4=y_2,\ q_5=\phi$

The equations of motion are

$$m_1\ddot{x}_1+c_{11}x_1+c_{12}x_2=m_1\varepsilon_1\dot{\phi}^2\cos\phi-(k_1+d_1)\dot{x}_1-d_1\dot{\phi}y_1-\Delta c_{11}(x_1\cos 2\phi+y_1\sin 2\phi)-\Delta c_{12}(x_2\cos 2\phi+y_2\sin 2\phi)$$

$$m_1\ddot{y}_1+c_{11}y_1+c_{12}y_2=m_1\varepsilon_1\dot{\phi}^2\sin\phi-(k_1+d_1)\dot{y}_1+d_1\dot{\phi}x_1-\Delta c_{11}(x_1\sin 2\phi-y_1\cos 2\phi)-\Delta c_{12}(x_2\sin 2\phi-y_2\cos 2\phi)$$

$$m_2\ddot{x}_2+c_{22}x_2+c_{12}x_1=m_2\varepsilon_2\dot{\phi}^2\cos(\phi+\beta)-(k_2+d_2)\dot{x}_2-d_2\dot{\phi}y_2-\Delta c_{22}(x_2\cos 2\phi+y_2\sin 2\phi)-\Delta c_{12}(x_1\cos 2\phi+y_1\sin 2\phi)$$

$$m_2\ddot{y}_2+c_{22}y_2+c_{12}y_1=m_2\varepsilon_2\dot{\phi}^2\sin(\phi+\beta)-(k_2+d_2)\dot{y}_2+d_2\dot{\phi}x_2-\Delta c_{22}(x_2\sin 2\phi-y_2\cos 2\phi)-\Delta c_{12}(x_1\sin 2\phi-y_1\cos 2\phi)$$

$$I\ddot{\phi}=M-S\dot{\phi}-R\dot{\phi}-c_{11}\varepsilon_1(x_1\sin\phi-y_1\cos\phi)-c_{12}\varepsilon_1(x_2\sin\phi-y_2\cos\phi)-c_{22}\varepsilon_2\{x_2\sin(\phi+\beta)-y_2\cos(\phi+\beta)\}-c_{12}\varepsilon_2\{x_1\sin(\phi+\beta)-y_1\cos(\phi+\beta)\}-d_1(\dot{x}_1y_1-\dot{y}_1x_1)-d_2(\dot{x}_2y_2-\dot{y}_2x_2)-d_1\dot{\phi}(x_1^2+y_1^2)-d_2\dot{\phi}(x_2^2+y_2^2)+\Delta c_{11}\{(x_1^2-y_1^2)\sin 2\phi-2x_1y_1\cos 2\phi\}+\Delta c_{12}\}(x_1x_2-y_1y_2)\sin 2\phi-(x_1x_2+y_1y_2)\cos 2\phi\}+\Delta c_{22}\{(x_2^2-y_2^2)\sin 2\phi-2x_2y_2\cos 2\phi\}\cdots\cdots(\text{A-5})$$

These equation can conveniently be represented by the complex variables $z_1=x_1+iy_1,\ z_2=x_2+iy_2$.

$$m_1\ddot{z}_1+c_{11}z_1+c_{12}z_2=m_1\varepsilon_1\dot{\phi}^2e^{i\phi}-(k_1+d_1)\dot{z}_1-id_1\dot{\phi}z_1-\Delta c_{11}\bar{z}_1e^{2i\phi}-\Delta c_{12}\bar{z}_2e^{2i\phi}$$

$$m_2\ddot{z}_2+c_{22}z_2+c_{12}z_1=m_2\varepsilon_2\dot{\phi}^2e^{i(\phi+\beta)}-(k_2+d_2)\dot{z}_2-id_2\dot{\phi}z_2-\Delta c_{22}\bar{z}_2e^{2i\phi}-\Delta c_{12}\bar{z}_1e^{2i\phi}$$

$$I\ddot{\phi}=M-S\dot{\phi}-R\dot{\phi}-\frac{i}{2}\{c_{11}\varepsilon_1(z_1e^{-i\phi}-\bar{z}_1e^{i\phi})+c_{12}\varepsilon_1(z_2e^{-i\phi}-\bar{z}_2e^{i\phi})+c_{22}\varepsilon_2(z_2e^{-i(\phi+\beta)}-\bar{z}_2e^{i(\phi+\beta)})+c_{12}\varepsilon_2(z_1e^{-i(\phi+\beta)}-\bar{z}_1e^{i(\phi+\beta)})-d_1(\dot{\bar{z}}_1z_1-\dot{z}_1\bar{z}_1)-d_2(\dot{\bar{z}}_2z_2-\dot{z}_2\bar{z}_2)\}-d_1\dot{\phi}z_1\bar{z}_1-d_2\dot{\phi}z_2\bar{z}_2+\frac{1}{2}\Delta c_{11}\{(z_1^2+\bar{z}_1^2)\sin 2\phi+i(z_1^2-\bar{z}_1^2)\cos 2\phi\}+\frac{1}{2}\Delta c_{22}\{z_2^2+\bar{z}_2^2)\sin 2\phi+i(z_2^2-\bar{z}_2^2)\cos 2\phi\}+\frac{1}{2}\Delta c_{12}\{(z_1z_2+\bar{z}_1\bar{z}_2)\sin 2\phi-(z_1z_2+z_1z_2)\cos 2\phi\}\cdots\cdots(\text{A-6})$$

References

Dimentberg, F. M. (1961): Flexural vibration of rotating shaft. Butteworths.

Hübner, W. (1965): Ing. Arch., 34, 411.

Ingermer Fernlund (1963): Running through the critical speed of a rotor. Report No. 22 from the Institute of Machine Elements, Chalmers University of Technology, Gothenburg, Sweden, 1—86.

Kawai, R. and Iwatsubo, T. (1974): A study on during its passing through critical speed of asymmetry shaft with limited power. Trans. Japan Soc. Mech. Engrs., Vol. 40, No. 335, 1908—1916.

Kononenko, V. O. (1964): Kolebatel'nye sistemys ogranichennym voz, Uzhdeniem, Nauka Moskva, 206—222.

Lewis, F. M. (1943): Vibration during acceleration through the critical speed. Trans. ASME., APM 54, 24, 253—261.

Nonami, K. and Miyashita, M (1978): Problems of rotor passing through critical speed with gyroscopic effect. Bull. JSME, Vol. 21, No. 151, 56—63.

B. S. Yang, (1980): A study on nonstationary vibration of rotor with asymmetry shaft-passing through critical speed. Master's thesis, NFUB, 1—18.